Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Core Mathematics C34

Advanced

Monday 27 January 2014 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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PEARSON

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1. $$f(x) = \frac{2x}{x^2 + 3}, \qquad x \in \mathbb{R}$$

Find the set of values of x for which $f'(x) > 0$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

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Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. Solve, for $0 \leqslant x \leqslant 270°$, the equation

$$\frac{\tan 2x + \tan 50°}{1 - \tan 2x \tan 50°} = 2$$

Give your answers in degrees to 2 decimal places.

(6)

Leave blank

Question 2 continued

Q2

(Total 6 marks)

Leave blank

3. Given that

$$4x^3 + 2x^2 + 17x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$$

(a) find the values of the constants A, B, C and D.

(4)

(b) Hence find

$$\int_1^4 \frac{4x^3 + 2x^2 + 17x + 8}{x^2 + 4} \, \mathrm{d}x$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(6)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

(Total 10 marks)

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4.

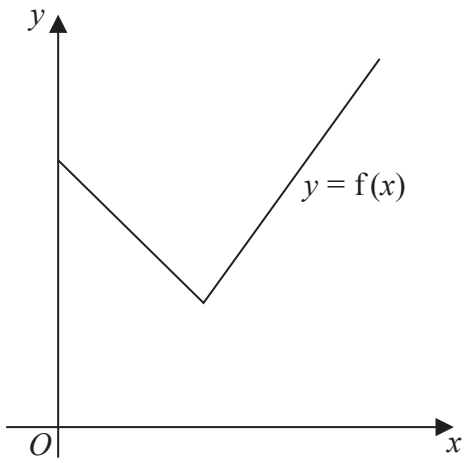

Figure 1

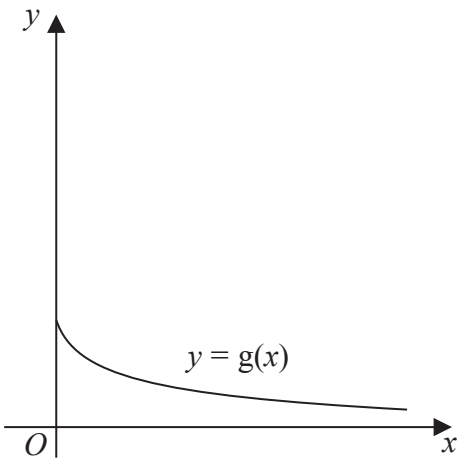

Figure 2

Figure 1 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

Figure 2 shows a sketch of part of the graph $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{x + 9}{2x + 3}, \quad x \geqslant 0$$

(a) Find the value of fg(1)

(2)

(b) State the range of g

(2)

(c) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(4)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly two roots,

(d) state the range of possible values of k.

(3)

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Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 11 marks)

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5. (a) Prove, by using logarithms, that

$$\frac{\mathrm{d}}{\mathrm{d}x}(2^x) = 2^x \ln 2$$

(3)

The curve C has the equation

$$2x + 3y^2 + 3x^2y + 12 = 4 \times 2^x$$

The point P, with coordinates (2, 0), lies on C.

(b) Find an equation of the tangent to C at P.

(6)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 9 marks)

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6. Given that the binomial expansion, in ascending powers of x, of

$$\frac{6}{\sqrt{(9 + Ax^2)}}, \qquad |x| < \frac{3}{\sqrt{|A|}}$$

is $$B - \frac{2}{3}x^2 + Cx^4 + \ldots$$

(a) find the values of the constants A, B and C.

(7)

(b) Hence find the coefficient of x^6

(2)

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Question 6 continued

Q6

(Total 9 marks)

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7.

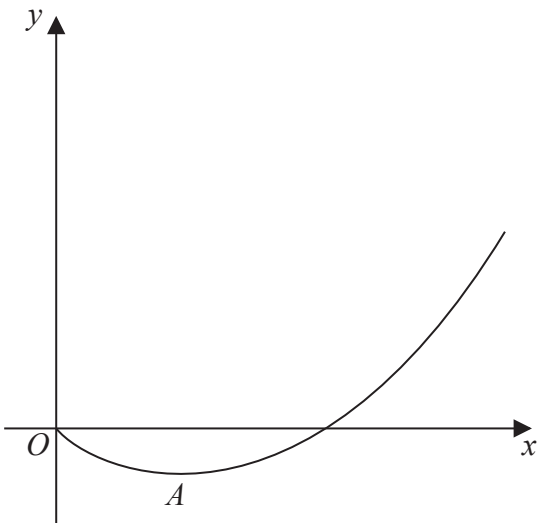

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2x(1 + x)\ln x, \quad x > 0$$

The curve has a minimum turning point at A.

(a) Find $\mathrm{f}'(x)$

(3)

(b) Hence show that the x coordinate of A is the solution of the equation

$$x = \mathrm{e}^{-\frac{1+x}{1+2x}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \mathrm{e}^{-\frac{1+x_n}{1+2x_n}}, \qquad x_0 = 0.46$$

to find the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(d) Use your answer to part (c) to estimate the coordinates of A to 2 decimal places.

(2)

Leave blank

Question 7 continued

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Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 11 marks)

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8. (a) Prove that

$$2\text{cosec } 2A - \cot A \equiv \tan A, \qquad A \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $0 \leqslant \theta \leqslant \frac{\pi}{2}$

(i) $2\text{cosec } 4\theta - \cot 2\theta = \sqrt{3}$

(ii) $\tan \theta + \cot \theta = 5$

Give your answers to 3 significant figures.

(6)

Leave blank

Question 8 continued

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Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 10 marks)

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9. (a) Use the substitution $u = 4 - \sqrt{x}$ to find

$$\int \frac{\mathrm{d}x}{4 - \sqrt{x}}$$

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{4 - \sqrt{h}}{20}$$

where h is the height in metres and t is the time measured in years after the tree is planted.

(b) Find the range in values of h for which the height of a tree in this species is increasing.
(2)

(c) Given that one of these trees is 1 metre high when it is planted, calculate the time it would take to reach a height of 10 metres. Write your answer to 3 significant figures.
(7)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 15 marks)

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10. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{j} + 12\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection.

(6)

(b) Show that l_1 and l_2 are perpendicular to each other.

(2)

The point A, with position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$, lies on l_1

The point B is the image of A after reflection in the line l_2

(c) Find the position vector of B.

(3)

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Question 10 continued

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Question 10 continued

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Question 10 continued

Q10

(Total 11 marks)

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11. The curve C has parametric equations

$$x = 10\cos 2t, \quad y = 6\sin t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

The point A with coordinates (5, 3) lies on C.

(a) Find the value of t at the point A.

(1)

(b) Show that an equation of the normal to C at A is

$$3y = 10x - 41$$

(6)

The normal to C at A cuts C again at the point B.

(c) Find the exact coordinates of B.

(8)

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Question 11 continued

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Question 11 continued

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Question 11 continued

Q11

(Total 15 marks)

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12.

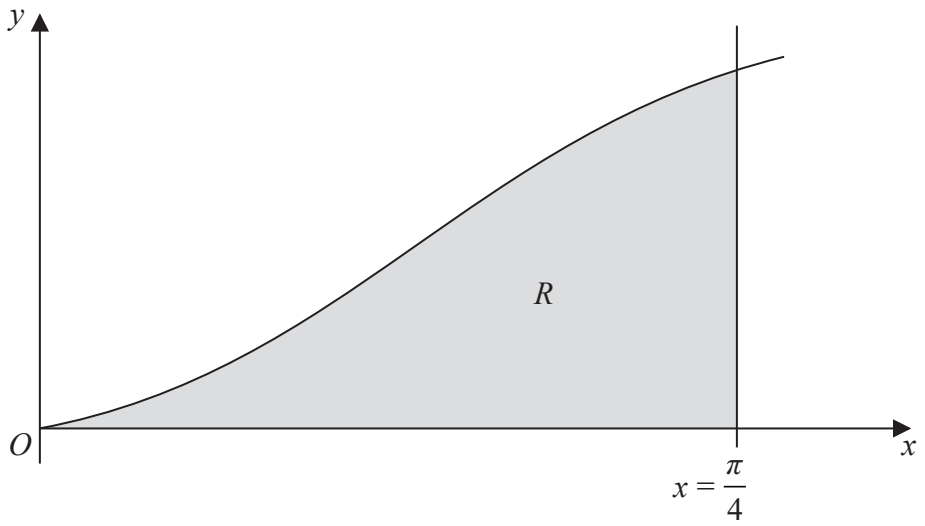

Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = x(\sin x + \cos x), \qquad 0 \leqslant x \leqslant \frac{\pi}{4}$$

The finite region R, shown shaded in Figure 4, is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution, with volume V.

(a) Assuming the formula for volume of revolution show that $V = \int_0^{\frac{\pi}{4}} \pi x^2 (1 + \sin 2x)\,\mathrm{d}x$

(3)

(b) Hence using calculus find the exact value of V.

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(9)

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Question 12 continued

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Question 12 continued

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Question 12 continued

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Question 12 continued

Q12

(Total 12 marks)

TOTAL FOR PAPER: 125 MARKS

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