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Question 1 continued

Lined area for writing the answer to Question 1.

Q1

(Total 6 marks)



3.

Figure 1

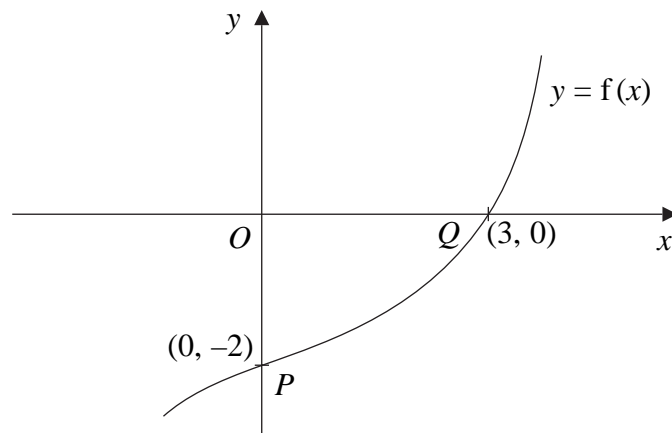


Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$, where f is an increasing function of x . The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, (3)

(b) $y = f^{-1}(x)$, (3)

(c) $y = \frac{1}{2}f(3x)$. (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.



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Question 3 continued



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Question 3 continued

Q3

(Total 9 marks)



4. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, $T^{\circ}\text{C}$, t minutes after it enters the liquid, is given by

$$T = 400 e^{-0.05t} + 25, \quad t \geq 0.$$

(a) Find the temperature of the ball as it enters the liquid. (1)

(b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures. (4)

(c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in $^{\circ}\text{C}$ per minute to 3 significant figures. (3)

(d) From the equation for temperature T in terms of t , given above, explain why the temperature of the ball can never fall to 20°C . (1)



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Question 4 continued

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Q4

(Total 9 marks)



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Question 5 continued

Lined area for writing the answer to Question 5.

Q5

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(Total 11 marks)



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6. (a) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$.

(2)

(b) Hence, or otherwise, prove that

$$\operatorname{cosec}^4\theta - \cot^4\theta \equiv \operatorname{cosec}^2\theta + \cot^2\theta.$$

(2)

(c) Solve, for $90^\circ < \theta < 180^\circ$,

$$\operatorname{cosec}^4\theta - \cot^4\theta = 2 - \cot \theta.$$

(6)



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Question 6 continued

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7. For the constant k , where $k > 1$, the functions f and g are defined by

$$f: x \mapsto \ln(x + k), \quad x > -k,$$

$$g: x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

- (a) On separate axes, sketch the graph of f and the graph of g .

On each sketch state, in terms of k , the coordinates of points where the graph meets the coordinate axes.

(5)

- (b) Write down the range of f .

(1)

- (c) Find $fg\left(\frac{k}{4}\right)$ in terms of k , giving your answer in its simplest form.

(2)

The curve C has equation $y = f(x)$. The tangent to C at the point with x -coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

- (d) Find the value of k .

(4)



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Question 7 continued

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(Total 12 marks)

Q7

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8. (a) Given that $\cos A = \frac{3}{4}$, where $270^\circ < A < 360^\circ$, find the exact value of $\sin 2A$. **(5)**

(b) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$. **(3)**

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

(ii) show that $\frac{dy}{dx} = \sin 2x$. **(4)**



