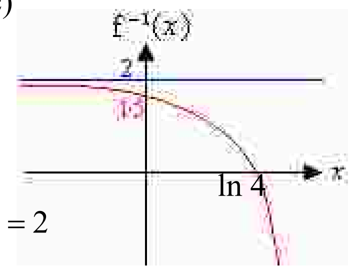
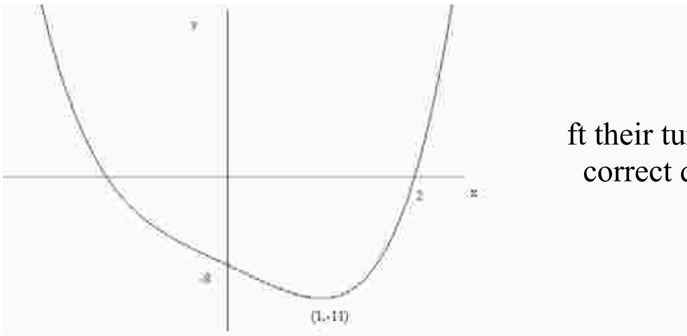
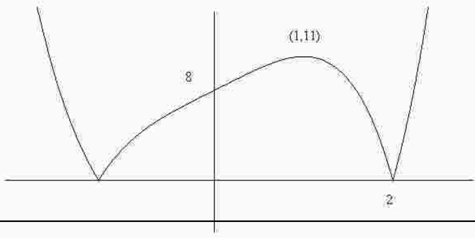
January 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme		Marks
1.	(a) $\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$		B1
	$= 2\sin\theta\cos^2\theta + (1 - 2\sin^2\theta)\sin\theta$		B1 B1
	$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$		M1
	$= 3\sin\theta - 4\sin^3\theta$ *	cso	A1 **(5)**
	(b) $\sin 3\theta = 3 \times \frac{\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ or exact equivalent		M1 A1 **(2)**
			[7]
2.	(a) $f(x) = \frac{(x+2)^2, -3(x+2) + 3}{(x+2)^2}$		M1 A1, A1
	$= \frac{x^2 + 4x + 4 - 3x - 6 + 3}{(x+2)^2} = \frac{x^2 + x + 1}{(x+2)^2}$ *	cso	A1 **(4)**
	(b) $x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}, \; > 0$ for all values of x.		M1 A1, A1 **(3)**
	(c) $f(x) = \frac{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}{(x+2)^2}$ Numerator is positive from (b) $x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive) Hence $f(x) > 0$		B1 **(1)**
			[8]
	Alternative to (b) $\frac{d}{dx}(x^2 + x + 1) = 2x + 1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2 + x + 1 = \frac{3}{4}$		M1 A1
	A parabola with positive coefficient of x^2 has a minimum $\Rightarrow x^2 + x + 1 > 0$ Accept equivalent arguments		A1 **(3)**

Question Number	Scheme	Marks
3.	(a) $y=\frac{\pi}{4} \Rightarrow x=2\sin\frac{\pi}{4}=2\times\frac{1}{\sqrt{2}}=\sqrt{2} \Rightarrow P\in C$	B1 **(1)**
	Accept equivalent (reversed) arguments. In any method it must be clear that $\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ or exact equivalent is used.	
	(b) $\frac{dx}{dy}=2\cos y$ *or* $1=2\cos y\frac{dy}{dx}$	M1 A1
	$\frac{dy}{dx}=\frac{1}{2\cos y}$ May be awarded after substitution	M1
	$y=\frac{\pi}{4} \Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{2}}$ * cso	A1 **(4)**
	(c) $m'=-\sqrt{2}$	B1
	$y-\frac{\pi}{4}=-\sqrt{2}\left(x-\sqrt{2}\right)$	M1 A1
	$y=-\sqrt{2}x+2+\frac{\pi}{4}$	A1 **(4)**
		[9]
4.	(i) $\frac{dy}{dx}=\frac{\left(9+x^2\right)-x(2x)}{\left(9+x^2\right)^2}$ $\left(=\frac{9-x^2}{\left(9+x^2\right)^2}\right)$	M1 A1
	$\frac{dy}{dx}=0 \Rightarrow 9-x^2=0 \Rightarrow x=\pm3$	M1 A1
	$\left(3,\frac{1}{6}\right),\ \left(-3,-\frac{1}{6}\right)$ Final two A marks depend on second M only	A1, A1 **(6)**
	(ii) $\frac{dy}{dx}=\frac{3}{2}\left(1+e^{2x}\right)^{\frac{1}{2}}\times2e^{2x}$	M1 A1 A1
	$x=\frac{1}{2}\ln3 \Rightarrow \frac{dy}{dx}=\frac{3}{2}\left(1+e^{\ln3}\right)^{\frac{1}{2}}\times2e^{\ln3}=3\times4^{\frac{1}{2}}\times3=18$	M1 A1 **(5)**
		[11]

Question Number	Scheme	Marks
5.	(a) $R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2$	M1 A1
	$\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$ accept awrt 1.05	M1 A1 **(4)**
	(b) $\sin(x + \text{ their } \alpha) = \frac{1}{2}$	M1
	$x + \text{ their } \alpha = \frac{\pi}{6}\ \left(\frac{5\pi}{6}, \frac{13\pi}{6}\right)$	A1
	$x = \frac{\pi}{2}, \frac{11\pi}{6}$ accept awrt 1.57, 5.76	M1 A1 **(4)**
		[8]
	The use of degrees loses only one mark in this question. Penalise the first time it occurs in an answer and then ignore.	

Question Number	Scheme	Marks
6.	(a) $y = \ln(4-2x)$	
	$e^y = 4-2x$ leading to $x = 2-\frac{1}{2}e^y$ Changing subject and removing ln	M1 A1
	$y = 2-\frac{1}{2}e^x \quad \Rightarrow \quad f^{-1} \mapsto 2-\frac{1}{2}e^x$ * cso	A1
	Domain of f^{-1} is $\mathbb{R}$	B1 **(4)**
	(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in \mathbb{R}$)	B1 **(1)**
	(c)  $y = 2$	
	Shape	B1
	1.5	B1
	ln 4	B1
		B1 **(4)**
	(d) $x_1 \approx -0.3704, \quad x_2 \approx -0.3452$ cao	B1, B1 **(2)**
	If more than 4 dp given in this part a maximum on one mark is lost. Penalise on the first occasion.	
	(e) $x_3 = -0.354\,030\,19 \ldots$	
	$x_4 = -0.350\,926\,88 \ldots$	
	$x_5 = -0.352\,017\,61 \ldots$	
	$x_6 = -0.351\,633\,86 \ldots$ Calculating to at least x_6 to at least four dp	M1
	$k \approx -0.352$ cao	A1 **(2)** **[13]**
	Alternative to (e)	
	$k \approx -0.352$ Found in any way	
	Let $g(x) = x + \frac{1}{2}e^x$	
	$g(-0.3515) \approx +0.0003, \quad g(-0.3525) \approx -0.001$	M1
	Change of sign (and continuity) $\Rightarrow k \in (-0.3525, -0.3515)$	
	$\Rightarrow k = -0.352$ (to 3 dp)	A1 **(2)**

Question Number	Scheme	Marks
7.	(a) $f(-2) = 16 + 8 - 8\ (= 16) > 0$	B1
	$f(-1) = 1 + 4 - 8\ (= -3) < 0$	B1
	Change of sign (and continuity) $\Rightarrow$ root in interval $(-2, -1)$	B1ft **(3)**
	ft their calculation as long as there is a sign change	
	(b) $\frac{dy}{dx} = 4x^3 - 4 = 0 \quad \Rightarrow \quad x = 1$	M1 A1
	Turning point is $(1, -11)$	A1 **(3)**
	(c) $a = 2, b = 4, c = 4$	B1 B1 B1 **(3)**
	(d) 	
	Shape	B1
	ft their turning point in correct quadrant only	B1 ft
	2 and −8	B1 **(3)**
	(e) 	
	Shape	B1 **(1)** **[13]**

Question Number	Scheme	Marks
8.	(i) $\sec^2 x - \text{cosec}^2 x = \left(1+\tan^2 x\right) - \left(1+\cot^2 x\right)$	M1 A1
	$= \tan^2 x - \cot^2 x$ ∗ cso	A1 **(3)**
	(ii)(a) $y = \arccos x \Rightarrow x = \cos y$	B1
	$x = \sin\left(\frac{\pi}{2} - y\right) \Rightarrow \arcsin x = \frac{\pi}{2} - y$	B1 **(2)**
	Accept $\arcsin x = \arcsin \cos y$	
	(b) $\arccos x + \arcsin x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$	B1 **(1)**
		[6]
	Alternatives for (i)	
	$\sec^2 x - \tan^2 x = 1 = \text{cosec}^2 x - \cot^2 x$	M1 A1
	Rearranging $\sec^2 x - \text{cosec}^2 x = \tan^2 x - \cot^2 x$ ∗ cso	A1 **(3)**
	$\left(\text{LHS} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}\right)$	
	RHS $= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x} = \frac{\left(\sin^2 x - \cos^2 x\right)\left(\sin^2 x + \cos^2 x\right)}{\cos^2 x \sin^2 x}$	M1
	$= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}$	A1
	$=$ LHS ∗ or equivalent	A1 **(3)**