

Mark Scheme (Results)

January 2008

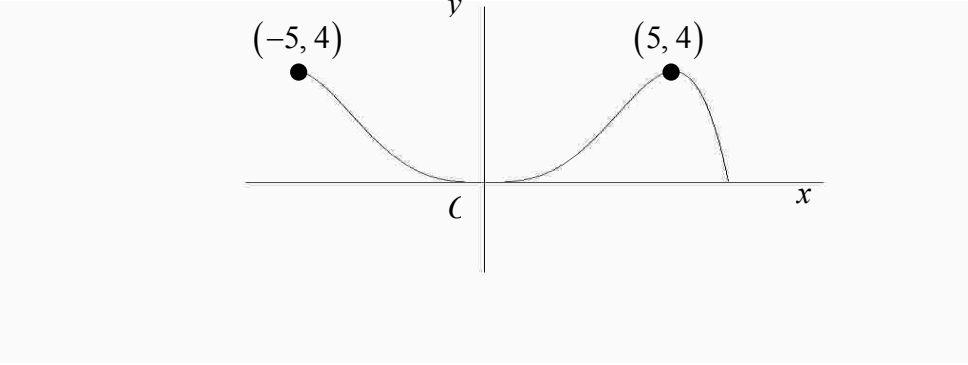
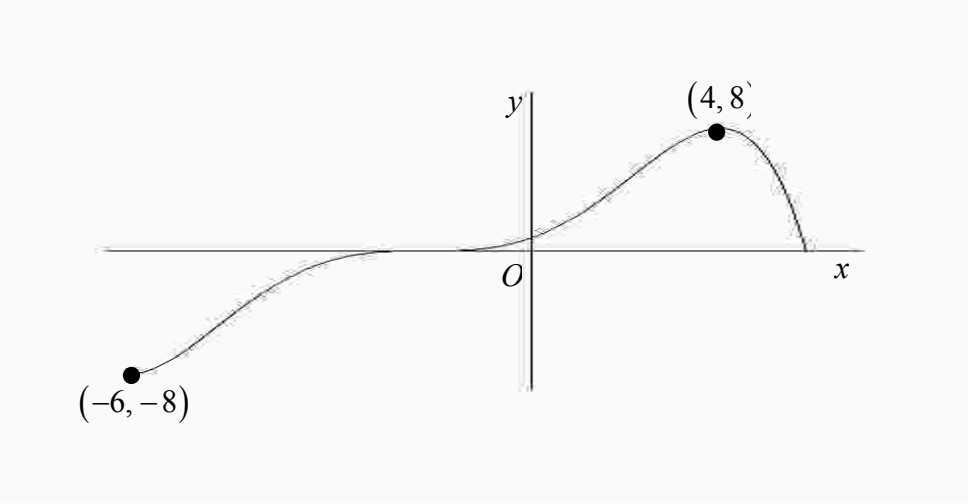
GCE

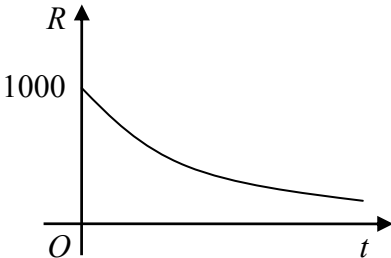
GCE Mathematics (6665/01)

January 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	$ \begin{array}{r} x^2 - 1 \quad \begin{array}{r} \overline{2x^2 \quad -1} \\ 2x^4 \quad -3x^2 + x + 1 \\ \underline{2x^4 \quad -2x^2} \\ -x^2 + x + 1 \\ \underline{-x^2 \quad +1} \\ x \end{array} \\ \\ 2x^2 - 1 + \frac{x}{x^2 - 1} \\ \\ a = 2, b = 0, c = -1, d = 1, e = 0 \\ d = 1 \text{ and } b = 0, e = 0 \text{ stated or implied} \end{array} $	<p>M1 A1 A1</p> <p>A1</p> <p>[4]</p>
2.	<p>(a)</p> $\frac{dy}{dx} = 2e^{2x} \tan x + e^{2x} \sec^2 x$ $\frac{dy}{dx} = 0 \Rightarrow 2e^{2x} \tan x + e^{2x} \sec^2 x = 0$ $2 \tan x + 1 + \tan^2 x = 0$ $(\tan x + 1)^2 = 0$ $\tan x = -1 \quad *$ <p>(b)</p> $\left(\frac{dy}{dx} \right)_0 = 1$ <p>Equation of tangent at $(0, 0)$ is $y = x$</p>	<p>M1 A1+A1</p> <p>M1</p> <p>A1</p> <p>cs0 A1 (6)</p> <p>M1</p> <p>A1 (2)</p> <p>[8]</p>

Question Number	Scheme	Marks
3.	(a) $f(2) = 0.38 \dots$ $f(3) = -0.39 \dots$ Change of sign (and continuity) \Rightarrow root in $(2, 3)$ *	M1 A1 (2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$ $x_2 \approx 2.50498$ $x_3 \approx 2.50518$	M1 A1 A1 (3)
	(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends. $f(2.5045) \approx 6 \times 10^{-4}$ $f(2.5055) \approx -2 \times 10^{-4}$ Change of sign (and continuity) \Rightarrow root $\in (2.5045, 2.5055)$ \Rightarrow root = 2.505 to 3 dp *	M1 A1 (2)
	Note: The root, correct to 5 dp, is 2.50524	[7]

Question Number	Scheme	Marks
4.	<div>(a)</div> <div>  </div> <div> <div>Shape</div> <div>(5, 4)</div> <div>(-5, 4)</div> </div> <div> <div>(b) For the purpose of marking this paper, the graph is identical to (a)</div> <div> <div>Shape</div> <div>(5, 4)</div> <div>(-5, 4)</div> </div> </div> <div>(c)</div> <div>  </div> <div> <div>General shape – unchanged</div> <div>Translation to left</div> <div>(4, 8)</div> <div>(-6, -8)</div> </div> <div> <p>In all parts of this question ignore any drawing outside the domains shown in the diagrams above.</p> </div>	<div> <div>B1</div> <div>B1</div> <div>B1</div> <div>(3)</div> </div> <div> <div>B1</div> <div>B1</div> <div>B1</div> <div>(3)</div> </div> <div> <div>B1</div> <div>B1</div> <div>B1</div> <div>B1</div> <div>(4)</div> </div> <div> <div>[10]</div> </div>

Question Number	Scheme	Marks
5.	(a) 1000	B1 (1)
	(b) $1000e^{-5730c} = 500$ $e^{-5730c} = \frac{1}{2}$ $-5730c = \ln \frac{1}{2}$ $c = 0.000121$	M1 A1 M1 A1 (4)
	(c) $R = 1000e^{-22920c} = 62.5$	Accept 62-63 M1 A1 (2)
	(d)	
		Shape 1000 B1 B1 (2) [9]

Question Number	Scheme	Marks
6.	<p>(a) $\cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x$</p> <p>$= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$</p> <p>$= (2\cos^2 x - 1)\cos x - 2(1 - \cos^2 x)\cos x$ any correct expression</p> <p>$= 4\cos^3 x - 3\cos x$</p> <p>(b)(i) $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = \frac{\cos^2 x + (1+\sin x)^2}{(1+\sin x)\cos x}$</p> <p>$= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$</p> <p>$= \frac{2(1+\sin x)}{(1+\sin x)\cos x}$</p> <p>$= \frac{2}{\cos x} = 2\sec x$ *</p> <p>(c) $\sec x = 2$ or $\cos x = \frac{1}{2}$</p> <p>$x = \frac{\pi}{3}, \frac{5\pi}{3}$ accept awrt 1.05, 5.24</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1 (4)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (4) cso</p> <p>M1</p> <p>A1, A1 (3)</p> <p>[11]</p>
7.	<p>(a) $\frac{dy}{dx} = 6\cos 2x - 8\sin 2x$</p> <p>$\left(\frac{dy}{dx}\right)_0 = 6$</p> <p>$y - 4 = -\frac{1}{6}x$ or equivalent</p> <p>(b) $R = \sqrt{3^2 + 4^2} = 5$</p> <p>$\tan \alpha = \frac{4}{3}, \alpha \approx 0.927$ awrt 0.927</p> <p>(c) $\sin(2x + \text{their } \alpha) = 0$</p> <p>$x = -2.03, -0.46, 1.11, 2.68$</p> <p>First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the y-coordinate.</p>	<p>M1 A1</p> <p>B1</p> <p>M1 A1 (5)</p> <p>M1 A1</p> <p>M1 A1 (4)</p> <p>M1</p> <p>A1 A1 A1 (4)</p> <p>[13]</p>

Question Number	Scheme	Marks
8.	$(a) \quad x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{\frac{1}{3}} \text{ or } \sqrt[3]{\frac{1-x}{2}}$	M1 A1 (2)
	$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$	Ignore domain
	$(b) \quad gf(x) = \frac{3}{1-2x^3} - 4$	M1 A1
	$= \frac{3-4(1-2x^3)}{1-2x^3}$	M1
	$= \frac{8x^3-1}{1-2x^3} *$	cso A1 (4)
	$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$	Ignore domain
	$(c) \quad 8x^3 - 1 = 0$	Attempting solution of numerator = 0 M1
	$x = \frac{1}{2}$	Correct answer and no additional answers A1 (2)
	$(d) \quad \frac{dy}{dx} = \frac{(1-2x^3) \times 24x^2 + (8x^3-1) \times 6x^2}{(1-2x^3)^2}$	M1 A1
	$= \frac{18x^2}{(1-2x^3)^2}$	A1
	Solving their numerator = 0 and substituting to find y.	M1
	$x = 0, y = -1$	A1 (5)
		[13]