

Mark Scheme (Results)

January 2009

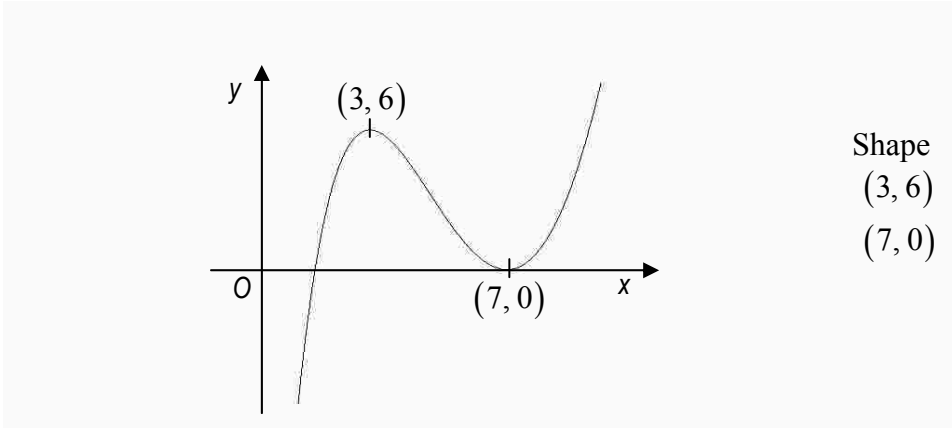
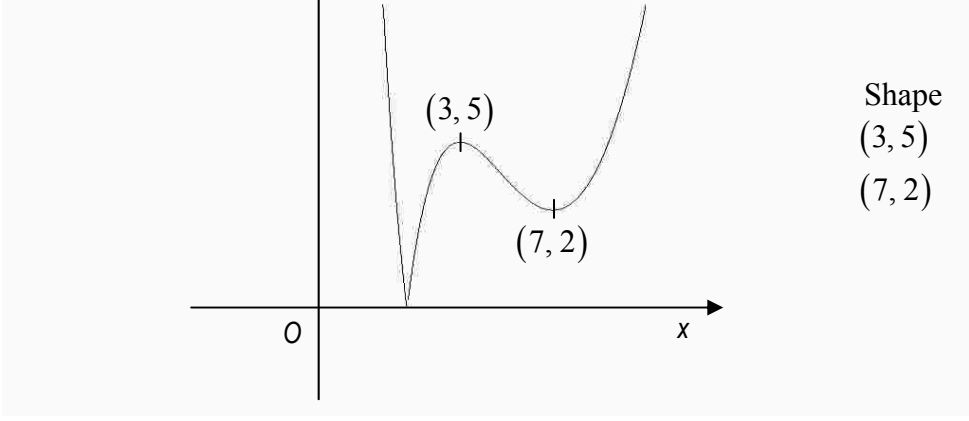
GCE

GCE Mathematics (6665/01)

January 2009
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$\frac{d}{dx}(\sqrt{5x-1}) = \frac{d}{dx}((5x-1)^{\frac{1}{2}})$ $= 5 \times \frac{1}{2}(5x-1)^{-\frac{1}{2}}$ $\frac{dy}{dx} = 2x\sqrt{5x-1} + \frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$ <p>At $x = 2$, $\frac{dy}{dx} = 4\sqrt{9} + \frac{10}{\sqrt{9}} = 12 + \frac{10}{3}$</p> $= \frac{46}{3}$	<p>M1 A1</p> <p>M1 A1ft</p> <p>M1</p> <p>A1 (6)</p>
(b)	$\frac{d}{dx}\left(\frac{\sin 2x}{x^2}\right) = \frac{2x^2 \cos 2x - 2x \sin 2x}{x^4}$	<p>M1 $\frac{A1+A1}{A1}$</p> <p>(4)</p> <p>[10]</p>
<p><i>Alternative to (b)</i></p> $\frac{d}{dx}(\sin 2x \times x^{-2}) = 2 \cos 2x \times x^{-2} + \sin 2x \times (-2)x^{-3}$ $= 2x^{-2} \cos 2x - 2x^{-3} \sin 2x \quad \left(= \frac{2 \cos 2x}{x^2} - \frac{2 \sin 2x}{x^3} \right)$		<p>M1 A1 + A1</p> <p>A1 (4)</p>

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<p>2 (a)</p> <p>(b)</p>	$\frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3} = \frac{2x+2}{(x-3)(x+1)} - \frac{x+1}{x-3}$ $= \frac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$ $= \frac{(x+1)(1-x)}{(x-3)(x+1)}$ $= \frac{1-x}{x-3}$ <p>Accept $-\frac{x-1}{x-3}, \frac{x-1}{3-x}$</p> $\frac{d}{dx}\left(\frac{1-x}{x-3}\right) = \frac{(x-3)(-1) - (1-x)1}{(x-3)^2}$ $= \frac{-x+3-1+x}{(x-3)^2} = \frac{2}{(x-3)^2} *$ <p>Alternative to (a)</p> $\frac{2x+2}{x^2-2x-3} = \frac{2(x+1)}{(x-3)(x+1)} = \frac{2}{x-3}$ $\frac{2}{x-3} - \frac{x+1}{x-3} = \frac{2-(x+1)}{x-3}$ $= \frac{1-x}{x-3}$ <p>Alternatives to (b)</p> <p>① $f(x) = \frac{1-x}{x-3} = -1 - \frac{2}{x-3} = -1 - 2(x-3)^{-1}$</p> $f'(x) = (-1)(-2)(x-3)^{-2}$ $= \frac{2}{(x-3)^2} *$ <p>② $f(x) = (1-x)(x-3)^{-1}$</p> $f'(x) = (-1)(x-3)^{-1} + (1-x)(-1)(x-3)^{-2}$ $= -\frac{1}{x-3} - \frac{1-x}{(x-3)^2} = \frac{-(x-3) - (1-x)}{(x-3)^2}$ $= \frac{2}{(x-3)^2} *$	<p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>CSO A1 (3)</p> <p>[7]</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>CSO A1 (3)</p> <p>M1</p> <p>A1</p> <p>A1 (3)</p>

Question Number	Scheme	Marks
<p>3</p> <p>(a)</p>	 <p>Shape (3, 6) (7, 0)</p>	<p>B1 B1 B1</p> <p>(3)</p>
<p>(b)</p>	 <p>Shape (3, 5) (7, 2)</p>	<p>B1 B1 B1</p> <p>(3) [6]</p>

Question Number	Scheme	Marks
4	$x = \cos(2y + \pi)$ $\frac{dx}{dy} = -2 \sin(2y + \pi)$ $\frac{dy}{dx} = -\frac{1}{2 \sin(2y + \pi)}$ <p>At $y = \frac{\pi}{4}$,</p> $\frac{dy}{dx} = -\frac{1}{2 \sin \frac{3\pi}{2}} = \frac{1}{2}$ $y - \frac{\pi}{4} = \frac{1}{2}x$ $y = \frac{1}{2}x + \frac{\pi}{4}$	<p>M1 A1</p> <p>A1ft</p> <p>Follow through their $\frac{dx}{dy}$ before or after substitution</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(6)</p> <p>[6]</p>

Question Number	Scheme	Marks
5 (a)	$g(x) \geq 1$	B1 (1)
5 (b)	$fg(x) = f(e^{x^2}) = 3e^{x^2} + \ln e^{x^2}$ $= x^2 + 3e^{x^2} \quad *$ $(fg : x \mapsto x^2 + 3e^{x^2})$	M1 A1 (2)
5 (c)	$fg(x) \geq 3$	B1 (1)
5 (d)	$\frac{d}{dx}(x^2 + 3e^{x^2}) = 2x + 6xe^{x^2}$ $2x + 6xe^{x^2} = x^2 e^{x^2} + 2x$ $e^{x^2}(6x - x^2) = 0$ $e^{x^2} \neq 0, \quad 6x - x^2 = 0$ $x = 0, 6$	M1 A1 M1 A1 A1 A1 (6) [10]

Question Number	Scheme	Marks
<p>6 (a)(i)</p> <p>(ii)</p> <p>(b)</p>	$\begin{aligned} \sin 3\theta &= \sin(2\theta + \theta) \\ &= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta \\ &= 2 \sin \theta \cos \theta \cdot \cos \theta + (1 - 2 \sin^2 \theta) \sin \theta \\ &= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta \\ &= 3 \sin \theta - 4 \sin^3 \theta \quad * \end{aligned}$ $\begin{aligned} 8 \sin^3 \theta - 6 \sin \theta + 1 &= 0 \\ -2 \sin 3\theta + 1 &= 0 \\ \sin 3\theta &= \frac{1}{2} \\ 3\theta &= \frac{\pi}{6}, \frac{5\pi}{6} \\ \theta &= \frac{\pi}{18}, \frac{5\pi}{18} \end{aligned}$ $\begin{aligned} \sin 15^\circ &= \sin(60^\circ - 45^\circ) = \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}} \\ &= \frac{1}{4} \sqrt{6} - \frac{1}{4} \sqrt{2} = \frac{1}{4} (\sqrt{6} - \sqrt{2}) \quad * \end{aligned}$	<p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>M1</p> <p>A1 A1 (5)</p> <p>M1</p> <p>M1 A1</p> <p>A1 (4)</p> <p>[13]</p>
	<p><i>Alternatives to (b)</i></p> <p>① $\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$</p> $\begin{aligned} &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{1}{4} \sqrt{6} - \frac{1}{4} \sqrt{2} = \frac{1}{4} (\sqrt{6} - \sqrt{2}) \quad * \end{aligned}$ <p>② Using $\cos 2\theta = 1 - 2 \sin^2 \theta$, $\cos 30^\circ = 1 - 2 \sin^2 15^\circ$</p> $2 \sin^2 15^\circ = 1 - \cos 30^\circ = 1 - \frac{\sqrt{3}}{2}$ $\sin^2 15^\circ = \frac{2 - \sqrt{3}}{4}$ $\left(\frac{1}{4} (\sqrt{6} - \sqrt{2}) \right)^2 = \frac{1}{16} (6 + 2 - 2\sqrt{12}) = \frac{2 - \sqrt{3}}{4}$ <p>Hence $\sin 15^\circ = \frac{1}{4} (\sqrt{6} - \sqrt{2}) \quad *$</p>	<p>M1</p> <p>M1 A1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p>

Question Number	Scheme	Marks
7 (a)	$f'(x) = 3e^x + 3xe^x$ $3e^x + 3xe^x = 3e^x(1+x) = 0$ $x = -1$ $f(-1) = -3e^{-1} - 1$	M1 A1 M1 A1 B1 (5)
7 (b)	$x_1 = 0.2596$ $x_2 = 0.2571$ $x_3 = 0.2578$	B1 B1 B1 (3)
7 (c)	Choosing (0.257 55, 0.257 65) or an appropriate tighter interval. $f(0.257\ 55) = -0.000\ 379 \dots$ $f(0.257\ 65) = 0.000\ 109 \dots$ Change of sign (and continuity) \Rightarrow root \in (0.257 55, 0.257 65) * ($\Rightarrow x = 0.2576$, is correct to 4 decimal places)	M1 A1 A1 cso A1 (3) [11]
<i>Note: $x = 0.257\ 627\ 65 \dots$ is accurate</i>		

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8 (a)	$R^2 = 3^2 + 4^2$ $R = 5$ $\tan \alpha = \frac{4}{3}$ $\alpha = 53 \dots^\circ$	M1 A1 M1 A1 (4) awrt 53°
(b)	Maximum value is 5 At the maximum, $\cos(\theta - \alpha) = 1$ or $\theta - \alpha = 0$ $\theta = \alpha = 53 \dots^\circ$	ft their R B1 ft M1 ft their α A1 ft (3)
(c)	$f(t) = 10 + 5 \cos(15t - \alpha)^\circ$ Minimum occurs when $\cos(15t - \alpha)^\circ = -1$ The minimum temperature is $(10 - 5)^\circ = 5^\circ$	M1 A1 ft (2)
(d)	$15t - \alpha = 180$ $t = 15.5$	awrt 15.5 M1 M1 A1 (3) [12]