

Mark Scheme (Results) January 2010

GCE

Core Mathematics C3 (6665)

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January 2010

Publications Code US022710

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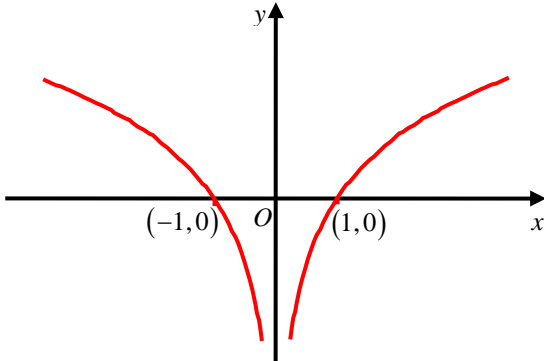
Question Number	Scheme	Marks
Q1	$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$ $= \frac{x+1}{3(x^2-1)} - \frac{1}{3x+1}$ $= \frac{x+1}{3(x+1)(x-1)} - \frac{1}{3x+1}$ $= \frac{1}{3(x-1)} - \frac{1}{3x+1}$ $= \frac{3x+1-3(x-1)}{3(x-1)(3x+1)}$ <p>or $\frac{3x+1}{3(x-1)(3x+1)} - \frac{3(x-1)}{3(x-1)(3x+1)}$</p> $= \frac{4}{3(x-1)(3x+1)}$	<p>Award below</p> <p>$x^2 - 1 \rightarrow (x+1)(x-1)$ or $3x^2 - 3 \rightarrow (x+1)(3x-3)$ or $3x^2 - 3 \rightarrow (3x+3)(x-1)$ seen or implied anywhere in candidate's working.</p> <p>Attempt to combine. M1</p> <p>Correct result. A1</p> <p>Decide to award M1 here!! M1</p> <p>Either $\frac{4}{3(x-1)(3x+1)}$ or $\frac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\frac{4}{(3x-3)(3x+1)}$ or $\frac{4}{9x^2-6x-3}$</p> <p>A1 aef</p> <p>[4]</p>

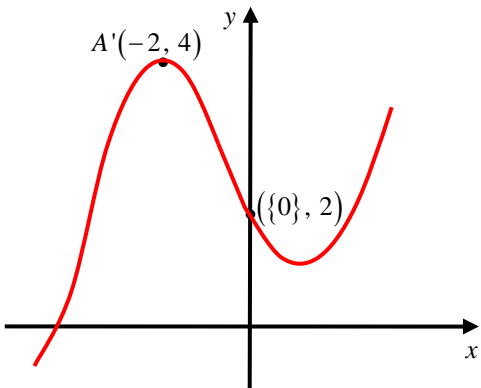

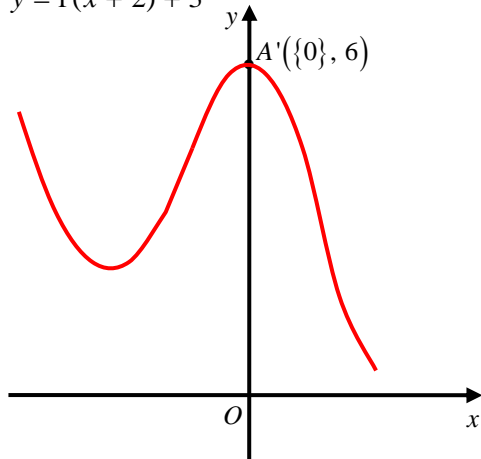
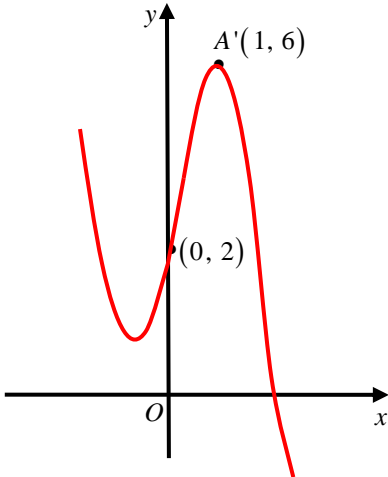

Question Number	Scheme	Marks
Q2	$f(x) = x^3 + 2x^2 - 3x - 11$	
(a)	$f(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x + 2) - 3x - 11 = 0$ $\Rightarrow x^2(x + 2) = 3x + 11$ $\Rightarrow x^2 = \frac{3x + 11}{x + 2}$ $\Rightarrow x = \sqrt{\left(\frac{3x + 11}{x + 2}\right)}$	<p>Sets $f(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).</p> <p>M1</p> <p>then rearranges to give the quoted result on the question paper.</p> <p>A1 AG</p> <p>(2)</p>
(b)	<p>Iterative formula: $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$, $x_1 = 0$</p> $x_2 = \sqrt{\left(\frac{3(0) + 11}{(0) + 2}\right)}$ $x_2 = 2.34520788...$ $x_3 = 2.037324945...$ $x_4 = 2.058748112...$	<p>An attempt to substitute $x_1 = 0$ into the iterative formula.</p> <p>Can be implied by $x_2 = \sqrt{5.5}$ or 2.35 or awrt 2.345</p> <p>M1</p> <p>Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037</p> <p>A1</p> <p>$x_4 =$ awrt 2.059</p> <p>A1</p> <p>(3)</p>
(c)	<p>Let $f(x) = x^3 + 2x^2 - 3x - 11 = 0$</p> $f(2.0565) = -0.013781637...$ $f(2.0575) = 0.0041401094...$ <p>Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)</p>	<div>Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter</div> <div>any one value awrt 1 sf</div> <div>both values correct awrt 1sf, sign change and conclusion</div> <div>As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root".</div> <p>M1</p> <p>dM1</p> <p>A1</p> <p>(3)</p>
		[8]

Question Number	Scheme	Marks
Q3 (a)	$5\cos x - 3\sin x = R\cos(x + \alpha), \quad R > 0, \quad 0 < x < \frac{\pi}{2}$ $5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$ <p>Equate $\cos x$: $5 = R\cos\alpha$ Equate $\sin x$: $3 = R\sin\alpha$</p> $R = \sqrt{5^2 + 3^2}; = \sqrt{34} \quad \{= 5.83095..\}$ $\tan\alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^{\circ}$ <p>Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$</p>	<p>M1; A1</p> <p>M1 A1</p>
(b)	$5\cos x - 3\sin x = 4$ $\sqrt{34}\cos(x + 0.5404) = 4$ $\cos(x + 0.5404) = \frac{4}{\sqrt{34}} \quad \{= 0.68599...\}$ $(x + 0.5404) = 0.814826916...^{\circ}$ $x = 0.2744...^{\circ}$ $(x + 0.5404) = 2\pi - 0.814826916...^{\circ} \quad \{= 5.468358...^{\circ}\}$ $x = 4.9279...^{\circ}$ <p>Hence, $x = \{0.27, 4.93\}$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>ddM1 A1</p> <p>(4)</p> <p>(5)</p> <p>[9]</p>

Part (b): If there are any EXTRA solutions inside the range $0 \leq x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \leq x < 2\pi$.

Question Number	Scheme	Marks
Q4 (i)	$y = \frac{\ln(x^2 + 1)}{x}$ $u = \ln(x^2 + 1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2 + 1}$ <p>Apply quotient rule: $\left\{ \begin{array}{l} u = \ln(x^2 + 1) \quad v = x \\ \frac{du}{dx} = \frac{2x}{x^2 + 1} \quad \frac{dv}{dx} = 1 \end{array} \right\}$</p> $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2 + 1} \right)(x) - \ln(x^2 + 1)}{x^2}$ $\left\{ \frac{dy}{dx} = \frac{2}{(x^2 + 1)} - \frac{1}{x^2} \ln(x^2 + 1) \right\}$	<p>$\ln(x^2 + 1) \rightarrow \frac{\text{something}}{x^2 + 1}$ M1</p> <p>$\ln(x^2 + 1) \rightarrow \frac{2x}{x^2 + 1}$ A1</p> <p>Applying $\frac{xu' - \ln(x^2 + 1)v'}{x^2}$ correctly. M1</p> <p>Correct differentiation with correct bracketing but allow recovery. A1</p> <p>{Ignore subsequent working.}</p> <p>(4)</p>
(ii)	$x = \tan y$ $\frac{dx}{dy} = \sec^2 y$ $\frac{dy}{dx} = \frac{1}{\sec^2 y} \{ = \cos^2 y \}$ $\frac{dy}{dx} = \frac{1}{1 + \tan^2 y}$ <p>Hence, $\frac{dy}{dx} = \frac{1}{1 + x^2}$, (as required)</p>	<p>$\tan y \rightarrow \sec^2 y$ or an attempt to differentiate $\frac{\sin y}{\cos y}$ using either the quotient rule or product rule. M1*</p> <p>$\frac{dx}{dy} = \sec^2 y$ A1</p> <p>Finding $\frac{dy}{dx}$ by reciprocating $\frac{dx}{dy}$. dM1*</p> <p>For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y. dM1*</p> <p>For the correct proof, leading on from the previous line of working. A1 AG</p> <p>(5)</p> <p>[9]</p>

Question Number	Scheme		Marks
Q5	<div>$y = \ln x$</div> <div></div>	<div>Right-hand branch in quadrants 4 and 1. Correct shape.</div> <div>B1</div> <div>Left-hand branch in quadrants 2 and 3. Correct shape.</div> <div>B1</div> <div>Completely correct sketch and both $(-1, \{0\})$ and $(1, \{0\})$</div> <div>B1</div> <div>(3)</div> <div>[3]</div>	

Question Number	Scheme	Marks
Q6 (i)	<p>$y = f(-x) + 1$</p>  <p>Shape of </p> <p>and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis.</p> <p>Either $(\{0\}, 2)$ or $A'(-2, 4)$</p> <p>Both $(\{0\}, 2)$ and $A'(-2, 4)$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p>
(ii)	<p>$y = f(x + 2) + 3$</p>  <p>Any translation of the original curve.</p> <p>The translated maximum has either x-coordinate of 0 (can be implied) or y-coordinate of 6.</p> <p>The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes.</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p>
(iii)	<p>$y = 2f(2x)$</p>  <p>Shape of </p> <p>with a minimum in quadrant 2 and a maximum in quadrant 1.</p> <p>Either $(\{0\}, 2)$ or $A'(1, 6)$</p> <p>Both $(\{0\}, 2)$ and $A'(1, 6)$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p> <p>[9]</p>

Question Number	Scheme	Marks
Q7 (a)	$y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$ $\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$ $\frac{dy}{dx} = \left\{ \frac{\sin x}{\cos^2 x} \right\} = \left(\frac{1}{\cos x} \right) \left(\frac{\sin x}{\cos x} \right) = \underline{\underline{\sec x \tan x}}$	<p>$\frac{dy}{dx} = \pm ((\cos x)^{-2}(\sin x))$ M1</p> <p>$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$ A1</p> <p>Convincing proof. Must see both <u>underlined steps</u>. A1 AG</p> <p>(3)</p>
(b)	$y = e^{2x} \sec 3x$ $\left\{ \begin{array}{l} u = e^{2x} \quad v = \sec 3x \\ \frac{du}{dx} = 2e^{2x} \quad \frac{dv}{dx} = 3 \sec 3x \tan 3x \end{array} \right\}$ $\frac{dy}{dx} = 2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan 3x$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">Seen or implied</div> <p>Either $e^{2x} \rightarrow 2e^{2x}$ or $\sec 3x \rightarrow 3 \sec 3x \tan 3x$ M1</p> <p>Both $e^{2x} \rightarrow 2e^{2x}$ and $\sec 3x \rightarrow 3 \sec 3x \tan 3x$ A1</p> <p>Applies $vu' + uv'$ correctly for their u, u', v, v' M1</p> <p>$2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan 3x$ A1 isw</p> <p>(4)</p>
(c)	<p>Turning point $\Rightarrow \frac{dy}{dx} = 0$</p> <p>Hence, $e^{2x} \sec 3x (2 + 3 \tan 3x) = 0$</p> <p>{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3 \tan 3x = 0$, }</p> <p>giving $\tan 3x = -\frac{2}{3}$</p> <p>$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600\dots$</p> <p>Hence, $y = \{b\} = e^{2(-0.196)} \sec(3 \times -0.196)$</p> <p>$= 0.812093\dots = 0.812$ (3sf)</p>	<p>Sets their $\frac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms. M1</p> <p>$\tan 3x = \pm k$; $k \neq 0$ M1</p> <p>Either awrt -0.196° or awrt -11.2° A1</p> <p>0.812 A1 cao</p> <p>(4)</p>

[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b), (for these values of x) then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

Question Number	Scheme	Marks
Q8	<p>$\operatorname{cosec}^2 2x - \cot 2x = 1, \text{ (eqn *) } 0 \leq x \leq 180^\circ$</p> <p>Using $\operatorname{cosec}^2 2x = 1 + \cot^2 2x$ gives</p> <p>$1 + \cot^2 2x - \cot 2x = 1$</p> <p>$\cot^2 2x - \cot 2x = 0 \quad \text{or} \quad \cot^2 2x = \cot 2x$</p> <p>$\cot 2x(\cot 2x - 1) = 0 \quad \text{or} \quad \cot 2x = 1$</p> <p>$\cot 2x = 0 \quad \text{or} \quad \cot 2x = 1$</p> <p>$\cot 2x = 0 \Rightarrow (\tan 2x \rightarrow \infty) \Rightarrow 2x = 90, 270$</p> <p>$\Rightarrow x = 45, 135$</p> <p>$\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$</p> <p>$\Rightarrow x = 22.5, 112.5$</p> <p>Overall, $x = \{22.5, 45, 112.5, 135\}$</p>	<p>M1</p> <p>A1</p> <p>dM1</p> <p>A1</p> <p>ddM1</p> <p>A1</p> <p>B1</p> <p>[7]</p>

If there are any EXTRA solutions inside the range $0 \leq x \leq 180^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question). Also ignore EXTRA solutions outside the range $0 \leq x \leq 180^\circ$.

Question Number	Scheme	Marks
Q9 (i)(a)	$\ln(3x - 7) = 5$ $e^{\ln(3x - 7)} = e^5$ $3x - 7 = e^5 \Rightarrow x = \frac{e^5 + 7}{3} \{= 51.804...\}$	<p>Takes e of both sides of the equation. This can be implied by $3x - 7 = e^5$. M1</p> <p>Then rearranges to make x the subject. dM1</p> <p>Exact answer of $\frac{e^5 + 7}{3}$. A1</p> <p>(3)</p>
(b)	$3^x e^{7x+2} = 15$ $\ln(3^x e^{7x+2}) = \ln 15$ $\ln 3^x + \ln e^{7x+2} = \ln 15$ $x \ln 3 + 7x + 2 = \ln 15$ $x(\ln 3 + 7) = -2 + \ln 15$ $x = \frac{-2 + \ln 15}{7 + \ln 3} \{= 0.0874...\}$	<p>Takes ln (or logs) of both sides of the equation. M1</p> <p>Applies the addition law of logarithms. M1</p> <p>$x \ln 3 + 7x + 2 = \ln 15$ A1 oe</p> <p>Factorising out at least two x terms on one side and collecting number terms on the other side. ddM1</p> <p>Exact answer of $\frac{-2 + \ln 15}{7 + \ln 3}$ A1 oe</p> <p>(5)</p>
(ii) (a)	$f(x) = e^{2x} + 3, x \in \mathbb{R}$ $y = e^{2x} + 3 \Rightarrow y - 3 = e^{2x}$ $\Rightarrow \ln(y - 3) = 2x$ $\Rightarrow \frac{1}{2} \ln(y - 3) = x$ Hence $f^{-1}(x) = \frac{1}{2} \ln(x - 3)$ $f^{-1}(x)$: Domain: $x > 3$ or $(3, \infty)$	<p>Attempt to make x (or swapped y) the subject M1</p> <p>Makes e^{2x} the subject and takes ln of both sides M1</p> <p>$\frac{1}{2} \ln(x - 3)$ or $\ln \sqrt{x - 3}$ A1 cao</p> <p>or $f^{-1}(y) = \frac{1}{2} \ln(y - 3)$ (see appendix)</p> <p>Either $x > 3$ or $(3, \infty)$ or Domain > 3. B1</p> <p>(4)</p>
(b)	$g(x) = \ln(x - 1), x \in \mathbb{R}, x > 1$ $fg(x) = e^{2 \ln(x-1)} + 3 \{= (x - 1)^2 + 3\}$ $fg(x)$: Range: $y > 3$ or $(3, \infty)$	<p>An attempt to put function g into function f. M1</p> <p>$e^{2 \ln(x-1)} + 3$ or $(x - 1)^2 + 3$ or $x^2 - 2x + 4$. A1 isw</p> <p>Either $y > 3$ or $(3, \infty)$ or Range > 3 or $fg(x) > 3$. B1</p> <p>(3)</p>
		[15]

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