

Mark Scheme (Results) January 2010

GCE

Core Mathematics C3 (6665)

Edexcel is one of the leading examining and awarding bodies in the UK and throughout the world. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers.

Through a network of UK and overseas offices, Edexcel's centres receive the support they need to help them deliver their education and training programmes to learners.

For further information, please call our GCE line on 0844 576 0025, our GCSE team on 0844 576 0027, or visit our website at www.edexcel.com.

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our **Ask The Expert** email service helpful.

Ask The Expert can be accessed online at the following link:

<http://www.edexcel.com/Aboutus/contact-us/>

January 2010

Publications Code US022710

All the material in this publication is copyright

© Edexcel Ltd 2010

January 2010
6665 Core Mathematics C3
Mark Scheme

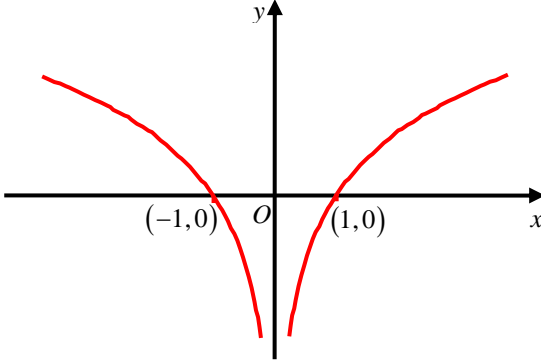
Question Number	Scheme	Marks
Q1	$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$ $= \frac{x+1}{3(x^2-1)} - \frac{1}{3x+1}$ $= \frac{x+1}{3(x+1)(x-1)} - \frac{1}{3x+1}$ $= \frac{1}{3(x-1)} - \frac{1}{3x+1}$ $= \frac{3x+1-3(x-1)}{3(x-1)(3x+1)}$ <p>or $\frac{3x+1}{3(x-1)(3x+1)} - \frac{3(x-1)}{3(x-1)(3x+1)}$</p> $= \frac{4}{3(x-1)(3x+1)}$ <p> <i>Decide to award M1 here!!</i> Either $\frac{4}{3(x-1)(3x+1)}$ or $\frac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\frac{4}{(3x-3)(3x+1)}$ or $\frac{4}{9x^2-6x-3}$ </p>	<p>Award below</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 aef</p> <p>[4]</p>

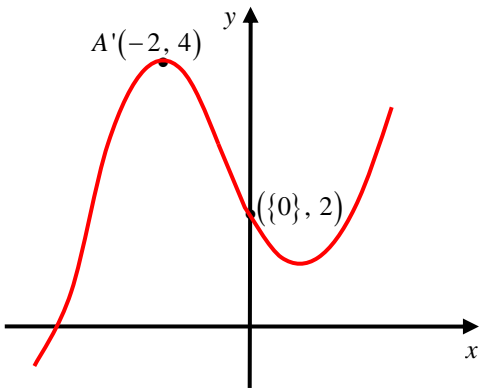

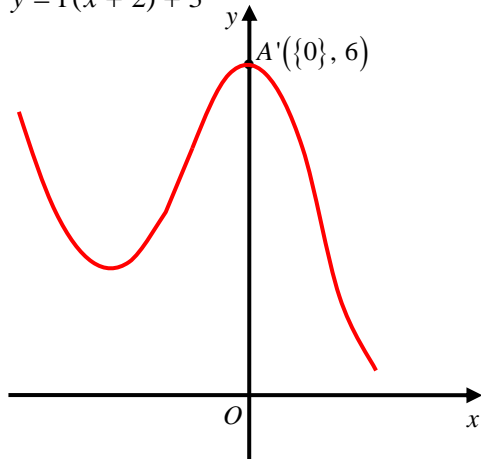
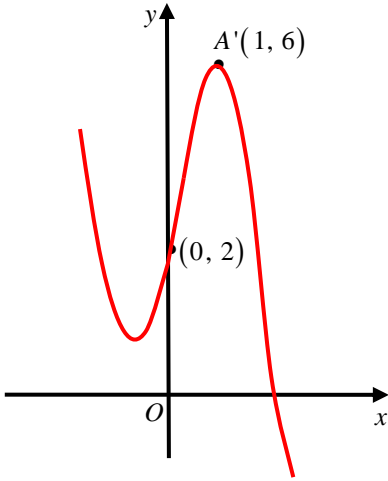

Question Number	Scheme	Marks
Q2	$f(x) = x^3 + 2x^2 - 3x - 11$	
(a)	$f(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x + 2) - 3x - 11 = 0$ $\Rightarrow x^2(x + 2) = 3x + 11$ $\Rightarrow x^2 = \frac{3x + 11}{x + 2}$ $\Rightarrow x = \sqrt{\left(\frac{3x + 11}{x + 2}\right)}$	<p>Sets $f(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).</p> <p>M1</p> <p>then rearranges to give the quoted result on the question paper.</p> <p>A1 AG</p> <p>(2)</p>
(b)	<p>Iterative formula: $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$, $x_1 = 0$</p> $x_2 = \sqrt{\left(\frac{3(0) + 11}{(0) + 2}\right)}$ $x_2 = 2.34520788...$ $x_3 = 2.037324945...$ $x_4 = 2.058748112...$	<p>An attempt to substitute $x_1 = 0$ into the iterative formula.</p> <p>Can be implied by $x_2 = \sqrt{5.5}$ or 2.35 or awrt 2.345</p> <p>M1</p> <p>Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037</p> <p>A1</p> <p>$x_4 =$ awrt 2.059</p> <p>A1</p> <p>(3)</p>
(c)	<p>Let $f(x) = x^3 + 2x^2 - 3x - 11 = 0$</p> $f(2.0565) = -0.013781637...$ $f(2.0575) = 0.0041401094...$ <p>Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)</p>	<div>Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter</div> <div>any one value awrt 1 sf</div> <div>both values correct awrt 1sf, sign change and conclusion</div> <div>As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root".</div> <p>M1</p> <p>dM1</p> <p>A1</p> <p>(3)</p>
		[8]

Question Number	Scheme	Marks
Q3 (a)	$5\cos x - 3\sin x = R\cos(x + \alpha), \quad R > 0, \quad 0 < x < \frac{\pi}{2}$ $5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$ <p>Equate $\cos x$: $5 = R\cos\alpha$ Equate $\sin x$: $3 = R\sin\alpha$</p> $R = \sqrt{5^2 + 3^2}; = \sqrt{34} \quad \{= 5.83095..\}$ $\tan\alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^{\circ}$ <p>Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$</p>	<p>M1; A1</p> <p>M1 A1</p>
(b)	$5\cos x - 3\sin x = 4$ $\sqrt{34}\cos(x + 0.5404) = 4$ $\cos(x + 0.5404) = \frac{4}{\sqrt{34}} \quad \{= 0.68599...\}$ $(x + 0.5404) = 0.814826916...^{\circ}$ $x = 0.2744...^{\circ}$ $(x + 0.5404) = 2\pi - 0.814826916...^{\circ} \quad \{= 5.468358...^{\circ}\}$ $x = 4.9279...^{\circ}$ <p>Hence, $x = \{0.27, 4.93\}$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>ddM1 A1</p> <p>(4)</p> <p>(5)</p> <p>[9]</p>

Part (b): If there are any EXTRA solutions inside the range $0 \leq x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \leq x < 2\pi$.

Question Number	Scheme	Marks
Q4 (i)	$y = \frac{\ln(x^2 + 1)}{x}$ $u = \ln(x^2 + 1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2 + 1}$ <p>Apply quotient rule: $\left\{ \begin{array}{l} u = \ln(x^2 + 1) \quad v = x \\ \frac{du}{dx} = \frac{2x}{x^2 + 1} \quad \frac{dv}{dx} = 1 \end{array} \right\}$</p> $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2 + 1} \right)(x) - \ln(x^2 + 1)}{x^2}$ $\left\{ \frac{dy}{dx} = \frac{2}{(x^2 + 1)} - \frac{1}{x^2} \ln(x^2 + 1) \right\}$	<p>$\ln(x^2 + 1) \rightarrow \frac{\text{something}}{x^2 + 1}$ M1</p> <p>$\ln(x^2 + 1) \rightarrow \frac{2x}{x^2 + 1}$ A1</p> <p>Applying $\frac{xu' - \ln(x^2 + 1)v'}{x^2}$ correctly. M1</p> <p>Correct differentiation with correct bracketing but allow recovery. A1</p> <p>{Ignore subsequent working.}</p> <p>(4)</p>
(ii)	$x = \tan y$ $\frac{dx}{dy} = \sec^2 y$ $\frac{dy}{dx} = \frac{1}{\sec^2 y} \{ = \cos^2 y \}$ $\frac{dy}{dx} = \frac{1}{1 + \tan^2 y}$ <p>Hence, $\frac{dy}{dx} = \frac{1}{1 + x^2}$, (as required)</p>	<p>$\tan y \rightarrow \sec^2 y$ or an attempt to differentiate $\frac{\sin y}{\cos y}$ using either the quotient rule or product rule. M1*</p> <p>$\frac{dx}{dy} = \sec^2 y$ A1</p> <p>Finding $\frac{dy}{dx}$ by reciprocating $\frac{dx}{dy}$. dM1*</p> <p>For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y. dM1*</p> <p>For the correct proof, leading on from the previous line of working. A1 AG</p> <p>(5)</p> <p>[9]</p>

Question Number	Scheme	Marks
Q5	<div><div><div>$y = \ln x$</div><div></div></div><div><div>Right-hand branch in quadrants 4 and 1. Correct shape.</div><div>Left-hand branch in quadrants 2 and 3. Correct shape.</div><div>Completely correct sketch and both $(-1, \{0\})$ and $(1, \{0\})$</div></div></div> <div><div>B1</div><div>B1</div><div>B1</div><div>(3)</div><div>[3]</div></div>	

Question Number	Scheme	Marks
Q6 (i)	<p>$y = f(-x) + 1$</p>  <p>Shape of </p> <p>and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis.</p> <p>Either $(\{0\}, 2)$ or $A'(-2, 4)$</p> <p>Both $(\{0\}, 2)$ and $A'(-2, 4)$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p>
(ii)	<p>$y = f(x + 2) + 3$</p>  <p>Any translation of the original curve.</p> <p>The translated maximum has either x-coordinate of 0 (can be implied) or y-coordinate of 6.</p> <p>The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes.</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p>
(iii)	<p>$y = 2f(2x)$</p>  <p>Shape of </p> <p>with a minimum in quadrant 2 and a maximum in quadrant 1.</p> <p>Either $(\{0\}, 2)$ or $A'(1, 6)$</p> <p>Both $(\{0\}, 2)$ and $A'(1, 6)$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p> <p>[9]</p>

Question Number	Scheme	Marks
Q7 (a)	$y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$ $\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$ $\frac{dy}{dx} = \left\{ \frac{\sin x}{\cos^2 x} \right\} = \left(\frac{1}{\cos x} \right) \left(\frac{\sin x}{\cos x} \right) = \underline{\underline{\sec x \tan x}}$	<p>$\frac{dy}{dx} = \pm ((\cos x)^{-2}(\sin x))$ M1</p> <p>$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$ A1</p> <p>Convincing proof. Must see both <u>underlined steps</u>. A1 AG</p> <p>(3)</p>
(b)	$y = e^{2x} \sec 3x$ $\left\{ \begin{array}{l} u = e^{2x} \quad v = \sec 3x \\ \frac{du}{dx} = 2e^{2x} \quad \frac{dv}{dx} = 3 \sec 3x \tan 3x \end{array} \right\}$ $\frac{dy}{dx} = 2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan 3x$	<p>Seen or implied</p> <p>Either $e^{2x} \rightarrow 2e^{2x}$ or $\sec 3x \rightarrow 3 \sec 3x \tan 3x$ M1</p> <p>Both $e^{2x} \rightarrow 2e^{2x}$ and $\sec 3x \rightarrow 3 \sec 3x \tan 3x$ A1</p> <p>Applies $vu' + uv'$ correctly for their u, u', v, v' M1</p> <p>$2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan 3x$ A1 isw</p> <p>(4)</p>
(c)	<p>Turning point $\Rightarrow \frac{dy}{dx} = 0$</p> <p>Hence, $e^{2x} \sec 3x (2 + 3 \tan 3x) = 0$</p> <p>{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3 \tan 3x = 0$, }</p> <p>giving $\tan 3x = -\frac{2}{3}$</p> <p>$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600\dots$</p> <p>Hence, $y = \{b\} = e^{2(-0.196)} \sec(3 \times -0.196)$</p> <p>$= 0.812093\dots = 0.812$ (3sf)</p>	<p>Sets their $\frac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms. M1</p> <p>$\tan 3x = \pm k$; $k \neq 0$ M1</p> <p>Either awrt -0.196° or awrt -11.2° A1</p> <p>0.812 A1 cao</p> <p>(4)</p>

[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b), (for these values of x) then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

Question Number	Scheme	Marks
Q8	<p>$\operatorname{cosec}^2 2x - \cot 2x = 1, \text{ (eqn *) } 0 \leq x \leq 180^\circ$</p> <p>Using $\operatorname{cosec}^2 2x = 1 + \cot^2 2x$ gives</p> <p>$1 + \cot^2 2x - \cot 2x = 1$</p> <p>$\cot^2 2x - \cot 2x = 0 \quad \text{or} \quad \cot^2 2x = \cot 2x$</p> <p>$\cot 2x(\cot 2x - 1) = 0 \quad \text{or} \quad \cot 2x = 1$</p> <p>$\cot 2x = 0 \quad \text{or} \quad \cot 2x = 1$</p> <p>$\cot 2x = 0 \Rightarrow (\tan 2x \rightarrow \infty) \Rightarrow 2x = 90, 270$</p> <p>$\Rightarrow x = 45, 135$</p> <p>$\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$</p> <p>$\Rightarrow x = 22.5, 112.5$</p> <p>Overall, $x = \{22.5, 45, 112.5, 135\}$</p>	<p>M1</p> <p>A1</p> <p>dM1</p> <p>A1</p> <p>ddM1</p> <p>A1</p> <p>B1</p> <p>[7]</p>

If there are any EXTRA solutions inside the range $0 \leq x \leq 180^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question). Also ignore EXTRA solutions outside the range $0 \leq x \leq 180^\circ$.

Question Number	Scheme	Marks
Q9 (i)(a)	$\ln(3x - 7) = 5$ $e^{\ln(3x - 7)} = e^5$ $3x - 7 = e^5 \Rightarrow x = \frac{e^5 + 7}{3} \{= 51.804...\}$	<p>Takes e of both sides of the equation. This can be implied by $3x - 7 = e^5$.</p> <p>Then rearranges to make x the subject.</p> <p>Exact answer of $\frac{e^5 + 7}{3}$.</p> <p>M1 dM1 A1 (3)</p>
(b)	$3^x e^{7x+2} = 15$ $\ln(3^x e^{7x+2}) = \ln 15$ $\ln 3^x + \ln e^{7x+2} = \ln 15$ $x \ln 3 + 7x + 2 = \ln 15$ $x(\ln 3 + 7) = -2 + \ln 15$ $x = \frac{-2 + \ln 15}{7 + \ln 3} \{= 0.0874...\}$	<p>Takes ln (or logs) of both sides of the equation.</p> <p>Applies the addition law of logarithms.</p> <p>$x \ln 3 + 7x + 2 = \ln 15$</p> <p>Factorising out at least two x terms on one side and collecting number terms on the other side.</p> <p>Exact answer of $\frac{-2 + \ln 15}{7 + \ln 3}$</p> <p>M1 M1 A1 oe ddM1 A1 oe (5)</p>
(ii) (a)	$f(x) = e^{2x} + 3, x \in \mathbb{R}$ $y = e^{2x} + 3 \Rightarrow y - 3 = e^{2x}$ $\Rightarrow \ln(y - 3) = 2x$ $\Rightarrow \frac{1}{2} \ln(y - 3) = x$ Hence $f^{-1}(x) = \frac{1}{2} \ln(x - 3)$ $f^{-1}(x)$: Domain: $x > 3$ or $(3, \infty)$	<p>Attempt to make x (or swapped y) the subject Makes e^{2x} the subject and takes ln of both sides</p> <p>$\frac{1}{2} \ln(x - 3)$ or $\ln \sqrt{x - 3}$ or $f^{-1}(y) = \frac{1}{2} \ln(y - 3)$ (see appendix) Either $x > 3$ or $(3, \infty)$ or <u>Domain</u> > 3.</p> <p>M1 M1 A1 cao B1 (4)</p>
(b)	$g(x) = \ln(x - 1), x \in \mathbb{R}, x > 1$ $fg(x) = e^{2 \ln(x-1)} + 3 \{= (x - 1)^2 + 3\}$ $fg(x)$: Range: $y > 3$ or $(3, \infty)$	<p>An attempt to put function g into function f. $e^{2 \ln(x-1)} + 3$ or $(x - 1)^2 + 3$ or $x^2 - 2x + 4$.</p> <p>Either $y > 3$ or $(3, \infty)$ or <u>Range</u> > 3 or <u>fg(x)</u> > 3.</p> <p>M1 A1 isw B1 (3)</p>
		[15]

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481

Email publications@linneydirect.com

Order Code US022710 January 2010

For more information on Edexcel qualifications, please visit www.edexcel.com/quals

Edexcel Limited. Registered in England and Wales no.4496750
Registered Office: One90 High Holborn, London, WC1V 7BH