Mark Scheme (Final)

Summer 2007

GCE

GCE Mathematics (6665/01)

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June 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1. (*a*)	$\ln 3x = \ln 6$ or $\ln x = \ln\left(\frac{6}{3}\right)$ or $\ln\left(\frac{3x}{6}\right) = 0$	M1
	$x = 2$ (only this answer)	A1 (cso) (2)
(*b*)	$(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form)	M1
	$(e^x - 3)(e^x - 1) = 0$	
	$e^x = 3$ or $e^x = 1$ Solving quadratic	M1 dep
	$x = \ln 3$, $x = 0$ (or ln 1)	M1 A1 (4)
		(6 marks)

Notes: (a) Answer $x = 2$ with no working or no incorrect working seen: M1A1

Note: $x = 2$ from $\ln x = \frac{\ln 6}{\ln 3} = \ln 2$ M0A0

$\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6 - \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1

(b) 1st M1 for attempting to multiply through by e^x : Allow y, X, even x, for e^x

2nd M1 is for solving quadratic as far as getting two values for e^x or y or X etc

3rd M1 is for converting their answer(s) of the form $e^x = k$ to $x = \ln k$ (must be exact)

A1 is for ln3 **and** ln1 or 0 (Both required and no further solutions)

2. (*a*)	$2x^2 + 3x - 2 = (2x - 1)(x + 2)$ at any stage	B1
	$f(x) = \dfrac{(2x+3)(2x-1)-(9+2x)}{(2x-1)(x+2)}$ f.t. on error in denominator factors (need not be single fraction)	M1, A1√
	Simplifying numerator to quadratic form	M1
	Correct **numerator** $= \dfrac{4x^2+2x-12}{[(2x-1)(x+2)]}$	A1
	Factorising numerator, **with** a denominator $= \dfrac{2(2x-3)(x+2)}{(2x-1)(x+2)}$ o.e.	M1
	$= \dfrac{4x-6}{2x-1}$ (✱)	A1 cso (7)
Alt.(*a*)	$2x^2 + 3x - 2 = (2x - 1)(x + 2)$ at any stage B1 $f(x) = \dfrac{(2x+3)(2x^2+3x-2)-(9+2x)(x+2)}{(x+2)(2x^2+3x-2)}$ M1A1 f.t. $= \dfrac{4x^3+10x^2-8x-24}{(x+2)(2x^2+3x-2)}$ $= \dfrac{2(x+2)(2x^2+x-6)}{(x+2)(2x^2+3x-2)}$ or $\dfrac{2(2x-3)(x^2+4x+4)}{(x+2)(2x^2+3x+2)}$ o.e. Any one linear factor × quadratic factor in **numerator** M1, A1 $= \dfrac{2(x+2)(x+2)(2x-3)}{(x+2)(2x^2+3x-2)}$ o.e. M1 $= \dfrac{2(2x-3)}{2x-1}$ $\dfrac{4x-6}{2x-1}$ (✱) A1	
(*b*)	Complete method for $f'(x)$; e.g $f'(x) = \dfrac{(2x-1)\times 4-(4x-6)\times 2}{(2x-1)^2}$ o.e	M1 A1
	$= \dfrac{8}{(2x-1)^2}$ or $8(2x-1)^{-2}$	A1 (3)
	Not treating f^{-1} (for f') as misread	**(10 marks)**

Notes: (a) 1[st] M1 in either version is for correct method

1[st] A1 Allow $\dfrac{2x+3(2x-1)-(9+2x)}{(2x-1)(x+2)}$ or $\dfrac{(2x+3)(2x-1)-9+2x}{(2x-1)(x+2)}$ or $\dfrac{2x+3(2x-1)-9+2x}{(2x-1)(x+2)}$ (fractions)

2[nd] M1 in (main a) is for forming 3 term quadratic in **numerator**

3[rd] M1 is for factorising their quadratic (usual rules) ; factor of 2 need not be extracted

(✱) A1 is given answer so is cso

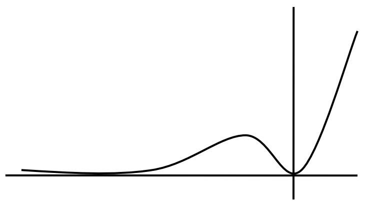
Alt :(a) 3[rd] M1 is for factorising resulting quadratic

(b) SC: For M allow ± given expression or one error in product rule

Alt: Attempt at $f(x) = 2 - 4(2x-1)^{-1}$ and diff. M1; $k(2x-1)^{-2}$ A1; A1 as above

Accept $8(4x^2 - 4x + 1)^{-1}$.

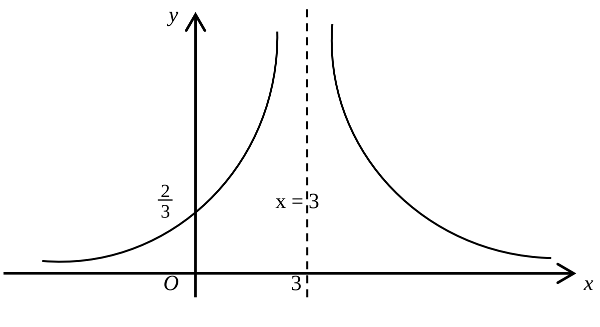
Differentiating original function – mark as scheme.

Question Number	Scheme	Marks
3. (*a*)	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2\mathrm{e}^x + 2x\mathrm{e}^x$	M1,A1,A1 (3)
(*b*)	If $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $\mathrm{e}^x(x^2 + 2x) = 0$ setting (*a*) = 0 [$\mathrm{e}^x \neq 0$] $x(x + 2) = 0$ $(x = 0)$ $x = -2$ $x = 0, y = 0$ **and** $x = -2, y = 4\mathrm{e}^{-2}$ (= 0.54…)	M1 A1 A1 √ (3)
(*c*)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x^2\mathrm{e}^x + 2x\mathrm{e}^x + 2x\mathrm{e}^x + 2\mathrm{e}^x$ $\left[= (x^2 + 4x + 2)\mathrm{e}^x\right]$	M1, A1 (2)
(*d*) Alt.(*d*)	$x = 0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ (=2) $x = -2$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$ [$= -2\mathrm{e}^{-2}$ (= –0.270…)] M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b) $\therefore$ minimum $\therefore$ maximum For M1: Evaluate, or state sign of, $\frac{\mathrm{d}y}{\mathrm{d}x}$ at two appropriate values – on either side of at least one of their answers from (b) or Evaluate y at two appropriate values – on either side of at least one of their answers from (b) or Sketch curve 	M1 A1 (cso) (2) **(10 marks)**

Notes: (a) M for attempt at $f(x)g'(x) + f'(x)g(x)$
1st A1 for one correct, 2nd A1 for the other correct.
Note that x^2e^x on its own scores no marks
(b) 1st A1 ($x = 0$) may be omitted, but for
2nd A1 both sets of coordinates needed ; f.t only on candidate's $x = -2$

(c) M1 requires complete method for candidate's (a), result may be unsimplified for A1

(d) A1 is cso; $x = 0$, min, and $x = -2$, max and no incorrect working seen,
or (in alternative) sign of $\frac{dy}{dx}$ either side correct, or values of y appropriate to t.p.
Need only consider the quadratic, as may assume $\mathrm{e}^x > 0$.
If all marks gained in (a) and (c), and correct x values, give M1A1 for correct statements with no working

Question Number	Scheme	Marks
4. (*a*)	$x^2(3-x)-1=0$ o.e. (e.g. $x^2(-x+3)=1$)	M1
	$x=\sqrt{\dfrac{1}{3-x}}$ (✱)	A1 (cso) (2)
	Note(✱), answer is given: need to see appropriate working and A1 is cso [Reverse process: Squaring and non-fractional equation M1, form f(x) A1]	
(*b*)	$x_2=0.6455$, $x_3=0.6517$, $x_4=0.6526$	B1; B1 (2)
	1st B1 is for one correct, 2nd B1 for other two correct If all three are to greater accuracy, award B0 B1	
(*c*)	Choose values in interval (0.6525, 0.6535) or tighter and evaluate both	M1
	f(0.6525) = –0.0005 (372… f(0.6535) = 0.002 (101…	
	At least one correct "up to bracket", i.e. -0.0005 or 0.002	A1
	Change of sign, $\therefore$ $x=0.653$ is a root (correct) to 3 d.p.	A1 (3)
	Requires both correct "up to bracket" and conclusion as above	
		(7 marks)
Alt (i)	Continued iterations at least as far as x_6 M1	
	$x_5=0.65268$, $x_6=0.6527$, $x_7=$ … two correct to at least 4 s.f. A1	
	Conclusion : Two values correct to 4 d.p., so 0.653 is root to 3 d.p. A1	
Alt (ii)	If use g(0.6525) = 0.6527..>0.6525 and g(0.6535) = 0.6528..<0.6535 M1A1	
	Conclusion : Both results correct, so 0.653 is root to 3 d.p. A1	
5. (*a*)	Finding g(4) = k and f(k) = …. or fg(x) = $\ln\left(\dfrac{4}{x-3}-1\right)$	M1
	[f(2) = ln(2x2 – 1) fg(4) = ln(4 – 1)] $=\ln 3$	A1 (2)
(*b*)	$y=\ln(2x-1)$ $\Rightarrow$ $e^y=2x-1$ or $e^x=2y-1$	M1, A1
	$f^{-1}(x)=\frac{1}{2}(e^x+1)$ Allow $y=\frac{1}{2}(e^x+1)$	A1
	Domain $x\in\Re$ [Allow $\Re$, all reals, $(-\infty,\infty)$] independent	B1 (4)
(*c*)	 Shape, and x-axis should appear to be asymptote	B1
	Equation $x = 3$ needed, may see in diagram (ignore others)	B1 ind.
	Intercept (0, $\frac{2}{3}$) no other; accept y = ⅔ (0.67) or on graph	B1 ind (3)
(*d*)	$\dfrac{2}{x-3}=3$ $\Rightarrow$ $x=3\frac{2}{3}$ or exact equiv.	B1
	$\dfrac{2}{x-3}=-3$, $\Rightarrow$ $x=2\frac{1}{3}$ or exact equiv.	M1, A1 (3)
	Note: $2=3(x+3)$ or $2=3(-x-3)$ o.e. is M0A0	
Alt:	Squaring to quadratic ($9x^2-54x+77=0$) **and** solving M1; B1A1	**(12 marks)**

6. (*a*)	Complete method for R: e.g. $R\cos\alpha = 3$, $R\sin\alpha = 2$, $R=\sqrt{(3^2+2^2)}$	M1
	$R = \sqrt{13}$ or 3.61 (or more accurate)	A1
	Complete method for $\tan\alpha = \frac{2}{3}$ [Allow $\tan\alpha = \frac{3}{2}$]	M1
	$\alpha = 0.588$ (Allow 33.7°)	A1 (4)
(*b*)	Greatest value $= \left(\sqrt{13}\right)^4 = 169$	M1, A1 (2)
(*c*)	$\sin(x+0.588) = \frac{1}{\sqrt{13}}$ (= 0.27735…) $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$	M1
	$(x + 0.588)$ = 0.281(03…) or 16.1°	A1
	$(x + 0.588)$ = π – 0.28103… Must be π – their 0.281 or 180° – their 16.1°	M1
	or $(x + 0.588)$ = 2π + 0.28103… Must be 2π + their 0.281 or 360° + their 16.1°	M1
	x = 2.273 or x = 5.976 (awrt) Both (radians only)	A1 (5)
	If 0.281 or 16.1° not seen, correct answers imply this A mark	**(11 marks)**

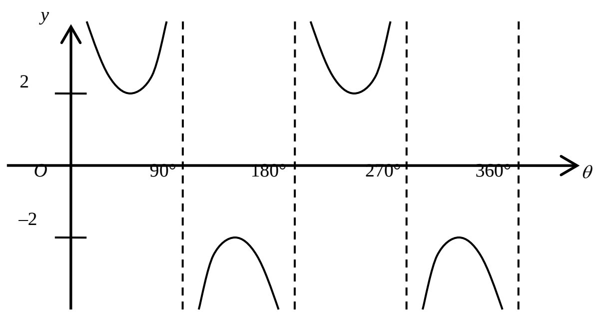
Notes: (a) 1st M1 for correct method for R
2nd M1 for correct method for $\tan\alpha$
No working at all: M1A1 for √13, M1A1 for 0.588 or 33.7°.
N.B. Rcos α = 2, Rsin α = 3 used, can still score M1A1 for R, but loses the A mark for α.
cosα = 3, sin α = 2: apply the same marking.

(b) M1 for realising $\sin(x + \alpha) = \pm1$, so finding R^4.

(c) Working in mixed degrees/rads : first two marks available
Working consistently in degrees: Possible to score first 4 marks
[Degree answers, just for reference only, are 130.2° and 342.4°]
Third M1 can be gained for candidate's 0.281 – candidate's 0.588 + 2π or equiv. in degrees
One of the answers correct in radians or degrees implies the corresponding M mark.

Alt: (c) (i) Squaring to form quadratic in sin x or cos x M1
$[13\cos^2 x - 4\cos x - 8 = 0,\quad 13\sin^2 x - 6\sin x - 3 = 0]$
Correct values for cos x = 0.953… , –0.646; or sin x = 0.767, 2.27 awrt A1
For any one value of cos x or sin$x$, correct method for two values of x M1
x = 2.273 or x = 5.976 (awrt) Both seen anywhere A1
Checking other values (0.307, 4.011 or 0.869, 3.449) and discarding M1

(ii) Squaring and forming equation of form $a\cos 2x + b\sin 2x = c$
$9\sin^2 x + 4\cos^2 x + 12\sin 2x = 1 \Rightarrow 12\sin 2x + 5\cos 2x = 11$
Setting up to solve using R formula e.g. $\sqrt{13}\cos(2x - 1.176) = 11$ M1
$$(2x - 1.176) = \cos^{-1}\left(\frac{11}{\sqrt{13}}\right) = 0.562(0... \qquad (\alpha)$$ A1
$$(2x - 1.176) = 2\pi - \alpha,\ 2\pi + \alpha, ...$$ M1
x = 2.273 or x = 5.976 (awrt) Both seen anywhere A1
Checking other values and discarding M1

Question Number	Scheme	Marks
7. (*a*)	$\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$ M1 Use of common denominator to obtain single fraction $=\dfrac{1}{\cos\theta\sin\theta}$ M1 Use of appropriate trig identity (in this case $\sin^2\theta+\cos^2\theta=1$) $=\dfrac{1}{\frac{1}{2}\sin 2\theta}$ Use of $\sin 2\theta=2\sin\theta\cos\theta$ $=2\text{cosec}2\theta$ (✱)	M1 M1 M1 A1 cso (4)
Alt.(*a*)	$\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\tan\theta+\dfrac{1}{\tan\theta}=\dfrac{\tan^2\theta+1}{\tan\theta}$ M1 $=\dfrac{\sec^2\theta}{\tan\theta}$ M1 $=\dfrac{1}{\cos\theta\sin\theta}=\dfrac{1}{\frac{1}{2}\sin 2\theta}$ M1 $=2\text{cosec}2\theta$ (✱) (cso) A1 If show two expressions are equal, need conclusion such as QED, tick, true.	
(*b*)	 Shape (May be translated but need to see 4"sections") T.P.s at $y=\pm 2$, asymptotic at correct x-values (dotted lines not required)	B1 B1 dep. (2)
(*c*)	$2\text{cosec}2\theta=3$ $\sin 2\theta=\dfrac{2}{3}$ Allow $\dfrac{2}{\sin 2\theta}=3$ [M1 for equation in $\sin 2\theta$] (2θ) = [41.810…°, 138.189…° ; 401.810…°, 498.189…°] 1st M1 for α, $180-\alpha$; 2nd M1 adding 360° to at least one of values θ = 20.9°, 69.1°, 200.9°, 249.1° (1 d.p.) awrt	M1, A1 M1; M1
Note	1st A1 for any two correct, 2nd A1 for other two Extra solutions in range lose final A1 only SC: Final 4 marks: θ = 20.9°, after M0M0 is B1; record as M0M0A1A0	A1,A1 (6)
Alt.(*c*)	$\tan\theta+\dfrac{1}{\tan\theta}=3$ and form quadratic , $\tan^2\theta-3\tan\theta+1=0$ M1, A1 (M1 for attempt to multiply through by tanθ, A1 for correct equation above) Solving quadratic [$\tan\theta=\dfrac{3\pm\sqrt{5}}{2}$ = 2.618… or = 0.3819…] M1 θ = 69.1°, 249.1° θ = 20.9°, 200.9° (1 d.p.) M1, A1, A1 (M1 is for one use of 180° + α °, A1A1 as for main scheme)	**(12 marks)**

Question Number	Scheme	Marks
8. (*a*)	$D = 10,\ t = 5,\quad x = 10e^{-\frac{1}{8}\times 5}$ $= 5.353$ awrt	M1 A1 (2)
(*b*)	$D = 10 + 10e^{-\frac{5}{8}},\ t = 1,\quad x = 15.3526\ldots \times e^{-\frac{1}{8}}$ $x = 13.549$ (✱)	M1 A1 cso (2)
Alt.(*b*)	$x = 10e^{-\frac{1}{8}\times 6} + 10e^{-\frac{1}{8}\times 1}$ M1 $x = 13.549$ (✱) A1 cso	
(*c*)	$15.3526\ldots e^{-\frac{1}{8}T} = 3$ $e^{-\frac{1}{8}T} = \dfrac{3}{15.3526\ldots} = 0.1954\ldots$ $-\dfrac{1}{8}T = \ln 0.1954\ldots$ $T = 13.06\ldots$ or 13.1 or 13	M1 M1 A1 (3) **(7 marks)**

Notes: (b) (main scheme) M1 is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{1}{8}}$, or $\{10 + \text{their(a)}\}\, e^{-\frac{1}{8}}$

N.B. The answer is given. There are many correct answers seen which deserve M0A0 or M1A0

(c) 1st M is for $(10 + 10e^{-\frac{5}{8}})\, e^{-\frac{T}{8}} = 3$ o.e.

2nd M is for converting $e^{-\frac{T}{8}} = k$ (k > 0) to $-\dfrac{T}{8} = \ln k$. This is independent of 1st M.

Trial and improvement: M1 as scheme,
M1 correct process for their equation (two equal to 3 s.f.)
A1 as scheme