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Mark Scheme (Results)
Summer 2008

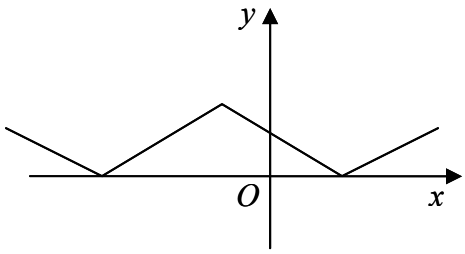
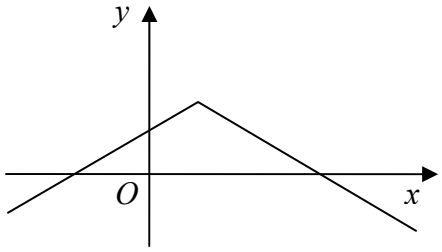
GCE

GCE Mathematics (6665/01)

June 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $e^{2x+1} = 2$	
	$2x+1 = \ln 2$	M1
	$x = \frac{1}{2}(\ln 2 - 1)$	A1 (2)
	(b) $\frac{dy}{dx} = 8e^{2x+1}$	B1
	$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$	B1
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$	M1
	$y = 16x + 16 - 8\ln 2$	A1 (4)
		[6]

Question Number	Scheme	Marks
2.	(a) $R^2 = 5^2 + 12^2$	M1
	$R = 13$	A1
	$\tan\alpha = \frac{12}{5}$	M1
	$\alpha \approx 1.176$ cao	A1 (4)
	(b) $\cos(x-\alpha) = \frac{6}{13}$	M1
	$x - \alpha = \arccos\frac{6}{13} = 1.091\ \ldots$	A1
	$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ \ldots$ awrt 2.3	A1
	$x - \alpha = -1.091\ \ldots$ accept … = 5.19 … for M	M1
	$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ \ldots$ awrt 0.084 or 0.085	A1 (5)
	(c)(i) $R_{\max} = 13$ ft their R	B1 ft
	(ii) At the maximum, $\cos(x-\alpha) = 1$ *or* $x - \alpha = 0$	M1
	$x = \alpha = 1.176\ \ldots$ awrt 1.2, ft their α	A1ft (3)
		[12]

Question Number	Scheme	Marks
3.	(a)  shape Vertices correctly placed	B1 B1 (2)
	(b)  shape Vertex and intersections with axes correctly placed	B1 B1 (2)
	(c) $P:(-1, 2)$ $Q:(0, 1)$ $R:(1, 0)$	B1 B1 B1 (3)
	(d) $x > -1$; $2 - x - 1 = \frac{1}{2}x$ Leading to $x = \frac{2}{3}$ $x < -1$; $2 + x + 1 = \frac{1}{2}x$ Leading to $x = -6$	M1 A1 A1 M1 A1 (5) **[12]**

Question Number	Scheme	Marks
4.	(a) $x^2-2x-3=(x-3)(x+1)$	B1
	$\mathrm{f}(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)}\right)$	M1 A1
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ ∗ cso	A1 (4)
	(b) $\left(0,\dfrac{1}{4}\right)$ Accept $0<y<\dfrac{1}{4}$, $0<\mathrm{f}(x)<\dfrac{1}{4}$ etc.	B1 B1 (2)
	(c) Let $y=\mathrm{f}(x)$ $y=\dfrac{1}{x+1}$	
	$x=\dfrac{1}{y+1}$	
	$yx+x=1$	
	$y=\dfrac{1-x}{x}$ or $\dfrac{1}{x}-1$	M1 A1
	$\mathrm{f}^{-1}(x)=\dfrac{1-x}{x}$	
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$ ft their part (b)	B1 ft (3)
	(d) $\mathrm{fg}(x)=\dfrac{1}{2x^2-3+1}$	
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$	M1
	$x^2=5$	A1
	$x=\pm\sqrt{5}$ both	A1 (3)
		[12]

Question Number	Scheme	Marks
5.	(a) $\sin^2\theta+\cos^2\theta=1$	
	$\div\sin^2\theta$ $\quad \frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$	M1
	$1+\cot^2\theta=\operatorname{cosec}^2\theta$ ✱ cso	A1 (2)
	Alternative for (a)	
	$1+\cot^2\theta=1+\frac{\cos^2\theta}{\sin^2\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$	M1
	$=\operatorname{cosec}^2\theta$ ✱ cso	A1
	(b) $2\left(\operatorname{cosec}^2\theta-1\right)-9\operatorname{cosec}\theta=3$	M1
	$2\operatorname{cosec}^2\theta-9\operatorname{cosec}\theta-5=0$ *or* $5\sin^2\theta+9\sin\theta-2=0$	M1
	$\left(2\operatorname{cosec}\theta+1\right)\left(\operatorname{cosec}\theta-5\right)=0$ *or* $\left(5\sin\theta-1\right)\left(\sin\theta+2\right)=0$	M1
	$\operatorname{cosec}\theta=5$ *or* $\sin\theta=\frac{1}{5}$	A1
	$\theta=11.5°, 168.5°$	A1 A1 (6)
		[8]

Question Number	Scheme	Marks
6.	(a)(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}\left(\sin x+2\cos x\right)\right)=3\mathrm{e}^{3x}\left(\sin x+2\cos x\right)+\mathrm{e}^{3x}\left(\cos x-2\sin x\right)$	M1 A1 A1 (3)
	$\left(=\mathrm{e}^{3x}\left(\sin x+7\cos x\right)\right)$	
	(ii) $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3\ln\left(5x+2\right)\right)=3x^2\ln\left(5x+2\right)+\frac{5x^3}{5x+2}$	M1 A1 A1 (3)
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(x+1\right)^2\left(6x+6\right)-2\left(x+1\right)\left(3x^2+6x-7\right)}{\left(x+1\right)^4}$	M1 $\frac{\text{A1}}{\text{A1}}$
	$=\frac{\left(x+1\right)\left(6x^2+12x+6-6x^2-12x+14\right)}{\left(x+1\right)^4}$	M1
	$=\frac{20}{\left(x+1\right)^3}$ * cso	A1 (5)
	(c) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{60}{\left(x+1\right)^4}=-\frac{15}{4}$	M1
	$\left(x+1\right)^4=16$	M1
	$x=1,-3$ both	A1 (3)
		[14]
	Note: The simplification in part (b) can be carried out as follows	
	$\frac{\left(x+1\right)^2\left(6x+6\right)-2\left(x+1\right)\left(3x^2+6x-7\right)}{\left(x+1\right)^4}$	
	$=\frac{\left(6x^3+18x^2+18x+6\right)-\left(6x^3+18x^2-2x-14\right)}{\left(x+1\right)^4}$	
	$=\frac{20x+20}{\left(x+1\right)^4}=\frac{20\left(x+1\right)}{\left(x+1\right)^4}=\frac{20}{\left(x+1\right)^3}$	M1 A1

Question Number	Scheme	Marks
7.	(a) $f(1.4) = -0.568 \ldots < 0$ $f(1.45) = 0.245 \ldots > 0$ Change of sign (and continuity) $\Rightarrow \alpha \in (1.4, 1.45)$	M1 A1 (2)
	(b) $3x^3 = 2x + 6$ $x^3 = \frac{2x}{3} + 2$ $x^2 = \frac{2}{3} + \frac{2}{x}$ $x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$ * cso	M1 A1 A1 (3)
	(c) $x_1 = 1.4371$ $x_2 = 1.4347$ $x_3 = 1.4355$	B1 B1 B1 (3)
	(d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval. $f(1.4345) = -0.01 \ldots$ $f(1.4355) = 0.003 \ldots$ Change of sign (and continuity) $\Rightarrow \alpha \in (1.4345, 1.4355)$ $\Rightarrow \alpha = 1.435$, correct to 3 decimal places * cso *Note*: $\alpha = 1.435\,304\,553 \ldots$	M1 M1 A1 (3) **[11]**