

Mark Scheme (Results)

Summer 2008

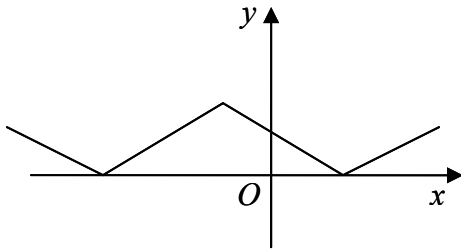
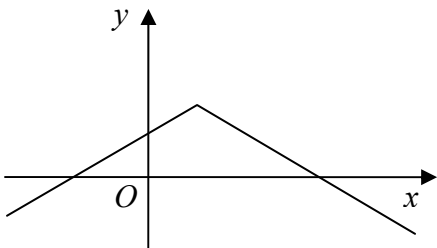
GCE

GCE Mathematics (6665/01)

June 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $e^{2x+1} = 2$ $2x+1 = \ln 2$ $x = \frac{1}{2}(\ln 2 - 1)$	M1 A1 (2)
	(b) $\frac{dy}{dx} = 8e^{2x+1}$ $x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$ $y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$ $y = 16x + 16 - 8\ln 2$	B1 B1 M1 A1 (4) [6]

Question Number	Scheme	Marks
2.	(a) $R^2 = 5^2 + 12^2$ $R = 13$ $\tan \alpha = \frac{12}{5}$ $\alpha \approx 1.176$	M1 A1 M1 A1 (4)
	(b) $\cos(x - \alpha) = \frac{6}{13}$ $x - \alpha = \arccos \frac{6}{13} = 1.091 \dots$ $x = 1.091 \dots + 1.176 \dots \approx 2.267 \dots$	M1 A1 A1
	$x - \alpha = -1.091 \dots$ $x = -1.091 \dots + 1.176 \dots \approx 0.0849 \dots$	accept ... = 5.19 ... for M A1 (5)
	(c)(i) $R_{\max} = 13$	fit their R B1 ft
	(ii) At the maximum, $\cos(x - \alpha) = 1$ or $x - \alpha = 0$ $x = \alpha = 1.176 \dots$	awrt 1.2, fit their α A1ft (3) [12]

Question Number	Scheme	Marks
3.	<p>(a) </p> <p style="text-align: right;">W shape Vertices correctly placed</p> <p>(b) </p> <p style="text-align: right;">M shape Vertex and intersections with axes correctly placed</p> <p>(c) $P: (-1, 2)$ $Q: (0, 1)$ $R: (1, 0)$</p> <p>(d) $x > -1; \quad 2 - x - 1 = \frac{1}{2}x$ Leading to $x = \frac{2}{3}$ $x < -1; \quad 2 + x + 1 = \frac{1}{2}x$ Leading to $x = -6$</p>	<p>B1 B1 (2)</p> <p>B1 B1 (2)</p> <p>B1 B1 B1 (3)</p> <p>M1 A1 A1 M1 A1 (5) [12]</p>

Question Number	Scheme	Marks
<p>4.</p>	<p>(a) $x^2 - 2x - 3 = (x - 3)(x + 1)$</p> $f(x) = \frac{2(x-1) - (x+1)}{(x-3)(x+1)} \left(\text{or } \frac{2(x-1)}{(x-3)(x+1)} - \frac{x+1}{(x-3)(x+1)} \right)$ $= \frac{x-3}{(x-3)(x+1)} = \frac{1}{x+1} *$	<p>B1</p> <p>M1 A1</p> <p>cso A1 (4)</p>
	<p>(b) $\left(0, \frac{1}{4}\right)$ Accept $0 < y < \frac{1}{4}$, $0 < f(x) < \frac{1}{4}$ etc.</p>	<p>B1 B1 (2)</p>
	<p>(c) Let $y = f(x)$</p> $y = \frac{1}{x+1}$ $x = \frac{1}{y+1}$ $yx + x = 1$ $y = \frac{1-x}{x}$ $f^{-1}(x) = \frac{1-x}{x}$	<p>or $\frac{1}{x} - 1$</p> <p>M1 A1</p>
	<p>Domain of f^{-1} is $\left(0, \frac{1}{4}\right)$</p> <p>(d) $fg(x) = \frac{1}{2x^2 - 3 + 1}$</p> $\frac{1}{2x^2 - 2} = \frac{1}{8}$ $x^2 = 5$ $x = \pm\sqrt{5}$	<p>ft their part (b)</p> <p>B1 ft (3)</p> <p>M1</p> <p>A1</p> <p>both A1 (3)</p> <p>[12]</p>

Question Number	Scheme	Marks
5.	<p>(a) $\sin^2 \theta + \cos^2 \theta = 1$ $\div \sin^2 \theta$ $\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$ $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$ *</p> <p><i>Alternative for (a)</i> $1 + \cot^2 \theta = 1 + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$ $= \operatorname{cosec}^2 \theta$ *</p> <p>(b) $2(\operatorname{cosec}^2 \theta - 1) - 9 \operatorname{cosec} \theta = 3$ $2 \operatorname{cosec}^2 \theta - 9 \operatorname{cosec} \theta - 5 = 0$ or $5 \sin^2 \theta + 9 \sin \theta - 2 = 0$ $(2 \operatorname{cosec} \theta + 1)(\operatorname{cosec} \theta - 5) = 0$ or $(5 \sin \theta - 1)(\sin \theta + 2) = 0$ $\operatorname{cosec} \theta = 5$ or $\sin \theta = \frac{1}{5}$ $\theta = 11.5^\circ, 168.5^\circ$</p>	<p>M1 A1 (2) cso M1 A1 cso M1 M1 M1 A1 A1 A1 (6) [8]</p>

Question Number	Scheme	Marks
6.	(a)(i) $\frac{d}{dx}(e^{3x}(\sin x + 2 \cos x)) = 3e^{3x}(\sin x + 2 \cos x) + e^{3x}(\cos x - 2 \sin x)$ $(= e^{3x}(\sin x + 7 \cos x))$	M1 A1 A1 (3)
	(ii) $\frac{d}{dx}(x^3 \ln(5x+2)) = 3x^2 \ln(5x+2) + \frac{5x^3}{5x+2}$	M1 A1 A1 (3)
	(b) $\frac{dy}{dx} = \frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$ $= \frac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$ $= \frac{20}{(x+1)^3} *$	M1 $\frac{A1}{A1}$ M1 cso A1 (5)
	(c) $\frac{d^2y}{dx^2} = -\frac{60}{(x+1)^4} = -\frac{15}{4}$ $(x+1)^4 = 16$ $x = 1, -3$	M1 M1 both A1 (3)
	<p><i>Note:</i> The simplification in part (b) can be carried out as follows</p> $\frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$ $= \frac{(6x^3+18x^2+18x+6) - (6x^3+18x^2-2x-14)}{(x+1)^4}$ $= \frac{20x+20}{(x+1)^4} = \frac{20(x+1)}{(x+1)^4} = \frac{20}{(x+1)^3}$	[14] M1 A1

Question Number	Scheme	Marks
7.	(a) $f(1.4) = -0.568 \dots < 0$ $f(1.45) = 0.245 \dots > 0$ Change of sign (and continuity) $\Rightarrow \alpha \in (1.4, 1.45)$	M1 A1 (2)
	(b) $3x^3 = 2x + 6$ $x^3 = \frac{2x}{3} + 2$ $x^2 = \frac{2}{3} + \frac{2}{x}$ $x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$ *	M1 A1 A1 (3)
	(c) $x_1 = 1.4371$ $x_2 = 1.4347$ $x_3 = 1.4355$	B1 B1 B1 (3)
	(d) Choosing the interval (1.4345, 1.4355) or appropriate tighter interval. $f(1.4345) = -0.01 \dots$ $f(1.4355) = 0.003 \dots$ Change of sign (and continuity) $\Rightarrow \alpha \in (1.4345, 1.4355)$ $\Rightarrow \alpha = 1.435$, correct to 3 decimal places * cso	M1 M1 A1 (3) [11]
<i>Note: $\alpha = 1.435\ 304\ 553 \dots$</i>		