

Mark Scheme (Results) Summer 2010

GCE

Core Mathematics C3 (6665)



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June 2010 6665 Core Mathematics C3 Mark Scheme

Question Number	Scheme	Marks	S
1. (a)	$\frac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$	M1	
	$\frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}\cos\theta\cos\theta} = \tan\theta \text{ (as required) AG}$	A1 cso	
			(2)
(b)	$2\tan\theta = 1 \implies \tan\theta = \frac{1}{2}$	M1	
	$\theta_1 = \text{awrt } 26.6^\circ$	A1	
	$\theta_2 = \operatorname{awrt} - 153.4^\circ$	A1√	(2)
			[5]
	(a) M1: Uses both a correct identity for $\sin 2\theta$ and a correct identity for $\cos 2\theta$. Also allow a candidate writing $1 + \cos 2\theta = 2\cos^2 \theta$ on the denominator. Also note that angles must be consistent in when candidates apply these identities. A1: Correct proof. No errors seen.		
	(b) 1^{st} M1 for either $2 \tan \theta = 1$ or $\tan \theta = \frac{1}{2}$, seen or implied.		
	A1: awrt 26.6 A1 $\int x^{-1} = 152.4^{\circ} = 100^{\circ} + 0$		
	Al $$: awrt -153.4 or $\theta_2 = -180 + \theta_1$		
	Special Case : For candidate solving, $\tan \theta = k$, where $k \neq \frac{1}{2}$, to give θ_1 and		
	$\theta_2 = -180^\circ + \theta_1$, then award M0A0B1 in part (b). Special Case: Note that those candidates who writes $\tan \theta = 1$ and gives ONLY		
	two answers of 45° and -135° that are inside the range will be awarded SC M0A0B1.		



Question Number	Scheme	Marks
2.	At P , $y = \underline{3}$	B1
	$\frac{dy}{dx} = \frac{3(-2)(5-3x)^{-3}(-3)}{(5-3x)^{3}} \left\{ \text{or } \frac{18}{(5-3x)^{3}} \right\}$	M1 <u>A1</u>
	$\frac{dy}{dx} = \frac{18}{(5-3(2))^3} \left\{ = -18 \right\}$	M1
	$m(\mathbf{N}) = \frac{-1}{-18}$ or $\frac{1}{18}$	M1
	N : $y-3 = \frac{1}{18}(x-2)$	M1
	N: $x - 18y + 52 = 0$	A1
		[7]
	1 st M1: $\pm k (5-3x)^{-3}$ can be implied. See appendix for application of the quotient rule. 2 nd M1: Substituting $x = 2$ into an equation involving their $\frac{dy}{dx}$:	
	3 rd M1: Uses m(N) = $-\frac{1}{\text{their m(T)}}$.	
	4 th M1: $y - y_1 = m(x - 2)$ with 'their NORMAL gradient' or a "changed" tangent	
	gradient and their y_1 . Or uses a complete method to express the equation of the tangent in the form $y = mx + c$ with 'their NORMAL ("changed" numerical) gradient', their y_1 and $x = 2$.	
	Note : To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.	



Question Number	Scheme		Marks	5
3 . (a	f (1.2) = 0.49166551, f (1.3) = -0.048719817 Sign change (and as f (x) is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$	M1A	.1	()
(1) $4\operatorname{cosec} x - 4x + 1 = 0 \implies 4x = 4\operatorname{cosec} x + 1$	M1		(2)
	$\Rightarrow x = \operatorname{cosec} x + \frac{1}{4} \Rightarrow \frac{1}{1} = \frac{1}{\sin x} + \frac{1}{4}$	A1	*	
				(2)
(0	$x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}$	M1		
	$x_1 = 1.303757858, x_2 = 1.286745793$	A1		
	x ₃ - 1.2717++015			(3)
((f (1.2905) = 0.00044566695, f (1.2915) = -0.00475017278 Sign change (and as f (x) is continuous) therefore a root α is such that	M1		
	$\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291 \ (3 \text{ dp})$	A1		
				(2) [9]
	 (a) M1: Attempts to evaluate both f(1.2) and f(1.3) and evaluates at least one of them correctly to awrt (or truncated) 1 sf. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion. (b) M1: Attempt to make 4x or x the subject of the equation. A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial f(x) = 0. 			
	(c) M1: An attempt to substitute $x_0 = 1.25$ into the iterative formula			
	$Eg = \frac{1}{\sin(1.25)} + \frac{1}{4}.$			
	Can be implied by $x_1 = awrt 1.3$ or $x_1 = awrt 46^\circ$.			
	A1: Both x_1 = awrt 1.3038 and x_2 = awrt 1.2867 A1: x_3 = awrt 1.2917			
	 (d) M1: Choose suitable interval for x, e.g. [1.2905, 1.2915] or tighter and at least one attempt to evaluate f(x). A1: both values correct to awrt (or truncated) 1 sf sign change and conclusion 			
	A1. both values confect to awrt (or truncated) 1 si, sign change and conclusion.			



Question Number	Scheme	Marks	
4. (a)	(0, 5) $(0, 5)$ $($	M1A1	
(b)	$\frac{x = 20}{2x - 5} = -(15 + x) \ ; \implies \underline{x = -\frac{10}{3}}$	B1 M1;A1 oe.	(2)
(c)	fg(2) = f(-3) = 2(-3) - 5 ; = -11 = 11	M1;A1	(3)
(d)	g(x) = $x^2 - 4x + 1 = (x - 2)^2 - 4 + 1 = (x - 2)^2 - 3$. Hence $g_{\min} = -3$ Either $g_{\min} = -3$ or $g(x) \ge -3$ or $g(5) = 25 - 20 + 1 = 6$ $-3 \le g(x) \le 6$ or $-3 \le y \le 6$ (a) M1: V or or graph with vertex on the <i>x</i> -axis. A1: $(\frac{5}{2}, \{0\})$ and $(\{0\}, 5)$ seen and the graph appears in both the first and second quadrants. (b) M1: Either $2x - 5 = -(15 + x)$ or $-(2x - 5) = 15 + x$ (c) M1: <i>Full method</i> of inserting g(2) into $f(x) = 2x - 5 $ or for inserting $x = 2$ into $ 2(x^2 - 4x + 1) - 5 $. There must be evidence of the modulus being applied. (d) M1: Full method to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $g_{\min} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find <i>x</i> and insert this value of <i>x</i> back into $f(x)$ in order to find the minimum. B1: For either finding the correct minimum value of g (can be implied by $g(x) \ge -3$ or $g(x) > -3$) or for stating that $g(5) = 6$. A1: $-3 \le g(x) \le 6$ or $-3 \le y \le 6$ or $-3 \le g \le 6$. Note that: $-3 \le x \le 6$ is A0. Note that: $-3 \le f(x) \le 6$ is A0. Note that: $-3 \ge g(x) \ge 6$ is A0. Note that: $g(x) \ge -3$ or $g(x) > -3$ or $x \ge -3$ or $x > -3$ with no working gains M1B1A0. Note that for the final Accuracy Mark: If a candidate writes down $-3 < g(x) < 6$ or $-3 < y < 6$, then award M1B1A0. If, however, a candidate writes down $g(x) \ge -3$ or $g(x) \le 6$, then award A0.	M1 B1 A1	(3)



Question Number	Scheme	Mark	S
5. (a)	Either $y = 2 \operatorname{or}(0, 2)$	B1	
			(1)
(b	When $x = 2$, $y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$	B1	
	$(2x^2 - 5x + 2) = 0 \implies (x - 2)(2x - 1) = 0$	M1	
	Either $x = 2$ (for possibly B1 above) or $x = \frac{1}{2}$.	A1	
			(3)
(c	$\frac{dy}{dx} = (4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	M1A1A1	
	dx		(3)
(d	$(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$	M1	(-)
	$2x^2 - 9x + 7 = 0 \implies (2x - 7)(x - 1) = 0$	M1	
	$x = \frac{7}{2}, 1$	A1	
	When $x = \frac{7}{2}$, $y = 9e^{-\frac{7}{2}}$, when $x = 1$, $y = -e^{-1}$	ddM1A1	
			(5)
			[12]
	(b) If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra solution		
	of $x = 0$, then withhold the final accuracy mark.		
	(c) M1: (their u')e ^{-x} + $(2x^2 - 5x + 2)$ (their v')		
	A1: Any one term correct. A1: Both terms correct.		
	(d) 1 st M1: For setting their $\frac{dy}{dx}$ found in part (c) equal to 0.		
	2^{nd} M1: Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term		
	quadratic or apply the formula to candidate's $ax^2 + bx + c$.		
	See rules for solving a three term quadratic equation on page 1 of this Appendix. 3^{rd} ddM1: An attempt to use at least one <i>x</i> -coordinate on $y = (2x^2 - 5x + 2)e^{-x}$.		
	Note that this method mark is dependent on the award of the two previous method		
	Some candidates write down corresponding <i>y</i> -coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one		
	of the two y-coordinates found is correct to awrt 2 sf.		
	Final A1: Both $\{x = 1\}$, $y = -e^{-1}$ and $\{x = \frac{7}{2}\}$, $y = 9e^{-\frac{7}{2}}$. cao		
	Note that both exact values of <i>y</i> are required.		







Question Number	Scheme	Marks
7. (a)	$R = \sqrt{6.25}$ or 2.5	B1
	$\tan \alpha = \frac{1.5}{2} = \frac{3}{4} \implies \alpha = \text{awrt } 0.6435$	M1A1
	Mar Malua 2.5	(3)
(D) (I)	$\max_{i=1}^{n} \sqrt{2} = 2.5$	B1√
(1)	$\frac{\sin(\theta - 0.0433) - 1}{2} 0 \frac{\theta - \tan \alpha - \frac{1}{2}}{2} \Rightarrow \theta - a \sin 2.21$	<u>M1</u> ;A1 √ (3)
(C)	$H_{\rm Max} = 8.5 \ ({\rm m})$	B1√
	$\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1 \text{ or } \frac{4\pi t}{25} = \text{ their (b) answer } \Rightarrow t = \text{ awrt } 4.41$	M1;A1
		(3)
(d)	$\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7; \Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$	M1;M1
	$\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4) \text{ or awrt } 0.41$	A1
	Either $t = awrt 2.1$ or awrt 6.7	A1
	So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \left\{\pi - 0.411517 \text{ or } 2.730076^{c}\right\}$	ddM1
	Times = $\{14:06, 18:43\}$	A1 (6)
	(a) B1: $R = 2.5$ or $R = \sqrt{6.25}$. For $R = \pm 2.5$, award B0.	[10]
	M1: $\tan \alpha = \pm \frac{1.5}{2}$ or $\tan \alpha = \pm \frac{2}{1.5}$	
	A1: $\alpha = awrt \ 0.6435$	
	(b) B1 $$: 2.5 or follow through the value of <i>R</i> in part (a).	
	M1: For $\sin(\theta - \text{their } \alpha) = 1$	
	A1 $$: awrt 2.21 or $\frac{\pi}{2}$ + their α rounding correctly to 3 sf.	
	(c) B1 $$: 8.5 or 6 + their <i>R</i> found in part (a) as long as the answer is greater than	
	N1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 1 \text{ or } \frac{4\pi t}{25} = \text{their (b) answer}$	
	A1: For $\sin^{-1}(0.4)$ This can be implied by awrt 4.41 or awrt 4.40.	
	(d) M1: $6 + (\text{their } R) \sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 7$, M1:	
	$\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = \frac{1}{\text{their } R}$	
	A1: For $\sin^{-1}(0.4)$. This can be implied by awrt 0.41 or awrt 2.73 or other values for	
	different α 's. Note this mark can be implied by seeing 1.055. A1: Either $t = awrt 2.1$ or $t = awrt 6.7$	
	ddM1: either π – their PV ^c . Note that this mark is dependent upon the two M marks.	
	This mark will usually be awarded for seeing either 2.730 or 3.373 A1: Both $t = 14:06$ and $t = 18:43$ or both 126 (min) and 403 (min) or both 2 hr 6	
	min and 6 hr 43 min.	



Question Number	Scheme	Marks
8. (a)	$\frac{(x+5)(2x-1)}{(x+5)(x-3)} = \frac{(2x-1)}{(x-3)}$	M1 B1 A1 aef (3)
(b)	$\ln\left(\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$	M1
	$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15} = e$	dM1
	$\frac{2x-1}{x-3} = e \implies 3e-1 = x(e-2)$	M1
	$\Rightarrow x = \frac{3e - 1}{e - 2}$	A1 aef cso
		(4) [7]
	(a) M1: An attempt to factorise the numerator. B1: Correct factorisation of denominator to give $(x + 5)(x - 3)$. Can be seen	
	anywhere. (b) M1: Uses a correct law of logarithms to combine at least two terms.	
	This usually is achieved by the subtraction law of logarithms to give $\begin{pmatrix} 2x^2 + 0x - 5 \end{pmatrix}$	
	$\ln\left(\frac{2x + 9x - 5}{x^2 + 2x - 15}\right) = 1.$	
	The product law of logarithms can be used to achieve $\ln (2 + 0 - 5) = \ln (2 + 2 - 15)$	
	$\ln(2x^2 + 9x - 5) = \ln(e(x^2 + 2x - 15)).$ The product and quotient law could also be used to achieve	
	$\ln\left(\frac{2x^2 + 9x - 5}{e(x^2 + 2x - 15)}\right) = 0.$	
	dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e.Note that this mark is dependent on the previous method mark being awarded.M1: Collect <i>x</i> terms together and factorise.Note that this is not a dependent method mark.	
	A1: $\frac{3e-1}{e-2}$ or $\frac{3e^{i}-1}{e^{i}-2}$ or $\frac{1-3e}{2-e}$. aef	
	Note that the answer needs to be in terms of e. The decimal answer is 9.9610559 Note that the solution must be correct in order for you to award this final accuracy mark.	
	Note: See Appendix for an alternative method of long division.	

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