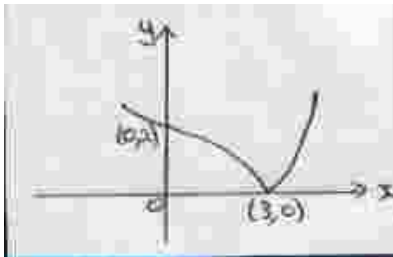
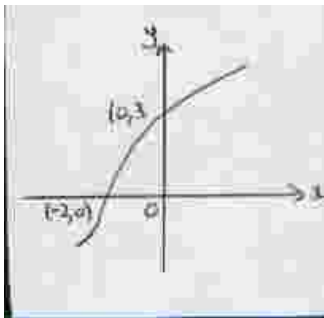
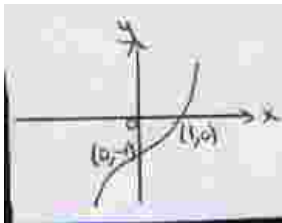


June 2006
6665 Core Mathematics C3
Mark Scheme

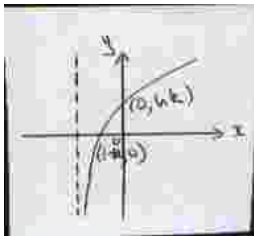
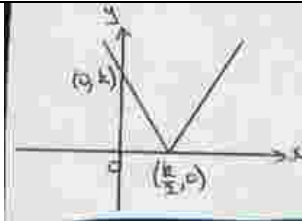
Question number	Scheme	Marks
1.	<p>(a) $\frac{(3x+2)(x-1)}{(x+1)(x-1)}, = \frac{3x+2}{x+1}$</p> <p>Notes M1 attempt to factorise numerator, <i>usual rules</i> B1 factorising denominator seen anywhere in (a), A1 given answer If factorisation of denom. not seen, correct answer implies B1</p> <p>(b) Expressing over common denominator</p> $\frac{3x+2}{x+1} - \frac{1}{x(x+1)} = \frac{x(3x+2)-1}{x(x+1)}$ <p>[Or “Otherwise” : $\frac{(3x^2 - x - 2)x - (x - 1)}{x(x^2 - 1)}$]</p> <p>Multiplying out numerator and attempt to factorise $[3x^2 + 2x - 1 \equiv (3x - 1)(x + 1)]$</p> <p>Answer: $\frac{3x-1}{x}$</p>	<p>M1B1, A1 (3)</p> <p>M1</p> <p>M1</p> <p>A1 (3)</p> <p>Total 6 marks</p>
2.	<p>(a) $\frac{dy}{dx} = 3e^{3x} + \frac{1}{x}$</p> <p>Notes B1 $3e^{3x}$ M1 : $\frac{a}{bx}$ A1: $3e^{3x} + \frac{1}{x}$</p> <p>(b) $(5+x^2)^{\frac{1}{2}}$</p> $\frac{3}{2} (5+x^2)^{\frac{1}{2}} \cdot 2x = 3x(5+x^2)^{\frac{1}{2}} \quad \text{M1 for}$ <p>$kx(5+x^2)^m$</p>	<p>B1M1A1 (3)</p> <p>B1</p> <p>M1 A1 (3)</p> <p>Total 6 marks</p>

Question Number	Scheme	Marks
3. (a)	 <p>Mod graph, reflect for $y < 0$</p> <p>$(0, 2), (3, 0)$ or marked on axes</p> <p>Correct shape, including cusp</p>	<p>M1</p> <p>A1</p> <p>A1 (3)</p>
	 <p>Attempt at reflection in $y = x$</p> <p>Curvature correct</p> <p>$-2, 0), (0, 3)$ or equiv.</p>	<p>M1</p> <p>A1</p> <p>B1 (3)</p>
	 <p>Attempt at 'stretches'</p> <p>$(0, -1)$ or equiv.</p> <p>$(1, 0)$</p>	<p>M1</p> <p>B1</p> <p>B1 (3)</p>
		Total 9 marks

Question Number	Scheme	Marks
4.	(a) 425 °C	B1 (1)
	(b) $300 = 400e^{-0.05t} + 25 \Rightarrow 400e^{-0.05t} = 275$ sub. $T = 300$ and attempt to rearrange to $e^{-0.05t} = a$, where $a \in \mathbb{Q}$ $e^{-0.05t} = \frac{275}{400}$ M1 correct application of logs $t = 7.49$	M1 A1 M1 A1 (4)
	(c) $\frac{dT}{dt} = -20 e^{-0.05t}$ (M1 for $ke^{-0.05t}$)	M1 A1
	At $t = 50$, rate of decrease = $(\pm) 1.64$ °C/min	A1 (3)
	(d) $T > 25$, (since $e^{-0.05t} \rightarrow 0$ as $t \rightarrow \infty$)	B1 (1)
		Total 9 marks

Question Number	Scheme	Marks
5. (a)	<p>Using product rule: $\frac{dy}{dx} = 2 \tan 2x + 2(2x - 1) \sec^2 2x$</p> <p>Use of "$\tan 2x = \frac{\sin 2x}{\cos 2x}$" and "$\sec 2x = \frac{1}{\cos 2x}$"</p> <p>$[= 2 \frac{\sin 2x}{\cos 2x} + 2(2x - 1) \frac{1}{\cos^2 2x}]$</p> <p>Setting $\frac{dy}{dx} = 0$ and multiplying through to eliminate fractions</p> <p>$[\Rightarrow 2 \sin 2x \cos 2x + 2(2x - 1) = 0]$</p> <p>Completion: producing $4k + \sin 4k - 2 = 0$ with no wrong working seen and at least previous line seen. AG</p>	<p>M1 A1 A1</p> <p>M1</p> <p>M1</p> <p>A1* (6)</p>
(b)	<p>$x_1 = 0.2670, x_2 = 0.2809, x_3 = 0.2746, x_4 = 0.2774,$</p> <p>Note: M1 for first correct application, first A1 for two correct, second A1 for all four correct</p> <p>Max -1 deduction, if ALL correct to > 4 d.p. M1 A0 A1</p> <p>SC: degree mode: M1 $x_1 = 0.4948$, A1 for $x_2 = 0.4914$, then A0; max 2</p>	M1 A1 A1 (3)
(c)	<p>Choose suitable interval for k: e.g. $[0.2765, 0.2775]$ and evaluate $f(x)$ at these values</p> <p>Show that $4k + \sin 4k - 2$ changes sign and deduction</p> <p>$[f(0.2765) = -0.000087..., f(0.2775) = +0.0057]$</p> <p>Note:</p> <p>Continued iteration: (no marks in degree mode)</p> <p>Some evidence of further iterations leading to 0.2765 or better</p> <p>M1;</p> <p>Deduction A1</p>	<p>M1</p> <p>A1 (2)</p> <p>(11 marks)</p>

Question Number	Scheme	Marks
6. (a)	Dividing $\sin^2 \theta + \cos^2 \theta \equiv 1$ by $\sin^2 \theta$ to give $\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} \equiv \frac{1}{\sin^2 \theta}$ Completion: $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta \Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta \equiv 1$ AG	M1 A1* (2)
(b)	$\operatorname{cosec}^4 \theta - \cot^4 \theta \equiv (\operatorname{cosec}^2 \theta - \cot^2 \theta)(\operatorname{cosec}^2 \theta + \cot^2 \theta)$ $\equiv (\operatorname{cosec}^2 \theta + \cot^2 \theta) \quad \text{using (a)} \quad \text{AG}$ <p>Notes: (i) Using LHS = $(1 + \cot^2 \theta)^2 - \cot^4 \theta$, using (a) & elim. $\cot^4 \theta$ M1, conclusion {using (a) again} A1* (ii) Conversion to sines and cosines: needs $\frac{(1 - \cos^2 \theta)(1 + \cos^2 \theta)}{\sin^4 \theta}$ for M1</p>	M1 A1* (2)
(c)	Using (b) to form $\operatorname{cosec}^2 \theta + \cot^2 \theta \equiv 2 - \cot \theta$ Forming quadratic in $\cot \theta$ $\Rightarrow 1 + \cot^2 \theta + \cot^2 \theta \equiv 2 - \cot \theta \quad \{\text{using (a)}\}$ $2 \cot^2 \theta + \cot \theta - 1 = 0$ Solving: $(2 \cot \theta - 1)(\cot \theta + 1) = 0$ to $\cot \theta =$ $\left(\cot \theta = \frac{1}{2} \right) \quad \text{or} \quad \cot \theta = -1$ $\theta = 135^\circ \quad (\text{or correct value(s) for candidate dep. on 3Ms})$ <p>Note: Ignore solutions outside range Extra “solutions” in range loses A1√, but candidate may possibly have more than one “correct” solution.</p>	M1 M1 A1 M1 A1 A1√ (6) (10 marks)

Question Number	Scheme	Marks
7.	(a)	
	 <p>Log graph: Shape</p> <p>Intersection with -ve x-axis</p> <p>$(0, \ln k), (1 - k, 0)$</p>	B1 dB1 B1
	 <p>Mod graph :V shape, vertex on +ve x-axis</p> <p>$(0, k)$ and $(\frac{k}{2}, 0)$</p>	B1 B1 (5)
	(b) $f(x) \in \mathbb{R} \quad , \quad -\infty < f(x) < \infty \quad , \quad -\infty < y < \infty$	B1 (1)
	(c) $\lg\left(\frac{k}{4}\right) = \ln\left\{k + \left \frac{2k}{4} - k\right \right\} \quad \text{or} \quad f\left(\left -\frac{k}{2}\right \right)$ $= \ln\left(\frac{3k}{2}\right)$	M1 A1 (2)
	(d) $\frac{dy}{dx} = \frac{1}{x + k}$ Equating (with $x = 3$) to grad. of line; $\frac{1}{3 + k} = \frac{2}{9}$ $k = 1\frac{1}{2}$	B1 M1; A1 A1✓ (4) (12 marks)

Question Number	Scheme	Marks
8. (a)	<p>Method for finding $\sin A$</p> $\sin A = -\frac{\sqrt{7}}{4}$ <p>Note: First A1 for $\frac{\sqrt{7}}{4}$, exact. Second A1 for sign (even if dec. answer given)</p> <p>Use of $\sin 2A \equiv 2 \sin A \cos A$</p> $\sin 2A = -\frac{3\sqrt{7}}{8} \text{ or equivalent exact}$ <p>Note: \pm f.t. Requires exact value, dependent on 2nd M</p>	<p>M1</p> <p>A1 A1</p> <p>M1</p> <p>A1√ (5)</p>
(b)(i)	$\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x \cos \frac{\pi}{3} - \sin 2x \sin \frac{\pi}{3} +$ $\cos 2x \cos \frac{\pi}{3} + \sin 2x \sin \frac{\pi}{3}$ $\equiv 2 \cos 2x \cos \frac{\pi}{3}$ <p>[This can be just written down (using factor formulae) for M1A1]</p> $\equiv \cos 2x \quad \text{AG}$	<p>M1</p> <p>A1</p> <p>A1* (3)</p>
(b)(ii)	<p>Note:</p> <p>M1A1 earned, if $\equiv 2 \cos 2x \cos \frac{\pi}{3}$ just written down, using factor theorem</p> <p>Final A1* requires some working after first result.</p> $\frac{dy}{dx} = 6 \sin x \cos x - 2 \sin 2x$ <p>or $6 \sin x \cos x - 2 \sin\left(2x + \frac{\pi}{3}\right) - 2 \sin\left(2x - \frac{\pi}{3}\right)$</p> $= 3 \sin 2x - 2 \sin 2x$ $= \sin 2x \quad \text{AG}$	<p>B1 B1</p> <p>M1</p> <p>A1* (4)</p>
	<p>Note: First B1 for $6 \sin x \cos x$; second B1 for remaining term(s)</p>	(12 marks)