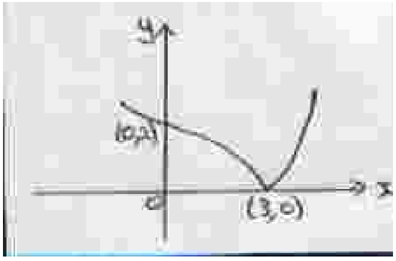
June 2006
6665 Core Mathematics C3
Mark Scheme

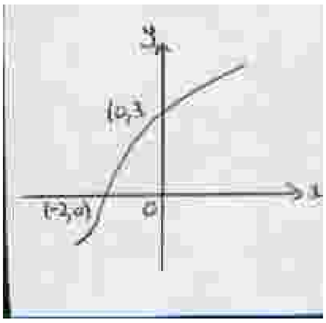
Question number	Scheme	Marks
1. (a)	$\frac{(3x+2)(x-1)}{(x+1)(x-1)}, \quad = \frac{3x+2}{x+1}$	M1B1, A1 (3)
	Notes M1 attempt to factorise numerator, *usual rules* B1 factorising denominator seen anywhere in (a), A1 given answer If factorisation of denom. not seen, correct answer implies B1	
(b)	Expressing over common denominator	
	$\frac{3x+2}{x+1} - \frac{1}{x(x+1)} = \frac{x(3x+2)-1}{x(x+1)}$	M1
	[Or "Otherwise" : $\frac{(3x^2-x-2)x-(x-1)}{x(x^2-1)}$]	
	Multiplying out numerator and attempt to factorise $[3x^2+2x-1 \equiv (3x-1)(x+1)]$	M1
	Answer: $\frac{3x-1}{x}$	A1 (3)
		Total 6 marks
2. *(a)*	$\frac{dy}{dx} = 3e^{3x} + \frac{1}{x}$	B1M1A1 (3)
	Notes B1 $3e^{3x}$ M1 : $\frac{a}{bx}$ A1: $3e^{3x} + \frac{1}{x}$	
(b)	$(5+x^2)^{\frac{1}{2}}$	B1
	$\frac{3}{2}(5+x^2)^{\frac{1}{2}} \, . \, 2x \quad = 3x(5+x^2)^{\frac{1}{2}}$ M1 for $kx(5+x^2)^m$	M1 A1 (3)
		Total 6 marks

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>3. (a)</td><td></td><td>Mod graph, reflect for $y < 0$

(0, 2), (3, 0) or marked on axes

Correct shape, including cusp</td><td>M1

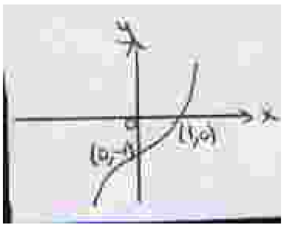
A1

A1 (3)</td></tr>
<tr><td>(b)</td><td></td><td>Attempt at reflection in $y = x$

Curvature correct

–2, 0), (0, 3) or equiv.</td><td>M1

A1

B1 (3)</td></tr>
<tr><td>(c)</td><td></td><td>Attempt at 'stretches'

(0, –1) or equiv.

(1, 0)</td><td>M1

B1

B1 (3)</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 9 marks</td></tr>
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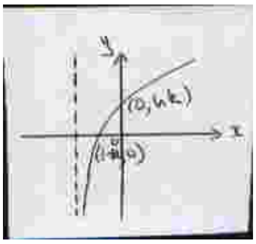
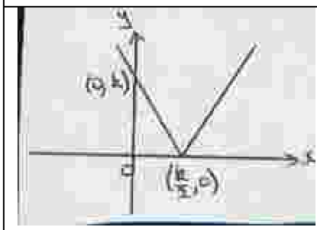
Question Number	Scheme	Marks
4. (*a*)	425 ºC	B1 (1)
(*b*)	$300 = 400\mathrm{e}^{-0.05t} + 25 \quad \Rightarrow 400\mathrm{e}^{-0.05t} = 275$ sub. $T = 300$ and attempt to rearrange to $\mathrm{e}^{-0.05t} = a$, where $a \in \mathbb{Q}$	M1
	$e^{-0.05t} = \dfrac{275}{400}$	A1
	M1 correct application of logs	M1
	$t = 7.49$	A1 (4)
(*c*)	$\dfrac{\mathrm{d}T}{\mathrm{d}t} = -20\ \mathrm{e}^{-0.05\,t}$ (M1 for $k\mathrm{e}^{-0.05\,t}$)	M1 A1
	At $t = 50$, rate of decrease = ($\pm$) 1.64 ºC/min	A1 (3)
(*d*)	$T > 25$, (since $\mathrm{e}^{-0.05\,t} \to 0$ as $t \to \infty$)	B1 (1)
		Total 9 marks

Question Number	Scheme	Marks
5. (*a*)	Using product rule: $\frac{dy}{dx} = 2\tan 2x + 2(2x-1)\sec^2 2x$	M1 A1 A1
	Use of "$\tan 2x = \frac{\sin 2x}{\cos 2x}$" and "$\sec 2x = \frac{1}{\cos 2x}$" $[= 2\frac{\sin 2x}{\cos 2x} + 2(2x-1)\frac{1}{\cos^2 2x}$]	M1
	Setting $\frac{dy}{dx} = 0$ and multiplying through to eliminate fractions $[\Rightarrow \quad 2\sin 2x\cos 2x + 2(2x-1) = 0]$	M1
	Completion: producing $4k + \sin 4k - 2 = 0$ with no wrong working seen and at least previous line seen. AG	A1* (6)
(*b*)	$x_1 = 0.2670$, $x_2 = 0.2809$, $x_3 = 0.2746$, $x_4 = 0.2774$, **Note:** M1 for first correct application, first A1 for two correct, second A1 for all four correct Max –1 deduction, if ALL correct to > 4 d.p. M1 A0 A1 SC: degree mode: M1 $x_1 = 0.4948$, A1 for $x_2 = 0.4914$, then A0; max 2	M1 A1 A1 (3)
(*c*)	Choose suitable interval for *k*: e.g. [0.2765, 0.2775] and evaluate f(*x*) at these values	M1
	Show that $4k + \sin 4k - 2$ changes sign and deduction [f(0.2765) = –0.000087.., f(0.2775) = +0.0057]	A1 (2)
	Note: Continued iteration: (no marks in degree mode) Some evidence of further iterations leading to 0.2765 or better M1; Deduction A1	**(11 marks)**

Question Number	Scheme	Marks
6. (*a*)	Dividing $\sin^2\theta + \cos^2\theta \equiv 1$ by $\sin^2\theta$ to give $\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} \equiv \frac{1}{\sin^2\theta}$	M1
	Completion: $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta \Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$ AG	A1* (2)
(*b*)	$\operatorname{cosec}^4\theta - \cot^4\theta \equiv (\operatorname{cosec}^2\theta - \cot^2\theta)(\operatorname{cosec}^2\theta + \cot^2\theta)$	M1
	$\equiv (\operatorname{cosec}^2\theta + \cot^2\theta)$ using (a) AG	A1* (2)
	Notes: (i) Using LHS = $(1 + \cot^2\theta)^2 - \cot^4\theta$, using (a) & elim. $\cot^4\theta$ M1, conclusion {using (a) again} A1* (ii) Conversion to sines and cosines: needs $\frac{(1-\cos^2\theta)(1+\cos^2\theta)}{\sin^4\theta}$ for M1	
(*c*)	Using (b) to form $\operatorname{cosec}^2\theta + \cot^2\theta \equiv 2 - \cot\theta$	M1
	Forming quadratic in $\cot\theta$	M1
	$\Rightarrow 1 + \cot^2\theta + \cot^2\theta \equiv 2 - \cot\theta$ {using (a)}	
	$2\cot^2\theta + \cot\theta - 1 = 0$	A1
	Solving: $(2\cot\theta - 1)(\cot\theta + 1) = 0$ to $\cot\theta =$	M1
	$\left(\cot\theta = \frac{1}{2}\right)$ or $\cot\theta = -1$	A1
	$\theta = 135^\circ$ (or correct value(s) for candidate dep. on 3Ms)	A1√ (6)
	Note: Ignore solutions outside range Extra "solutions" in range loses A1√, but candidate may possibly have more than one "correct" solution.	**(10 marks)**

Question Number	Scheme		Marks	
7. (*a*)		Log graph: Shape	B1	
		Intersection with –ve x-axis	dB1	
		$(0, \ln k)$, $(1 - k, 0)$	B1	
		Mod graph :V shape, vertex on +ve x-axis	B1	
		$(0, k)$ and $\left(\frac{k}{2}, 0\right)$	B1	(5)
(*b*)	$f(x) \in \mathbb{R}$, $-\infty < f(x) < \infty$, $-\infty < y < \infty$		B1	(1)
(*c*)	$fg\left(\frac{k}{4}\right) = \ln\{k + \lvert\frac{2k}{4} - k\rvert\}$ or $f\left(\lvert-\frac{k}{2}\rvert\right)$		M1	
	$= \ln\left(\frac{3k}{2}\right)$		A1	(2)
(*d*)	$\frac{dy}{dx} = \frac{1}{x + k}$		B1	
	Equating (with $x = 3$) to grad. of line; $\frac{1}{3 + k} = \frac{2}{9}$		M1; A1	
	$k = 1\frac{1}{2}$		A1√	(4)
			(12 marks)	

Question Number	Scheme	Marks
8. (*a*)	Method for finding sin A	M1
	$\sin A = -\frac{\sqrt{7}}{4}$	A1 A1
	Note: First A1 for $\frac{\sqrt{7}}{4}$, exact. Second A1 for sign (even if dec. answer given)	
	Use of $\sin 2A \equiv 2\sin A\cos A$	M1
	$\sin 2A = -\frac{3\sqrt{7}}{8}$ or equivalent exact	A1√ (5)
	Note: ± f.t. Requires exact value, dependent on 2nd M	
(*b*)(i)	$\cos\left(2x+\frac{\pi}{3}\right) + \cos\left(2x-\frac{\pi}{3}\right) \equiv \cos 2x \cos\frac{\pi}{3} - \sin 2x \sin\frac{\pi}{3} +$ $\cos 2x \cos\frac{\pi}{3} + \sin 2x \sin\frac{\pi}{3}$	M1
	$\equiv 2\cos 2x \cos\frac{\pi}{3}$	A1
	[This can be just written down (using factor formulae) for M1A1]	
	$\equiv \cos 2x$ AG	A1* (3)
	Note: M1A1 earned, if $\equiv 2\cos 2x\cos\frac{\pi}{3}$ just written down, using factor theorem. Final A1* requires some working after first result.	
(*b*)(ii)	$\frac{dy}{dx} = 6\sin x\cos x - 2\sin 2x$	B1 B1
	or $6\sin x\cos x - 2\sin\left(2x+\frac{\pi}{3}\right) - 2\sin\left(2x-\frac{\pi}{3}\right)$	
	$= 3\sin 2x - 2\sin 2x$	M1
	$= \sin 2x$ AG	A1* (4)
	Note: First B1 for $6\sin x\cos x$; second B1 for remaining term(s)	
		(12 marks)