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- 1. A curve  $C$  is described by the equation

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0.$$

Find an equation of the tangent to  $C$  at the point  $(1, -2)$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

(7)

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**Question 1 continued**

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**(Total 7 marks)**

**Q1**



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- 2. (a) Given that  $y = \sec x$ , complete the table with the values of  $y$  corresponding to  $x = \frac{\pi}{16}$ ,  $\frac{\pi}{8}$  and  $\frac{\pi}{4}$ .

$x$	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
$y$	1			1.20269	

(2)

- (b) Use the trapezium rule, with all the values for  $y$  in the completed table, to obtain an estimate for  $\int_0^{\frac{\pi}{4}} \sec x \, dx$ . Show all the steps of your working, and give your answer to 4 decimal places.

(3)

The exact value of  $\int_0^{\frac{\pi}{4}} \sec x \, dx$  is  $\ln(1 + \sqrt{2})$ .

- (c) Calculate the % error in using the estimate you obtained in part (b).
- (2)

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4.

Figure 1

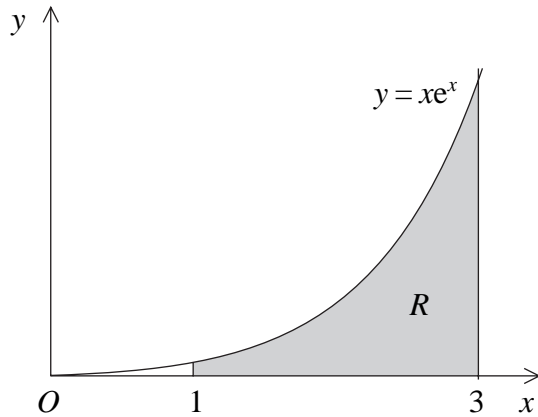


Figure 1 shows the finite shaded region,  $R$ , which is bounded by the curve  $y = xe^x$ , the line  $x = 1$ , the line  $x = 3$  and the  $x$ -axis.

The region  $R$  is rotated through 360 degrees about the  $x$ -axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(8)

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6. The line  $l_1$  has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where  $\lambda$  is a parameter.

The point  $A$  has coordinates  $(4, 8, a)$ , where  $a$  is a constant. The point  $B$  has coordinates  $(b, 13, 13)$ , where  $b$  is a constant. Points  $A$  and  $B$  lie on the line  $l_1$ .

(a) Find the values of  $a$  and  $b$ .

(3)

Given that the point  $O$  is the origin, and that the point  $P$  lies on  $l_1$  such that  $OP$  is perpendicular to  $l_1$ ,

(b) find the coordinates of  $P$ .

(5)

(c) Hence find the distance  $OP$ , giving your answer as a simplified surd.

(2)

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**Question 6 continued**

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**(Total 10 marks)**

**Q6**

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7. The volume of a spherical balloon of radius  $r$  cm is  $V$  cm<sup>3</sup>, where  $V = \frac{4}{3}\pi r^3$ .

(a) Find  $\frac{dV}{dr}$ . (1)

The volume of the balloon increases with time  $t$  seconds according to the formula

$$\frac{dV}{dt} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

(b) Using the chain rule, or otherwise, find an expression in terms of  $r$  and  $t$  for  $\frac{dr}{dt}$ . (2)

(c) Given that  $V = 0$  when  $t = 0$ , solve the differential equation  $\frac{dV}{dt} = \frac{1000}{(2t+1)^2}$ , to obtain  $V$  in terms of  $t$ . (4)

(d) Hence, at time  $t = 5$ ,

(i) find the radius of the balloon, giving your answer to 3 significant figures, (3)

(ii) show that the rate of increase of the radius of the balloon is approximately  $2.90 \times 10^{-2}$  cm s<sup>-1</sup>. (2)

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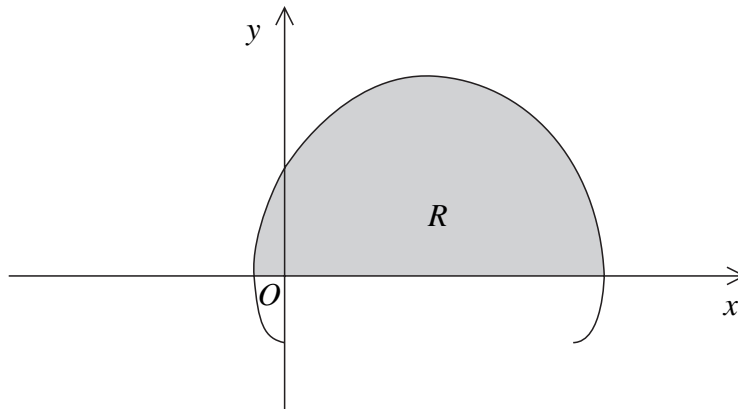






8.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = t - 2 \sin t, \quad y = 1 - 2 \cos t, \quad 0 \leq t \leq 2\pi.$$

- (a) Show that the curve crosses the  $x$ -axis where  $t = \frac{\pi}{3}$  and  $t = \frac{5\pi}{3}$ . (2)

The finite region  $R$  is enclosed by the curve and the  $x$ -axis, as shown shaded in Figure 2.

- (b) Show that the area of  $R$  is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos t)^2 dt. \quad \text{(3)}$$

- (c) Use this integral to find the exact value of the shaded area. (7)

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**Question 8 continued**

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**Q8**

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**(Total 12 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

