



1.

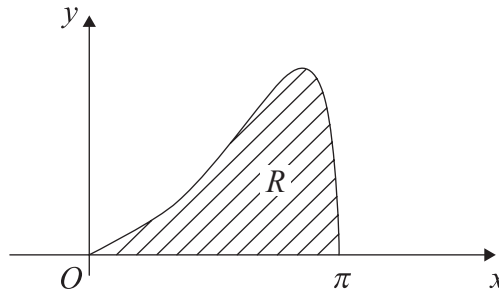


Figure 1

The curve shown in Figure 1 has equation  $y = e^x \sqrt{\sin x}$ ,  $0 \leq x \leq \pi$ . The finite region  $R$  bounded by the curve and the  $x$ -axis is shown shaded in Figure 1.

(a) Complete the table below with the values of  $y$  corresponding to  $x = \frac{\pi}{4}$  and  $\frac{\pi}{2}$ , giving your answers to 5 decimal places.

$x$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	$\pi$
$y$	0			8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region  $R$ . Give your answer to 4 decimal places.

(4)

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Question 1 continued

A series of horizontal lines for writing answers.

(Total 6 marks)

Q1







3.

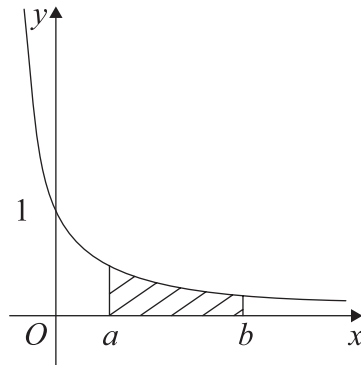


Figure 2

The curve shown in Figure 2 has equation  $y = \frac{1}{(2x+1)}$ . The finite region bounded by the curve, the  $x$ -axis and the lines  $x = a$  and  $x = b$  is shown shaded in Figure 2. This region is rotated through  $360^\circ$  about the  $x$ -axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of  $a$  and  $b$ .

(5)

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4. (i) Find  $\int \ln\left(\frac{x}{2}\right) dx$ . (4)

(ii) Find the exact value of  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x dx$ . (5)

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5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where  $x = -8$ .

(3)

(b) Find the gradient of the curve at each of these points.

(6)

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Question 6 continued

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Question 6 continued

A series of horizontal lines for writing the answer to Question 6.

Q6

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(Total 11 marks)



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7.

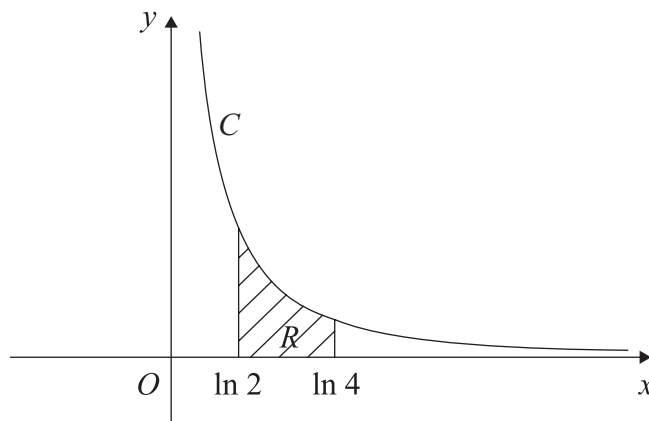


Figure 3

The curve *C* has parametric equations

$$x = \ln(t + 2), \quad y = \frac{1}{(t + 1)}, \quad t > -1.$$

The finite region *R* between the curve *C* and the *x*-axis, bounded by the lines with equations  $x = \ln 2$  and  $x = \ln 4$ , is shown shaded in Figure 3.

(a) Show that the area of *R* is given by the integral

$$\int_0^2 \frac{1}{(t + 1)(t + 2)} dt.$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve *C*, in the form  $y = f(x)$ .

(4)

(d) State the domain of values for  $x$  for this curve.

(1)

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Question 7 continued

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**Question 7 continued**

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(Total 15 marks)

Q7



8. Liquid is pouring into a large vertical circular cylinder at a constant rate of  $1600 \text{ cm}^3 \text{ s}^{-1}$  and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is  $4000 \text{ cm}^2$ .

- (a) Show that at time  $t$  seconds, the height  $h$  cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.} \quad (3)$$

When  $h = 25$ , water is leaking out of the hole at  $400 \text{ cm}^3 \text{ s}^{-1}$ .

- (b) Show that  $k = 0.02$  (1)

- (c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}} dh. \quad (2)$$

Using the substitution  $h = (20 - x)^2$ , or otherwise,

- (d) find the exact value of  $\int_0^{100} \frac{50}{20 - \sqrt{h}} dh$ . (6)

- (e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second. (1)

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