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Question 1 continued

Lined area for writing the answer to Question 1.

(Total 7 marks)

Q1



2. (a) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \quad |x| < 1 \quad (6)$$

- (b) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)



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Question 2 continued

Lined writing area for question 2.



3.

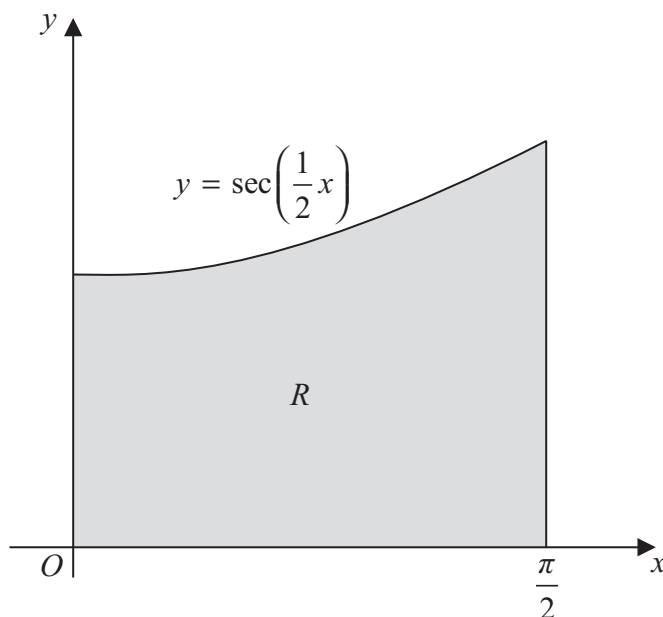


Figure 1

Figure 1 shows the finite region R bounded by the x -axis, the y -axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

- (a) Complete the table above giving the missing value of y to 6 decimal places. (1)
- (b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R , giving your answer to 4 decimal places. (3)

Region R is rotated through 2π radians about the x -axis.

- (c) Use calculus to find the exact volume of the solid formed. (4)



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Question 3 continued

A large rectangular area containing 30 horizontal lines for writing answers.

(Total 8 marks)

Q3



P 4 3 1 3 7 A 0 1 1 3 2

4. A curve *C* has parametric equations

$$x = 2\sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ at the point where $t = \frac{\pi}{6}$ (4)

(b) Find a cartesian equation for *C* in the form

$$y = f(x), \quad -k \leq x \leq k,$$

stating the value of the constant *k*. (3)

(c) Write down the range of *f(x)*. (2)



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5. (a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x} - 1)} dx = \int \frac{2}{u(2u - 1)} du \quad (3)$$

(b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x} - 1)} dx = 2 \ln \left(\frac{a}{b} \right)$$

where a and b are integers to be determined.

(7)



6. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at any time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \quad \theta \leq 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t} \tag{8}$$

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off. (3)



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Question 6 continued

Lined area for writing the answer to Question 6.

(Total 11 marks)

Q6



8. With respect to a fixed origin O , the line l has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \text{ where } \lambda \text{ is a scalar parameter.}$$

The point A lies on l and has coordinates $(3, -2, 6)$.

The point P has position vector $(-p\mathbf{i} + 2p\mathbf{k})$ relative to O , where p is a constant.

Given that vector \vec{PA} is perpendicular to l ,

(a) find the value of p .

(4)

Given also that B is a point on l such that $\angle BPA = 45^\circ$,

(b) find the coordinates of the two possible positions of B .

(5)



