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1. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y .

(5)

(b) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(2)

Horizontal lines for writing answers.



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Question 1 continued

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(Total 7 marks)

Q1



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2. Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \dots$$

(a) find the value of the constant k ,

(2)

(b) find the value of the constant A , giving your answer in its simplest form.

(3)



3.

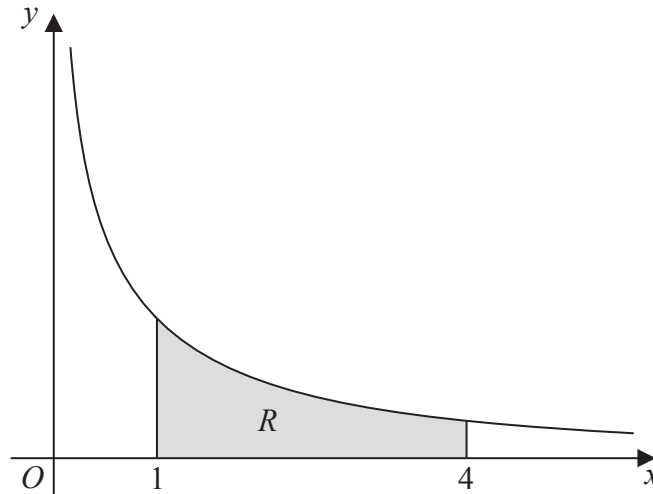


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{10}{2x + 5\sqrt{x}}$, $x > 0$

The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis, and the lines with equations $x = 1$ and $x = 4$

The table below shows corresponding values of x and y for $y = \frac{10}{2x + 5\sqrt{x}}$

x	1	2	3	4
y	1.42857	0.90326		0.55556

- (a) Complete the table above by giving the missing value of y to 5 decimal places. (1)
- (b) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R , giving your answer to 4 decimal places. (3)
- (c) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (b) is an overestimate or an underestimate for the area of R . (1)
- (d) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}} dx$$
(6)



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Question 3 continued

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(Total 11 marks)

Q3

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4.

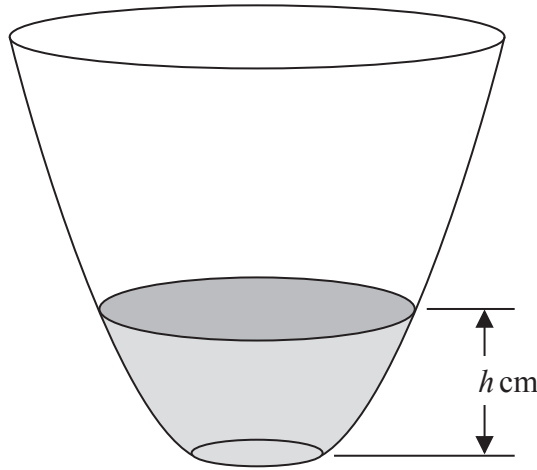


Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is h cm, the volume of water V cm³ is given by

$$V = 4\pi h(h + 4), \quad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of 80π cm³s⁻¹

Find the rate of change of the depth of the water, in cm s⁻¹, when $h = 6$

(5)



5.

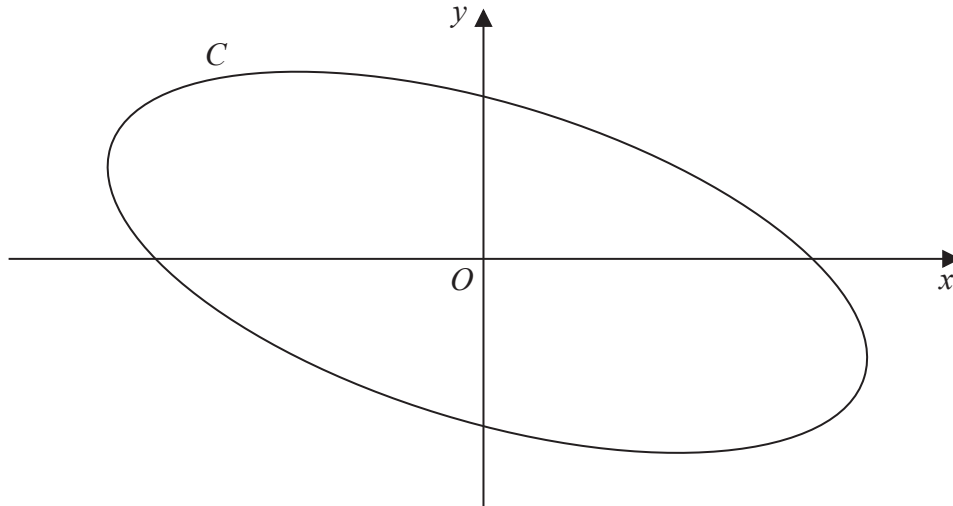


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \quad 0 \leq t < 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3} \cos t \tag{3}$$

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined.

(2)



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Question 5 continued

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(Total 5 marks)

Q5



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6. (i) Find

$$\int x e^{4x} dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x - 1)^3} dx, \quad x > \frac{1}{2}$$

(2)

(iii) Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

Lined area for student answer.



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Question 6 continued

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(Total 12 marks)

Q6

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7.

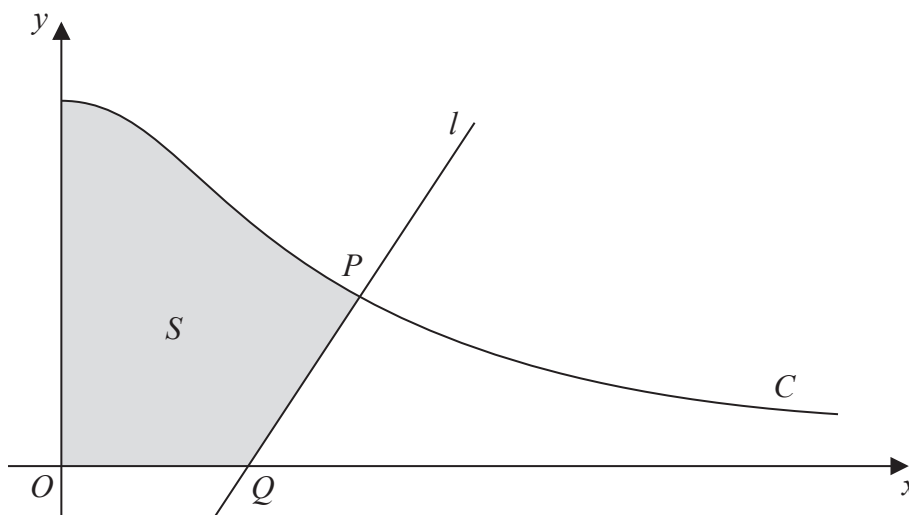


Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3 \tan \theta, \quad y = 4 \cos^2 \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $(3, 2)$.

The line l is the normal to C at P . The normal cuts the x -axis at the point Q .

- (a) Find the x coordinate of the point Q . (6)

The finite region S , shown shaded in Figure 4, is bounded by the curve C , the x -axis, the y -axis and the line l . This shaded region is rotated 2π radians about the x -axis to form a solid of revolution.

- (b) Find the exact value of the volume of the solid of revolution, giving your answer in the form $p\pi + q\pi^2$, where p and q are rational numbers to be determined.

[You may use the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.] (9)



8. Relative to a fixed origin O , the point A has position vector $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point B has position vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

The line l_1 passes through the points A and B .

(a) Find the vector \vec{AB} . (2)

(b) Hence find a vector equation for the line l_1 . (1)

The point P has position vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

Given that angle PBA is θ ,

(c) show that $\cos \theta = \frac{1}{3}$. (3)

The line l_2 passes through the point P and is parallel to the line l_1

(d) Find a vector equation for the line l_2 . (2)

The points C and D both lie on the line l_2

Given that $AB = PC = DP$ and the x coordinate of C is positive,

(e) find the coordinates of C and the coordinates of D . (3)

(f) find the exact area of the trapezium $ABCD$, giving your answer as a simplified surd. (4)



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Question 8 continued

Lined writing area for the answer to Question 8.



