

Mark Scheme (Results)

January 2008

GCE

GCE Mathematics (6666/01)

January 2008
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks												
1. (a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>0</td> <td>$\frac{\pi}{4}$</td> <td>$\frac{\pi}{2}$</td> <td>$\frac{3\pi}{4}$</td> <td>π</td> </tr> <tr> <td>y</td> <td>0</td> <td>1.844321332...</td> <td>4.810477381...</td> <td>8.87207</td> <td>0</td> </tr> </table>	x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	y	0	1.844321332...	4.810477381...	8.87207	0	
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π									
y	0	1.844321332...	4.810477381...	8.87207	0									
(b) Way 1	<p style="text-align: center;">0 can be implied</p> <p style="text-align: center;">↓</p> $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{4} \times \{0 + 2(1.84432 + 4.81048 + 8.87207) + 0\}$	<p>awrt 1.84432 B1 awrt 4.81048 or 4.81047 B1</p> <p>Outside brackets awrt 0.39 or $\frac{1}{2} \times$ awrt 0.79 B1 $\frac{1}{2} \times \frac{\pi}{4}$ or $\frac{\pi}{8}$</p> <p>For structure of trapezium <u>rule</u> {.....}; M1 $\sqrt{\quad}$</p> <p>Correct expression <u>inside brackets</u> which all must be multiplied by their "outside constant". A1 $\sqrt{\quad}$</p> <p>$= \frac{\pi}{8} \times 31.05374... = 12.19477518... = \underline{12.1948}$ (4dp) A1 cao</p>	[2]											
<i>Aliter</i> (b) Way 2	$\text{Area} \approx \frac{\pi}{4} \times \left\{ \frac{0+1.84432}{2} + \frac{1.84432+4.81048}{2} + \frac{4.81048+8.87207}{2} + \frac{8.87207+0}{2} \right\}$ <p>which is equivalent to:</p> $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{4} \times \{0 + 2(1.84432 + 4.81048 + 8.87207) + 0\}$	<p>$\frac{\pi}{4}$ (or awrt 0.79) and a divisor of 2 on all terms inside brackets. B1</p> <p>One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. M1 $\sqrt{\quad}$</p> <p>Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out. A1 $\sqrt{\quad}$</p> <p>$= \frac{\pi}{4} \times 15.52687... = 12.19477518... = \underline{12.1948}$ (4dp) A1 cao</p>	[4]											
		6 marks												

Note an expression like $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{4} + 2(1.84432 + 4.81048 + 8.87207)$ would score B1M1A0A0

Question Number	Scheme	Marks
<p>2. (a)</p> <p>(b)</p>	<p>** represents a constant (which must be consistent for first accuracy mark)</p> $(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$ <p>with $** \neq 1$</p> $= 2 \left\{ 1 + \frac{(\frac{1}{3})(**x)}{1} + \frac{(\frac{1}{3})(-\frac{2}{3})(**x)^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(**x)^3}{3!} + \dots \right\}$ <p>Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})(**x)^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(**x)^3}{3!}$</p> $= 2 \left\{ 1 + \frac{(\frac{1}{3})(-\frac{3x}{8})}{1} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{3x}{8})^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(-\frac{3x}{8})^3}{3!} + \dots \right\}$ $= 2 \left\{ 1 - \frac{1}{8}x; -\frac{1}{64}x^2 - \frac{5}{1536}x^3 - \dots \right\}$ $= 2 - \frac{1}{4}x; -\frac{1}{32}x^2 - \frac{5}{768}x^3 - \dots$ <p>Either $2\{1 - \frac{1}{8}x \dots\}$ or anything that cancels to $2 - \frac{1}{4}x$; Simplified $-\frac{1}{32}x^2 - \frac{5}{768}x^3$</p> <p><i>Attempt to substitute</i> $x = 0.1$ into a candidate's binomial expansion.</p> $(7.7)^{\frac{1}{3}} \approx 2 - \frac{1}{4}(0.1) - \frac{1}{32}(0.1)^2 - \frac{5}{768}(0.1)^3 - \dots$ $= 2 - 0.025 - 0.0003125 - 0.0000065104166\dots$ $= 1.97468099\dots$ <p>awrt 1.9746810</p>	<p>Takes 8 outside the bracket to give any of $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$. B1</p> <p>Expands $(1+**x)^{\frac{1}{3}}$ to give a simplified or an un-simplified $1 + \frac{1}{3}(**x)$; M1;</p> <p>A correct simplified or an un-simplified $\{ \dots \}$ expansion with candidate's followed through $(**x)$ A1 ✓</p> <p>[5]</p> <p>M1</p> <p>A1</p> <p>[2]</p> <p>7 marks</p>

You would award B1M1A0 for

$$= 2 \left\{ 1 + \frac{(\frac{1}{3})(-\frac{3x}{8})}{1} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{3x}{8})^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(-\frac{3x}{8})^3}{3!} + \dots \right\}$$

because ** is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822\dots$

Question Number	Scheme	Marks
<p>Aliter 2. (a) Way 2</p>	$(8-3x)^{\frac{1}{3}}$ $= \left\{ \begin{aligned} &(8)^{\frac{1}{3}} + \frac{(\frac{1}{3})(8)^{-\frac{2}{3}}(**x)}{1!} + \frac{(\frac{1}{3})(-\frac{2}{3})(8)^{-\frac{5}{3}}(**x)^2}{2!} \\ &+ \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(8)^{-\frac{8}{3}}(**x)^3 + \dots}{3!} \end{aligned} \right\}$ <p>with $** \neq 1$</p> $= \left\{ \begin{aligned} &(8)^{\frac{1}{3}} + \frac{(\frac{1}{3})(8)^{-\frac{2}{3}}(-3x)}{1!} + \frac{(\frac{1}{3})(-\frac{2}{3})(8)^{-\frac{5}{3}}(-3x)^2}{2!} \\ &+ \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(8)^{-\frac{8}{3}}(-3x)^3 + \dots}{3!} \end{aligned} \right\}$ $= \left\{ 2 + \frac{(\frac{1}{3})(1)}{4}(-3x) + \frac{(-\frac{1}{9})(\frac{1}{32})(9x^2)}{2} + \frac{(\frac{5}{81})(\frac{1}{256})(-27x^3)}{6} + \dots \right\}$ $= 2 - \frac{1}{4}x - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \dots$	<p>2 or $(8)^{\frac{1}{3}}$ (See note ↓) B1</p> <p>Expands $(8-3x)^{\frac{1}{3}}$ to give an un-simplified or simplified M1;</p> <p>$(8)^{\frac{1}{3}} + \frac{(\frac{1}{3})(8)^{-\frac{2}{3}}(**x)}{1!}$; A correct un-simplified or simplified {.....} expansion with A1 ✓ candidate's followed through $(**x)$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Award SC M1 if you see</p> $\frac{(\frac{1}{3})(-\frac{2}{3})(8)^{-\frac{5}{3}}(**x)^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(8)^{-\frac{8}{3}}(**x)^3}{3!}$ </div> <p>Anything that cancels to $2 - \frac{1}{4}x$; A1;</p> <p>or $2\{1 - \frac{1}{8}x \dots\}$</p> <p>Simplified $-\frac{1}{32}x^2 - \frac{5}{768}x^3$ A1</p> <p style="text-align: right;">[5]</p>

Attempts using Maclaurin expansion should be escalated up to your team leader.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822\dots$

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Question Number	Scheme	Marks
3.	$\text{Volume} = \pi \int_a^b \left(\frac{1}{2x+1} \right)^2 dx = \pi \int_a^b \frac{1}{(2x+1)^2} dx$ $= \pi \int_a^b (2x+1)^{-2} dx$ $= (\pi) \left[\frac{(2x+1)^{-1}}{(-1)(2)} \right]_a^b$ $= (\pi) \left[\frac{-\frac{1}{2}(2x+1)^{-1}}{1} \right]_a^b$ $= (\pi) \left[\left(\frac{-1}{2(2b+1)} \right) - \left(\frac{-1}{2(2a+1)} \right) \right]$ $= \frac{\pi}{2} \left[\frac{-2a - 1 + 2b + 1}{(2a+1)(2b+1)} \right]$ $= \frac{\pi}{2} \left[\frac{2(b-a)}{(2a+1)(2b+1)} \right]$ $= \frac{\pi(b-a)}{(2a+1)(2b+1)}$	<p>Use of $V = \pi \int y^2 dx$. Can be implied. Ignore limits.</p> <p>B1</p> <p>Integrating to give $\frac{\pm p(2x+1)^{-1}}{-\frac{1}{2}(2x+1)^{-1}}$</p> <p>M1 A1</p> <p>Substitutes limits of b and a and subtracts the correct way round.</p> <p>dM1</p> <p>$\frac{\pi(b-a)}{(2a+1)(2b+1)}$</p> <p>A1 aef</p> <p>[5]</p> <p>5 marks</p>

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}$$

Note that π is not required for the middle three marks of this question.

Question Number	Scheme	Marks
<p><i>Aliter</i> 3. Way 2</p>	<p>Volume = $\pi \int_a^b \left(\frac{1}{2x+1} \right)^2 dx = \pi \int_a^b \frac{1}{(2x+1)^2} dx$</p> <p>$= \pi \int_a^b (2x+1)^{-2} dx$</p> <p>Applying substitution $u = 2x+1 \Rightarrow \frac{du}{dx} = 2$ and changing limits $x \rightarrow u$ so that $a \rightarrow 2a+1$ and $b \rightarrow 2b+1$, gives</p> <p>$= (\pi) \int_{2a+1}^{2b+1} \frac{u^{-2}}{2} du$</p> <p>$= (\pi) \left[\frac{u^{-1}}{(-1)(2)} \right]_{2a+1}^{2b+1}$</p> <p>$= (\pi) \left[\frac{-\frac{1}{2}u^{-1}}{2a+1} \right]_{2a+1}^{2b+1}$</p> <p>$= (\pi) \left[\left(\frac{-1}{2(2b+1)} \right) - \left(\frac{-1}{2(2a+1)} \right) \right]$</p> <p>$= \frac{\pi}{2} \left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)} \right]$</p> <p>$= \frac{\pi}{2} \left[\frac{2(b-a)}{(2a+1)(2b+1)} \right]$</p> <p>$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$</p>	<p>Use of $V = \pi \int y^2 dx$. Can be implied. Ignore limits.</p> <p>B1</p> <p>Integrating to give $\frac{\pm pu^{-1}}{-\frac{1}{2}u^{-1}}$</p> <p>M1 A1</p> <p>Substitutes limits of $2b+1$ and $2a+1$ and subtracts the correct way round.</p> <p>dM1</p> <p>$\frac{\pi(b-a)}{(2a+1)(2b+1)}$</p> <p>A1 aef</p> <p>[5]</p> <p>5 marks</p>

Note that π is not required for the middle three marks of this question.

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}$$

Question Number	Scheme	Marks
4. (i)	$\int \ln\left(\frac{x}{2}\right) dx = \int 1 \cdot \ln\left(\frac{x}{2}\right) dx \Rightarrow \left\{ \begin{array}{l} u = \ln\left(\frac{x}{2}\right) \Rightarrow \frac{du}{dx} = \frac{1}{2} = \frac{1}{x} \\ \frac{dv}{dx} = 1 \Rightarrow v = x \end{array} \right\}$ $\int \ln\left(\frac{x}{2}\right) dx = x \ln\left(\frac{x}{2}\right) - \int x \cdot \frac{1}{x} dx$ $= x \ln\left(\frac{x}{2}\right) - \int 1 dx$ $= x \ln\left(\frac{x}{2}\right) - x + c$	<p>Use of 'integration by parts' formula in the correct direction. M1 Correct expression. A1 An attempt to multiply x by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$. <u>dM1</u> Correct integration with $+ c$ A1 aef</p> <p style="text-align: right;">[4]</p>
(ii)	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x dx$ <p>[NB: $\cos 2x = \pm 1 \pm 2\sin^2 x$ or $\sin^2 x = \frac{1}{2}(\pm 1 \pm \cos 2x)$]</p> $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx = \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 - \cos 2x) dx$ $= \frac{1}{2} \left[x - \frac{1}{2} \sin 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \frac{1}{2} \left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2} \right) - \left(\frac{\pi}{4} - \frac{\sin\left(\frac{\pi}{2}\right)}{2} \right) \right]$ $= \frac{1}{2} \left[\left(\frac{\pi}{2} - 0 \right) - \left(\frac{\pi}{4} - \frac{1}{2} \right) \right]$ $= \frac{1}{2} \left(\frac{\pi}{4} + \frac{1}{2} \right) = \frac{\pi}{8} + \frac{1}{4}$	<p>Consideration of double angle formula for $\cos 2x$ M1</p> <p><u>Integrating to give $\pm ax \pm b \sin 2x$; $a, b \neq 0$</u> dM1 Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$ A1</p> <p>Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round. ddM1</p> <p>$\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)$ or $\frac{\pi}{8} + \frac{1}{4}$ or $\frac{\pi}{8} + \frac{2}{8}$ A1 aef, cso</p> <p>Candidate must collect their π term and constant term together for A1 No fluked answers, hence cso.</p> <p style="text-align: right;">[5]</p>
9 marks		

Note: $\int \ln\left(\frac{x}{2}\right) dx = (\text{their } v)\ln\left(\frac{x}{2}\right) - \int (\text{their } v) \cdot (\text{their } \frac{du}{dx}) dx$ for M1 in part (i).

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269\dots$

Question Number	Scheme	Marks
<p><i>Aliter</i> 4. (i) Way 2</p>	$\int \ln\left(\frac{x}{2}\right) dx = \int (\ln x - \ln 2) dx = \int \ln x dx - \int \ln 2 dx$ $\int \ln x dx = \int 1 \cdot \ln x dx \Rightarrow \left\{ \begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 \Rightarrow v = x \end{array} \right\}$ $\int \ln x dx = x \ln x - \int x \cdot \frac{1}{x} dx$ $= x \ln x - x + c$ $\int \ln 2 dx = x \ln 2 + c$ <p>Hence, $\int \ln\left(\frac{x}{2}\right) dx = x \ln x - x - x \ln 2 + c$</p>	<p>Use of 'integration by parts' formula in the correct direction. M1</p> <p>Correct integration of $\ln x$ with or without $+ c$ A1</p> <p>Correct integration of $\ln 2$ with or without $+ c$ M1</p> <p>Correct integration with $+ c$ A1 aef</p> <p>[4]</p>

Note: $\int \ln x dx = (\text{their } v) \ln x - \int (\text{their } v) \cdot (\text{their } \frac{du}{dx}) dx$ for M1 in part (i).

Question Number	Scheme	Marks
<p><i>Aliter</i> 4. (i) Way 3</p>	$\int \ln\left(\frac{x}{2}\right) dx$ $u = \frac{x}{2} \Rightarrow \frac{du}{dx} = \frac{1}{2}$ $\int \ln\left(\frac{x}{2}\right) dx = 2 \int \ln u \, du$ $\int \ln u \, dx = \int 1 \cdot \ln u \, du$ $\int \ln u \, dx = u \ln u - \int u \cdot \frac{1}{u} \, du$ $= u \ln u - u + c$ $\int \ln\left(\frac{x}{2}\right) dx = 2(u \ln u - u) + c$ <p>Hence, $\int \ln\left(\frac{x}{2}\right) dx = x \ln\left(\frac{x}{2}\right) - x + c$</p>	<p>Applying substitution correctly to give</p> $\int \ln\left(\frac{x}{2}\right) dx = 2 \int \ln u \, du$ <p>Decide to award 2nd M1 here!</p> <p>Use of ‘integration by parts’ formula in the correct direction. M1</p> <p>Correct integration of $\ln u$ with or without $+ c$ A1</p> <p>Decide to award 2nd M1 here! M1</p> <p>Correct integration with $+ c$ A1 aef</p> <p>[4]</p>

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>4. (ii) Way 2</p>	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x \cdot \sin x \, dx \quad \text{and} \quad I = \int \sin^2 x \, dx$ $\left\{ \begin{array}{l} u = \sin x \Rightarrow \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x \Rightarrow v = -\cos x \end{array} \right\}$ $\therefore I = \left\{ -\sin x \cos x + \int \cos^2 x \, dx \right\}$ $\therefore I = \left\{ -\sin x \cos x + \int (1 - \sin^2 x) \, dx \right\}$ $\int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx \right\}$ $2 \int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx \right\}$ $2 \int \sin^2 x \, dx = \left\{ -\sin x \cos x + x \right\}$ $\int \sin^2 x \, dx = \left\{ -\frac{1}{2} \sin x \cos x + \frac{x}{2} \right\}$ $\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \left[\left(-\frac{1}{2} \sin\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right) + \frac{\left(\frac{\pi}{2}\right)}{2} \right) - \left(-\frac{1}{2} \sin\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{4}\right) + \frac{\left(\frac{\pi}{4}\right)}{2} \right) \right]$ $= \left[\left(0 + \frac{\pi}{4} \right) - \left(-\frac{1}{4} + \frac{\pi}{8} \right) \right]$ $= \frac{\pi}{8} + \frac{1}{4}$	<p>An attempt to use the correct by parts formula. M1</p> <p>For the LHS becoming 2I dM1</p> <p><u>Correct integration</u> A1</p> <p>Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round. ddM1</p> <p>$\frac{1}{2} \left(\frac{\pi}{4} + \frac{1}{2} \right)$ or $\frac{\pi}{8} + \frac{1}{4}$ or $\frac{\pi}{8} + \frac{2}{8}$ A1 aef cso [5]</p> <p>Candidate must collect their π term and constant term together for A1</p> <p>No fluked answers, hence cso.</p>

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269\dots$

Question Number	Scheme	Marks
5. (a)	$x^3 - 4y^2 = 12xy \quad (\text{eqn } *)$ $x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$ $4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$ $(y - 16)(y - 8) = 0$ $y = \frac{24 \pm \sqrt{576 - 4(128)}}{2}$ $y = 16 \text{ or } y = 8.$	<p>Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.</p> <p>M1</p> <p>An attempt to solve the quadratic in y by either factorising or by the formula or by completing the square.</p> <p>dM1</p> <p>Both $y = 16$ and $y = 8$. or $(-8, 8)$ and $(-8, 16)$.</p> <p>A1</p> <p>[3]</p>
(b)	$\left\{ \begin{array}{l} \cancel{\frac{dy}{dx}} \\ \cancel{\frac{dy}{dx}} \end{array} \right\} \times 3x^2 - 8y \frac{dy}{dx} = \left(12y + 12x \frac{dy}{dx} \right)$ $\left\{ \frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y} \right\}$ $\text{@ } (-8, 8), \frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = -3,$ $\text{@ } (-8, 16), \frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = 0.$	<p>Differentiates implicitly to include either $\pm ky \frac{dy}{dx}$ or $12x \frac{dy}{dx}$. Ignore $\frac{dy}{dx} = \dots$</p> <p>M1</p> <p>Correct LHS equation; <u>Correct application of product rule</u></p> <p>A1; (B1)</p> <p><i>not necessarily required.</i></p> <p>Substitutes $x = -8$ and <i>at least one</i> of their y-values to attempt to find any one of $\frac{dy}{dx}$.</p> <p>dM1</p> <p>One gradient found. Both gradients of <u>-3</u> and <u>0</u> correctly found.</p> <p>A1 A1 cso</p> <p>[6]</p>
		9 marks

Question Number	Scheme	Marks
<p><i>Aliter</i> 5. (b) Way 2</p>	$\left\{ \frac{\cancel{dx}}{\cancel{dx}} \times \right\} 3x^2 \frac{dx}{dy} - 8y; = \left(12y \frac{dx}{dy} + 12x \right)$ $\left\{ \frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y} \right\}$ <p>@ (-8, 8), $\frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = -3,$</p> <p>@ (-8, 16), $\frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = 0.$</p>	<p>Differentiates implicitly to include either $\pm kx^2 \frac{dx}{dy}$ or $12y \frac{dx}{dy}$. Ignore $\frac{dx}{dy} = \dots$</p> <p>Correct LHS equation</p> <p><u>Correct application of product rule</u></p> <p><i>not necessarily required.</i></p> <p>Substitutes $x = -8$ and <i>at least one</i> of their y-values to attempt to find any one of $\frac{dy}{dx}$ or $\frac{dx}{dy}$.</p> <p>One gradient found.</p> <p>Both gradients of <u>-3</u> and <u>0</u> correctly found.</p> <p>M1 A1; (B1)</p> <p>dM1 A1 A1 cs0 [6]</p>

Question Number	Scheme	Marks
<p><i>Aliter</i> 5. (b) Way 3</p>	$x^3 - 4y^2 = 12xy \text{ (eqn *)}$ $4y^2 + 12xy - x^3 = 0$ $y = \frac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$ $y = \frac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$ $y = \frac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$ $y = -\frac{3}{2}x \pm \frac{1}{2}(9x^2 + x^3)^{\frac{1}{2}}$ $\frac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}(\frac{1}{2})(9x^2 + x^3)^{-\frac{1}{2}}; (18x + 3x^2)$ $\frac{dy}{dx} = -\frac{3}{2} \pm \frac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$ <p>@ $x = -8$ $\frac{dy}{dx} = -\frac{3}{2} \pm \frac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$</p> $= -\frac{3}{2} \pm \frac{48}{4\sqrt{(64)}} = -\frac{3}{2} \pm \frac{48}{32}$ $\therefore \frac{dy}{dx} = -\frac{3}{2} \pm \frac{3}{2} = \underline{-3}, \underline{0}$	 <p>A credible attempt to make y the subject and an attempt to differentiate either $-\frac{3}{2}x$ or $\frac{1}{2}(9x^2 + x^3)^{\frac{1}{2}}$. M1</p> <p>$\frac{dy}{dx} = -\frac{3}{2} \pm k(9x^2 + x^3)^{-\frac{1}{2}}(g(x))$ A1</p> <p>$\frac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}(\frac{1}{2})(9x^2 + x^3)^{-\frac{1}{2}}; (18x + 3x^2)$ A1</p> <p>Substitutes $x = -8$ find any one of $\frac{dy}{dx}$. dM1</p> <p>One gradient correctly found. A1</p> <p>Both gradients of <u>-3</u> and <u>0</u> correctly found. A1</p> <p style="text-align: right;">[6]</p>

Question Number	Scheme	Marks
6. (a)	$\overline{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} \quad \& \quad \overline{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ $\overline{AB} = \overline{OB} - \overline{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$	<p>Finding the difference between \overline{OB} and \overline{OA}. Correct answer.</p> <p>M1 ± A1</p> <p style="text-align: right;">[2]</p>
(b)	$l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ $l_2: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$	<p>An expression of the form (vector) ± λ(vector) $\mathbf{r} = \overline{OA} \pm \lambda(\text{their } \overline{AB})$ or $\mathbf{r} = \overline{OB} \pm \lambda(\text{their } \overline{AB})$ or $\mathbf{r} = \overline{OA} \pm \lambda(\text{their } \overline{BA})$ or $\mathbf{r} = \overline{OB} \pm \lambda(\text{their } \overline{BA})$ (r is needed.)</p> <p>M1 A1 √ aef</p> <p style="text-align: right;">[2]</p>
(c)	$l_2: \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ <p>$\overline{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}$ & θ is angle</p> $\cos \theta = \frac{\overline{AB} \cdot \mathbf{d}_2}{(\overline{AB} \cdot \mathbf{d}_2)} = \frac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{(\sqrt{(1)^2 + (-2)^2 + (2)^2} \cdot \sqrt{(1)^2 + (0)^2 + (1)^2})}$ $\cos \theta = \frac{1 + 0 + 2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} \cdot \sqrt{(1)^2 + (0)^2 + (1)^2}}$ $\cos \theta = \frac{3}{3 \cdot \sqrt{2}} \Rightarrow \theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79.$	<p>← Considers dot product between \mathbf{d}_2 and their \overline{AB}.</p> <p>M1 √</p> <p>Correct followed through expression or equation.</p> <p>A1 √</p> <p>$\theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79$</p> <p>A1 cao</p> <p style="text-align: right;">[3]</p>

This means that $\cos \theta$ does not necessarily have to be the subject of the equation. It could be of the form $3\sqrt{2} \cos \theta = 3$.

Question Number	Scheme	Marks
<p>6. (d)</p> <p>Aliter 6. (d) Way 2</p>	<p>If l_1 and l_2 intersect then: $\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$</p> <p>i: $2 + \lambda = \mu$ (1) j: $6 - 2\lambda = 0$ (2) k: $-1 + 2\lambda = \mu$ (3)</p> <p>(2) yields $\lambda = 3$ Any two yields $\lambda = 3, \mu = 5$</p> <p>$l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $\mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$</p> <p>If l_1 and l_2 intersect then: $\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$</p> <p>i: $3 + \lambda = \mu$ (1) j: $4 - 2\lambda = 0$ (2) k: $1 + 2\lambda = \mu$ (3)</p> <p>(2) yields $\lambda = 2$ Any two yields $\lambda = 2, \mu = 5$</p> <p>$l_1: \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $\mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$</p>	<p>Either seeing equation (2) written down correctly with or without any other equation or seeing equations (1) and (3) written down correctly. M1 $\sqrt{}$</p> <p>Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find ... dM1</p> <p>either one of λ or μ correct. A1</p> <p>$\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $5\mathbf{i} + 5\mathbf{k}$ A1 cso</p> <p>Fully correct solution & no incorrect values of λ or μ seen earlier.</p> <p>Either seeing equation (2) written down correctly with or without any other equation or seeing equations (1) and (3) written down correctly. M1 $\sqrt{}$</p> <p>Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find ... dM1</p> <p>either one of λ or μ correct. A1</p> <p>$\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $5\mathbf{i} + 5\mathbf{k}$ A1 cso</p> <p>Fully correct solution & no incorrect values of λ or μ seen earlier.</p> <p>[4]</p> <p>[4]</p> <p>11 marks</p>

Note: Be careful! λ and μ are not defined in the question, so a candidate could interchange these or use different scalar parameters.

Question Number	Scheme	Marks
<p>Aliter 6. (d) Way 3</p>	<p>If l_1 and l_2 intersect then: $\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$</p> <p>i: $2 - \lambda = \mu$ (1) j: $6 + 2\lambda = 0$ (2) k: $-1 - 2\lambda = \mu$ (3)</p> <p>(2) yields $\lambda = -3$ Any two yields $\lambda = -3, \mu = 5$</p> <p>$l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} - 3 \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $\mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$</p>	<p>Either seeing equation (2) written down correctly with or without any other equation or seeing equations (1) and (3) written down correctly. M1 $\sqrt{}$</p> <p>Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find ... dM1</p> <p>either one of λ or μ correct. A1</p> <p>$\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $5\mathbf{i} + 5\mathbf{k}$ A1 cso</p> <p>Fully correct solution & no incorrect values of λ or μ seen earlier.</p> <p>[4]</p>
<p>Aliter 6. (d) Way 4</p>	<p>If l_1 and l_2 intersect then: $\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$</p> <p>i: $3 - \lambda = \mu$ (1) j: $4 + 2\lambda = 0$ (2) k: $1 - 2\lambda = \mu$ (3)</p> <p>(2) yields $\lambda = -2$ Any two yields $\lambda = -2, \mu = 5$</p> <p>$l_1: \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $\mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$</p>	<p>Either seeing equation (2) written down correctly with or without any other equation or seeing equations (1) and (3) written down correctly. M1 $\sqrt{}$</p> <p>Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find ... dM1</p> <p>either one of λ or μ correct. A1</p> <p>$\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $5\mathbf{i} + 5\mathbf{k}$ A1 cso</p> <p>Fully correct solution & no incorrect values of λ or μ seen earlier.</p> <p>[4]</p>
		<p>11 marks</p>

Question Number	Scheme	Marks
7. (a)	<p> $\left[x = \ln(t+2), y = \frac{1}{t+1} \right], \Rightarrow \frac{dx}{dt} = \frac{1}{t+2}$ </p> <p> $\text{Area}(R) = \int_{\ln 2}^{\ln 4} \frac{1}{t+1} dx; = \int_0^2 \left(\frac{1}{t+1} \right) \left(\frac{1}{t+2} \right) dt$ </p> <p> Changing limits, when: $x = \ln 2 \Rightarrow \ln 2 = \ln(t+2) \Rightarrow 2 = t+2 \Rightarrow t = 0$ $x = \ln 4 \Rightarrow \ln 4 = \ln(t+2) \Rightarrow 4 = t+2 \Rightarrow t = 2$ </p> <p> Hence, $\text{Area}(R) = \int_0^2 \frac{1}{(t+1)(t+2)} dt$ </p>	<p>Must state $\frac{dx}{dt} = \frac{1}{t+2}$ B1</p> <p>Area = $\int \frac{1}{t+1} dx$. M1; Ignore limits.</p> <p>$\int \left(\frac{1}{t+1} \right) \times \left(\frac{1}{t+2} \right) dt$. Ignore limits. A1 AG</p> <p>changes limits $x \rightarrow t$ so that $\ln 2 \rightarrow 0$ and $\ln 4 \rightarrow 2$ B1</p> <p>[4]</p>
(b)	<p> $\left(\frac{1}{(t+1)(t+2)} \right) = \frac{A}{(t+1)} + \frac{B}{(t+2)}$ </p> <p> $1 = A(t+2) + B(t+1)$ </p> <p> Let $t = -1, 1 = A(1) \Rightarrow \underline{A = 1}$ </p> <p> Let $t = -2, 1 = B(-1) \Rightarrow \underline{B = -1}$ </p> <p> $\int_0^2 \frac{1}{(t+1)(t+2)} dt = \int_0^2 \frac{1}{(t+1)} - \frac{1}{(t+2)} dt$ </p> <p> $= [\ln(t+1) - \ln(t+2)]_0^2$ </p> <p> $= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$ </p> <p> $= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$ </p>	<p>$\frac{A}{(t+1)} + \frac{B}{(t+2)}$ with A and B found M1</p>
	<p>Finds both A and B correctly. Can be implied. (See note below)</p>	<p>A1</p>
	<p> Either $\pm a \ln(t+1)$ or $\pm b \ln(t+2)$ dM1 Both ln terms correctly ft. A1 $\sqrt{\quad}$ </p> <p> Substitutes both limits of 2 and 0 and subtracts the correct way round. ddM1 </p> <p> $\ln 3 - \ln 4 + \ln 2$ or $\ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)$ or $\ln 3 - \ln 2$ or $\ln\left(\frac{3}{2}\right)$ (must deal with ln 1) </p>	<p>A1 aef isw [6]</p>

Takes out brackets.

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} + \frac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} - \frac{1}{(t+2)}$ means first M1A1 in (b).

Question Number	Scheme	Marks
7. (c)	$x = \ln(t+2), \quad y = \frac{1}{t+1}$ $e^x = t+2 \Rightarrow t = e^x - 2$ $y = \frac{1}{e^x - 2 + 1} \Rightarrow y = \frac{1}{e^x - 1}$	Attempt to make $t = \dots$ the subject giving $t = e^x - 2$ M1 A1 Eliminates t by substituting in y giving $y = \frac{1}{e^x - 1}$ dM1 A1 [4]
Aliter 7. (c) Way 2	$t+1 = \frac{1}{y} \Rightarrow t = \frac{1}{y} - 1 \text{ or } t = \frac{1-y}{y}$ $y(t+1) = 1 \Rightarrow yt + y = 1 \Rightarrow yt = 1 - y \Rightarrow t = \frac{1-y}{y}$ $x = \ln\left(\frac{1}{y} - 1 + 2\right) \text{ or } x = \ln\left(\frac{1-y}{y} + 2\right)$ $x = \ln\left(\frac{1}{y} + 1\right)$ $e^x = \frac{1}{y} + 1$ $e^x - 1 = \frac{1}{y}$ $y = \frac{1}{e^x - 1}$	Attempt to make $t = \dots$ the subject M1 Giving either $t = \frac{1}{y} - 1$ or $t = \frac{1-y}{y}$ A1 Eliminates t by substituting in x dM1 giving $y = \frac{1}{e^x - 1}$ A1 [4]
(d)	Domain : $x > 0$	$x > 0$ or just > 0 B1 [1]
		15 marks

Question Number	Scheme	Marks
<p><i>Aliter</i> 7. (c) Way 3</p>	$e^x = t + 2 \Rightarrow t + 1 = e^x - 1$ $y = \frac{1}{t+1} \Rightarrow y = \frac{1}{e^x - 1}$	<p>Attempt to make $t + 1 = \dots$ the subject giving $t + 1 = e^x - 1$ M1 A1</p> <p>Eliminates t by substituting in y giving $y = \frac{1}{e^x - 1}$ dM1 A1</p> <p style="text-align: right;">[4]</p>
<p><i>Aliter</i> 7. (c) Way 4</p>	$t + 1 = \frac{1}{y} \Rightarrow t + 2 = \frac{1}{y} + 1 \text{ or } t + 2 = \frac{1 + y}{y}$ $x = \ln\left(\frac{1}{y} + 1\right) \text{ or } x = \ln\left(\frac{1 + y}{y}\right)$ $x = \ln\left(\frac{1}{y} + 1\right)$ $e^x = \frac{1}{y} + 1 \Rightarrow e^x - 1 = \frac{1}{y}$ $y = \frac{1}{e^x - 1}$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Attempt to make $t + 2 = \dots$ the subject Either $t + 2 = \frac{1}{y} + 1$ or $t + 2 = \frac{1 + y}{y}$</p> </div> <p>M1 A1</p> <p>Eliminates t by substituting in x dM1</p> <p>giving $y = \frac{1}{e^x - 1}$ A1</p> <p style="text-align: right;">[4]</p>

Question Number	Scheme	Marks
8. (a)	$\frac{dV}{dt} = 1600 - c\sqrt{h} \quad \text{or} \quad \frac{dV}{dt} = 1600 - k\sqrt{h},$ $(V = 4000h \Rightarrow) \frac{dV}{dh} = 4000$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$	<p>Either of these statements M1</p> <p>$\frac{dV}{dh} = 4000$ or $\frac{dh}{dV} = \frac{1}{4000}$ M1</p>
	<p>Either, $\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$</p> <p>or $\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$</p>	<div style="border: 1px solid black; padding: 10px; width: fit-content; margin: auto;"> <p>Convincing proof of $\frac{dh}{dt}$</p> </div> <p>A1 AG [3]</p>
(b)	<p>When $h = 25$ water <i>leaks out such that</i> $\frac{dV}{dt} = 400$</p> $400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$ <p>From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required</p>	<p>Proof that $k = 0.02$ B1 AG [1]</p>
<i>Aliter</i> (b) Way 2	$400 = 4000k\sqrt{h}$ $\Rightarrow 400 = 4000k\sqrt{25}$ $\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$	<p>Using 400, 4000 and $h = 25$ or $\sqrt{h} = 5$. Proof that $k = 0.02$ B1 AG [1]</p>
(c)	$\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$ $\therefore \text{time required} = \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}} dh \quad \frac{\div 0.02}{\div 0.02}$ $\text{time required} = \int_0^{100} \frac{50}{20 - \sqrt{h}} dh$	<p><i>Separates the variables</i> with $\int \frac{dh}{0.4 - k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary. M1 oe</p> <p>Correct proof A1 AG [2]</p>

Question Number	Scheme	Marks
8. (d)	$\int_0^{100} \frac{50}{20-\sqrt{h}} dh \quad \text{with substitution } h = (20-x)^2$ $\frac{dh}{dx} = 2(20-x)(-1) \quad \text{or} \quad \frac{dh}{dx} = -2(20-x)$ $h = (20-x)^2 \Rightarrow \sqrt{h} = 20-x \Rightarrow x = 20-\sqrt{h}$ $\int \frac{50}{20-\sqrt{h}} dh = \int \frac{50}{x} \cdot -2(20-x) dx$ $= 100 \int \frac{x-20}{x} dx$ $= 100 \int \left(1 - \frac{20}{x}\right) dx$ $= 100(x - 20 \ln x) (+c)$ <p>change limits: when $h=0$ then $x=20$ and when $h=100$ then $x=10$</p> $\int_0^{100} \frac{50}{20-\sqrt{h}} dh = [100x - 2000 \ln x]_{20}^{10}$ <p>or</p> $\int_0^{100} \frac{50}{20-\sqrt{h}} dh = [100(20-\sqrt{h}) - 2000 \ln(20-\sqrt{h})]_0^{100}$ $= (1000 - 2000 \ln 10) - (2000 - 2000 \ln 20)$ $= 2000 \ln 20 - 2000 \ln 10 - 1000$ $= 2000 \ln 2 - 1000$	<p>Correct $\frac{dh}{dx}$ B1 aef</p> <p>$\pm \lambda \int \frac{20-x}{x} dx$ or $\pm \lambda \int \frac{20-x}{20-(20-x)} dx$ where λ is a constant M1</p> <p>$\pm \alpha x \pm \beta \ln x ; \alpha, \beta \neq 0$ M1 $100x - 2000 \ln x$ A1</p> <p>Correct use of limits, ie. putting them in the correct way round Either $x=10$ and $x=20$ or $h=100$ and $h=0$ ddM1</p> <p>Combining logs to give... $2000 \ln 2 - 1000$ or $-2000 \ln(\frac{1}{2}) - 1000$ A1 aef</p> <p>[6]</p>
(e)	<p>Time required = $2000 \ln 2 - 1000 = 386.2943611... \text{ sec}$</p> <p>= 386 seconds (nearest second)</p> <p>= 6 minutes and 26 seconds (nearest second)</p>	<p><u>6 minutes, 26 seconds</u> B1</p> <p>[1]</p> <p>13 marks</p>