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Mark Scheme (Results)
January 2009

GCE

GCE Mathematics (6666/01)

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6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$C: \; y^2 - 3y = x^3 + 8$	
	$\left\{\frac{\xcancel{\mathrm{d}}\xcancel{y}}{\xcancel{\mathrm{d}}\xcancel{x}}\xcancel{=}\right\} \quad 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$ Differentiates implicitly to include either $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm 3\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$.)	M1
	Correct equation.	A1
	$(2y-3)\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$ A correct (condoning sign error) attempt to combine or factorise their '$2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x}$'. Can be implied.	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$ $\underline{\frac{3x^2}{2y-3}}$	A1 oe
		(4)
(b)	$y = 3 \Rightarrow \; 9 - 3(3) = x^3 + 8$ Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \;\Rightarrow\; \underline{x = -2}$ Only $\underline{x = -2}$	A1
	$(-2, 3) \;\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(4)}{6-3} \;\Rightarrow\; \frac{\mathrm{d}y}{\mathrm{d}x} = 4$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$	A1 $\sqrt{}$
		(3)
	1(b) final A1 $\sqrt{}$. Note if the candidate inserts their x value and $y = 3$ into $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$, then an answer of $\frac{\mathrm{d}y}{\mathrm{d}x}$ = their x^2, ***may*** indicate a correct follow through.	
		[7]

Question Number	Scheme		Marks
2 (a)	Area$(R) = \int_0^2 \frac{3}{\sqrt{(1+4x)}}\,\mathrm{d}x = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,\mathrm{d}x$		
	$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$	***Integrating*** $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$. <u>Correct integration.</u> Ignore limits.	M1 A1
	$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$		
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}$ (units)^2	$\underline{3}$	<u>A1</u> (4)
	(Answer of 3 with no working scores M0A0M0A0.)		
(b)	Volume $= \underline{\pi \int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2 \mathrm{d}x}$	Use of $V = \underline{\pi\int y^2}\,\mathrm{d}x$. Can be implied. Ignore limits and $\mathrm{d}x$.	B1
	$= (\pi)\int_0^2 \frac{9}{1+4x}\,\mathrm{d}x$		
	$= (\pi)\left[\frac{9}{4}\ln\lvert 1+4x\rvert\right]_0^2$	$\pm k\ln\lvert 1+4x\rvert$ $\frac{9}{4}\ln\lvert 1+4x\rvert$	M1 A1
	$= (\pi)\left[\left(\frac{9}{4}\ln 9\right) - \left(\frac{9}{4}\ln 1\right)\right]$ Note that $\ln 1$ can be implied as equal to 0.	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \underline{\frac{9}{4}\pi\ln 9}$	$\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working here.	Note that $= \frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.	(5)
			[9]

Question Number	Scheme		Marks
3 (a)	$27x^2+32x+16 \equiv A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$	Forming this identity	M1
	$x=-\frac{2}{3},\quad 12-\frac{64}{3}+16=\left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3}=\left(\frac{5}{3}\right)B \Rightarrow B=4$ $x=1,\quad 27+32+16=25C \Rightarrow 75=25C \Rightarrow C=3$	Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations.	M1
		Both $B=4$ and $C=3$ **(Note the A1 is dependent on both method marks in this part.)**	A1
	Equate x^2: $27=-3A+9C \Rightarrow 27=-3A+27 \Rightarrow 0=-3A \Rightarrow A=0$ $x=0,\quad 16=2A+B+4C \Rightarrow 16=2A+4+12 \Rightarrow 0=2A \Rightarrow A=0$	Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A=0$.	B1
			(4)
(b)	$\mathrm{f}(x)=\frac{4}{(3x+2)^2}+\frac{3}{(1-x)}$		
	$=4(3x+2)^{-2}+3(1-x)^{-1}$	Moving powers to top on any one of the two expressions	M1
	$=4\left[2\left(1+\frac{3}{2}x\right)^{-2}\right]+3(1-x)^{-1}$ $=1\left(1+\frac{3}{2}x\right)^{-2}+3(1-x)^{-1}$		
	$=1\left\{\underline{1+(-2)(\tfrac{3x}{2});+\frac{(-2)(-3)}{2!}(\tfrac{3x}{2})^2+\dots}\right\}$	Either $1\pm(-2)(\frac{3x}{2})$ or $1\pm(-1)(-x)$ from either first or second expansions respectively	dM1;
	$+3\left\{\underline{1+(-1)(-x);+\frac{(-1)(-2)}{2!}(-x)^2+\dots}\right\}$	Ignoring 1 and 3, any one correct $\{\underline{\dots\dots\dots}\}$ expansion.	A1
		Both $\{\underline{\dots\dots\dots}\}$ correct.	A1
	$=\left\{1-3x+\frac{27}{4}x^2+\dots\right\}+3\left\{1+x+x^2+\dots\right\}$		
	$=4+0x\ ;+\frac{39}{4}x^2$	$4+(0x)\ ;\ \frac{39}{4}x^2$	A1; A1
			(6)

Question Number	Scheme		Marks
(c)	$\text{Actual} = \text{f}(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$	Attempt to find the actual value of f(0.2) or seeing awrt 4.3 and believing it is candidate's actual f(0.2).	M1
	Or $\text{Actual} = \text{f}(0.2) = \dfrac{4}{(3(0.2)+2)^2} + \dfrac{3}{(1-0.2)}$ $= \dfrac{4}{6.76} + 3.75 = 4.341715976... = \dfrac{2935}{676}$	Candidates can also attempt to find the actual value by using $\dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$ with their A, B and C.	
	$\text{Estimate} = \text{f}(0.2) = 4 + \tfrac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$	Attempt to find an estimate for f(0.2) using their answer to (b)	M1 $\sqrt{}$
	$\%\text{age error} = \dfrac{\lvert 4.39 - 4.341715976...\rvert}{4.341715976...} \times 100$	$\left\lvert \dfrac{\text{their estimate - actual}}{\text{actual}} \right\rvert \times 100$	M1
	$= 1.112095408... = 1.1\%(2\text{sf})$	1.1%	A1 **cao** (4) **[14]**

Question Number	Scheme		Marks
4 (a)	$\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$		
	As $\left\{ \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix}-2\\1\\-4\end{pmatrix} \bullet \begin{pmatrix}q\\2\\2\end{pmatrix} \right\} = \underline{(-2\times q)+(1\times 2)+(-4\times 2)}$	Apply dot product calculation between two direction vectors, ie. $\underline{(-2\times q)+(1\times 2)+(-4\times 2)}$	M1
	$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG	Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$	A1 cso
			(2)
(b)	Lines meet where:		
	$\begin{pmatrix}11\\2\\17\end{pmatrix} + \lambda\begin{pmatrix}-2\\1\\-4\end{pmatrix} = \begin{pmatrix}-5\\11\\p\end{pmatrix} + \mu\begin{pmatrix}q\\2\\2\end{pmatrix}$		
	First two of $\mathbf{i}: \ 11 - 2\lambda = -5 + q\mu$ (1) $\mathbf{j}: \ 2 + \lambda = 11 + 2\mu$ (2) $\mathbf{k}: \ 17 - 4\lambda = p + 2\mu$ (3)	Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)	M1
	(1) + 2**(2)** gives: $15 = 17 + \mu \Rightarrow \mu = -2$	Attempts to solve (1) and (2) to find one of either λ or μ	dM1
		Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$	A1
	(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$	Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$	A1
	(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$	Attempt to substitute their λ and μ into their **k** component to give an equation in p alone.	ddM1
	$\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$	$\underline{p = 1}$	A1 cso
			(6)
(c)	$\mathbf{r} = \begin{pmatrix}11\\2\\17\end{pmatrix} + 5\begin{pmatrix}-2\\1\\-4\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}-5\\11\\1\end{pmatrix} - 2\begin{pmatrix}-3\\2\\2\end{pmatrix}$	Substitutes their value of λ or μ into the correct line l_1 or l_2 .	M1
	Intersect at $\mathbf{r} = \underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1, 7, -3)}$	$\underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1, 7, -3)}$	A1
			(2)

Question Number	Scheme		Marks
(d)	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$	Finding vector $\overrightarrow{AX}$ by finding the difference between $\overrightarrow{OX}$ and $\overrightarrow{OA}$. Can be ft using candidate's $\overrightarrow{OX}$.	M1 $\sqrt{\ }$ ±
	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$		
	$\overrightarrow{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$	$\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} \\ \text{their } \overrightarrow{AX} \\ \\ \end{pmatrix}$	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			(3)
			[13]

Question Number	Scheme		Marks
5 (a)	Similar triangles $\Rightarrow \underline{\frac{r}{h}=\frac{16}{24}} \Rightarrow \underline{r=\frac{2h}{3}}$	Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V=\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h = \frac{4\pi h^3}{27}$ **AG**	Substitutes $r=\frac{2h}{3}$ into the formula for the volume of water V.	A1
			(2)
(b)	From the question, $\frac{\mathrm{d}V}{\mathrm{d}t}=8$	$\frac{\mathrm{d}V}{\mathrm{d}t}=8$	B1
	$\underline{\frac{\mathrm{d}V}{\mathrm{d}h}}=\underline{\frac{12\pi h^2}{27}}=\underline{\frac{4\pi h^2}{9}}$	$\underline{\frac{\mathrm{d}V}{\mathrm{d}h}}=\underline{\frac{12\pi h^2}{27}}$ or $\underline{\frac{4\pi h^2}{9}}$	B1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{dh} = \underline{8\times\frac{9}{4\pi h^2}} = \underline{\frac{18}{\pi h^2}}$	Candidate's $\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{\mathrm{d}h}$;	M1;
		$\underline{8\div\left(\frac{12\pi h^2}{27}\right)}$ or $\underline{8\times\frac{9}{4\pi h^2}}$ or $\underline{\frac{18}{\pi h^2}}$ oe	A1
	When $h=12$, $\frac{\mathrm{d}h}{\mathrm{d}t}=\underline{\frac{18}{144\pi}}=\underline{\frac{1}{8\pi}}$	$\underline{\frac{18}{144\pi}}$ or $\underline{\frac{1}{8\pi}}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working after $\underline{\frac{18}{144\pi}}$.		(5)
			[7]

Question Number	Scheme	Marks
6 (a)	$\int \tan^2 x \, \mathrm{d}x$	
	$\left[NB: \underline{\sec^2 A = 1 + \tan^2 A} \text{ gives } \underline{\tan^2 A = \sec^2 A - 1}\right]$ The correct <u>underlined identity</u>.	M1 oe
	$= \int \sec^2 x - 1 \, \mathrm{d}x$	
	$= \underline{\tan x - x} \; (+ c)$ Correct integration with/without + c	A1
		(2)
(b)	$\int \frac{1}{x^3} \ln x \, \mathrm{d}x$	
	$\left\{ \begin{array}{l} u = \ln x \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = x^{-3} \quad \Rightarrow \quad v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array} \right\}$	
	$= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} . \frac{1}{x} \, \mathrm{d}x$ Use of 'integration by parts' formula in the correct direction. Correct direction means that $u = \ln x$.	M1
	Correct expression.	A1
	$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} \, \mathrm{d}x$ An attempt to multiply through $\frac{k}{x^n}$, $n \in$ □ , $n \ldots 2$ by $\frac{1}{x}$ and an attempt to ...	
	$= \underline{-\frac{1}{2x^2} \ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)} \; (+ c)$ … "integrate"(process the result);	M1
	<u>correct solution</u> with/without + c	A1 oe
		(4)

Question Number	Scheme		Marks
(c)	$\int \frac{\mathrm{e}^{3x}}{1+\mathrm{e}^x}\,\mathrm{d}x$		
	$\left\{u = 1+\mathrm{e}^x \Rightarrow \underline{\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x}\,,\ \underline{\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{\mathrm{e}^x}}\,,\ \underline{\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{u-1}}\right\}$	Differentiating to find any one of the <u>three underlined</u>	<u>B1</u>
	$= \int \frac{\mathrm{e}^{2x}.\mathrm{e}^x}{1+\mathrm{e}^x}\,\mathrm{d}x = \int \frac{(u-1)^2.\mathrm{e}^x}{u}.\frac{1}{\mathrm{e}^x}\,\mathrm{d}u$ or $= \int \frac{(u-1)^3}{u}.\frac{1}{(u-1)}\,\mathrm{d}u$	Attempt to substitute for $\mathrm{e}^{2x} = \mathrm{f}(u)$, their $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{\mathrm{e}^x}$ and $u = 1+\mathrm{e}^x$ or $\mathrm{e}^{3x} = \mathrm{f}(u)$, their $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{u-1}$ and $u = 1+\mathrm{e}^x$.	M1*
	$= \int \frac{(u-1)^2}{u}\,\mathrm{d}u$	$\underline{\int \frac{(u-1)^2}{u}\,\mathrm{d}u}$	A1
	$= \int \frac{u^2-2u+1}{u}\,\mathrm{d}u$ $= \int u - 2 + \frac{1}{u}\,\mathrm{d}u$	An attempt to multiply out their numerator to give at least three terms and divide through each term by u	dM1*
	$= \frac{u^2}{2} - 2u + \ln u\ (+c)$	Correct integration with/without +c	A1
	$= \frac{(1+\mathrm{e}^x)^2}{2} - 2(1+\mathrm{e}^x) + \ln(1+\mathrm{e}^x) + c$	Substitutes $u = 1+\mathrm{e}^x$ back into their integrated expression with at least two terms.	dM1*
	$= \frac{1}{2} + \mathrm{e}^x + \frac{1}{2}\mathrm{e}^{2x} - 2 - 2\mathrm{e}^x + \ln(1+\mathrm{e}^x) + c$		
	$= \frac{1}{2} + \mathrm{e}^x + \frac{1}{2}\mathrm{e}^{2x} - 2 - 2\mathrm{e}^x + \ln(1+\mathrm{e}^x) + c$		
	$= \frac{1}{2}\mathrm{e}^{2x} - \mathrm{e}^x + \ln(1+\mathrm{e}^x) - \frac{3}{2} + c$		
	$= \frac{1}{2}\mathrm{e}^{2x} - \mathrm{e}^x + \ln(1+\mathrm{e}^x) + k$ **AG**	$\underline{\frac{1}{2}\mathrm{e}^{2x} - \mathrm{e}^x + \ln(1+\mathrm{e}^x) + k}$ must use a $+c$ and "$-\frac{3}{2}$" combined.	A1 **cso** (7) **[13]**

Question Number	Scheme	Marks
7 (a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$ $A(7,1)$	B1 (1)
(b)	$x=t^3-8t$, $y=t^2$,	
	$\frac{\mathrm{d}x}{\mathrm{d}t}=3t^2-8$, $\frac{\mathrm{d}y}{\mathrm{d}t}=2t$	
	$\therefore \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2t}{3t^2-8}$ Their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1
	At A, $\mathrm{m}(\mathbf{T})=\underline{\frac{2(-1)}{3(-1)^2-8}}=\underline{\frac{-2}{3-8}}=\underline{\frac{-2}{-5}}=\underline{\frac{2}{5}}$ Substitutes for t to give any of the four underlined oe:	
	$\mathbf{T}$: $y-\left(\text{their } 1\right)=m_T\left(x-\left(\text{their } 7\right)\right)$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$ Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+\text{"}c\text{"}$.	dM1
	Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$	
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG** $\underline{2x-5y-9=0}$	A1 **cso** (5)
(c)	$2(t^3-8t)-5t^2-9=0$ Substitution of both $x=t^3-8t$ and $y=t^2$ into $\mathbf{T}$	M1
	$2t^3-5t^2-16t-9=0$	
	$(t+1)\left\{(2t^2-7t-9)=0\right\}$ $(t+1)\left\{(t+1)(2t-9)=0\right\}$ A realisation that $(t+1)$ is a factor.	dM1
	$\left\{t=-1 \text{ (at } A)\right\}$ $t=\frac{9}{2}$ at B $t=\frac{9}{2}$	A1
	$x=\left(\frac{9}{2}\right)^2-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}=55.125$ or awrt 55.1 Candidate uses their value of t to find either the x or y coordinate	ddM1
	$y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3 One of either x or y correct. Both x and y correct.	A1 A1
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$ awrt	(6)
		[12]