Mark Scheme (Results)

January 2013

GCE Mathematics
6666 Core Mathematics 4

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2013
Publications Code UA034368

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes .

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ or AG: The answer is printed on the paper
- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ddM1 denotes a method mark which is dependent upon the award of the previous 2 method marks.
- dM1* denotes a method mark which is dependent upon the award of the M1* mark.

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Misreads

A misread must be consistent for the whole question to be interpreted as such.
These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.

January 2013
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1.	$(2+3x)^{-3} = \underline{(2)^{-3}}\left(1+\frac{3x}{2}\right)^{-3} = \underline{\frac{1}{8}}\left(1+\frac{3x}{2}\right)^{-3}$	$\underline{(2)^{-3}}$ or $\underline{\frac{1}{8}}$	<u>B1</u>
	$=\left\{\frac{1}{8}\right\}\left[1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3+\dots\right]$	see notes	M1 A1
	$=\left\{\frac{1}{8}\right\}\left[\underline{1+(-3)\left(\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x}{2}\right)^3+\dots}\right]$		
	$=\frac{1}{8}\left[1-\frac{9}{2}x;+\frac{27}{2}x^2-\frac{135}{4}x^3+\dots\right]$	See notes below!	
	$=\frac{1}{8}-\frac{9}{16}x;+\frac{27}{16}x^2-\frac{135}{32}x^3+\dots$		A1; A1
			[5] **5**

<u>B1</u>: $\underline{(2)^{-3}}$ or $\underline{\frac{1}{8}}$ outside brackets or $\underline{\frac{1}{8}}$ as constant term in the binomial expansion.

M1: Expands $(\dots+kx)^{-3}$ to give any 2 terms out of 4 terms simplified or un-simplified,

Eg: $1+(-3)(kx)$ or $(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2$ or $1+\dots\dots+\frac{(-3)(-4)}{2!}(kx)^2$

or $\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ **where** $k\neq 1$ are ok for M1.

A1: A correct simplified or un-simplified $1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent (kx) where $k\neq 1$.

"Incorrect bracketing" $\left\{\frac{1}{8}\right\}\left[\underline{1+(-3)\left(\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x^2}{2}\right)+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x^3}{2}\right)+\dots}\right]$ is M1A0

unless recovered.

A1: For $\frac{1}{8}-\frac{9}{16}x$ **(simplified fractions)** or also allow $0.125-0.5625x$.

Allow Special Case A1 for either SC: $\frac{1}{8}\left[1-\frac{9}{2}x;\dots\right]$ or **SC:** $K\left[1-\frac{9}{2}x+\frac{27}{2}x^2-\frac{135}{4}x^3+\dots\right]$

(where K can be 1 or omitted), with each term in the $[\dots\dots\dots]$ either a simplified fraction or a decimal.

A1: Accept only $\frac{27}{16}x^2-\frac{135}{32}x^3$ or $1\frac{11}{16}x^2-4\frac{7}{32}x^3$ or $1.6875x^2-4.21875x^3$

1. ctd	Candidates who write $=\frac{1}{8}\left[1+(-3)\left(-\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(-\frac{3x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(-\frac{3x}{2}\right)^3+\ldots\right]$ where $k=-\frac{3}{2}$ and not $\frac{3}{2}$ and achieve $\frac{1}{8}+\frac{9}{16}x+\frac{27}{16}x^2+\frac{135}{32}x^3+\ldots$ will get B1M1A1A0A0. ***Alternative method:*** Candidates can apply an alternative form of the binomial expansion. $(2+3x)^{-3} = (2)^{-3}+(-3)(2)^{-4}(3x)+\frac{(-3)(-4)}{2!}(2)^{-5}(3x)^2+\frac{(-3)(-4)(-5)}{3!}(2)^{-6}(3x)^3$ **B1:** $\frac{1}{8}$ or $(2)^{-3}$ **M1:** Any two of four (un-simplified) terms correct. **A1:** All four (un-simplified) terms correct. **A1:** $\frac{1}{8}-\frac{9}{16}x$ **A1:** $+\frac{27}{16}x^2-\frac{135}{32}x^3$ **Note:** The terms in C need to be evaluated, so ${}^{-3}C_0(2)^{-3}+{}^{-3}C_1(2)^{-4}(3x)+{}^{-3}C_2(2)^{-5}(3x)^2+{}^{-3}C_3(2)^{-6}(3x)^3$ without further working is B0M0A0.

Question Number	Scheme	
2. (a)	$\int \frac{1}{x^3}\ln x\,\mathrm{d}x$, $\left\{\begin{array}{l} u=\ln x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x}=x^{-3} \Rightarrow v=\frac{x^{-2}}{-2}=\frac{-1}{2x^2}\end{array}\right\}$	
	In the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu\frac{1}{x^2}.\frac{1}{x}$	M1
	$=\underline{\frac{-1}{2x^2}\ln x} - \underline{\underline{\int \frac{-1}{2x^2}.\frac{1}{x}\,\mathrm{d}x}}$ — $\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified.	A1
	$\underline{\underline{-\int \frac{-1}{2x^2}.\frac{1}{x}}}$ simplified or un-simplified.	A1
	$\left\{= \frac{-1}{2x^2}\ln x + \frac{1}{2}\int \frac{1}{x^3}\,\mathrm{d}x\right\}$	
	$=-\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)\{+c\}$ — $\pm\int \mu\frac{1}{x^2}.\frac{1}{x} \to \pm\beta x^{-2}$.	dM1
	Correct answer, with/without $+c$	A1
		[5]
(b)	$\left\{\left[-\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\right]_1^2\right\} = \left(-\frac{1}{2(2)^2}\ln 2 - \frac{1}{4(2)^2}\right) - \left(-\frac{1}{2(1)^2}\ln 1 - \frac{1}{4(1)^2}\right)$ — Applies limits of 2 and 1 to their part (a) answer and subtracts the correct way round.	M1
	$=\frac{3}{16}-\frac{1}{8}\ln 2$ or $\frac{3}{16}-\ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3-2\ln 2)$, etc, or awrt 0.1 — or equivalent.	A1
		[2] **7**
(a)	**M1**: Integration by parts is applied in the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu\frac{1}{x^2}.\frac{1}{x}$ or equivalent. **A1**: $\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified. **A1**: $\underline{\underline{-\int \frac{-1}{2x^2}.\frac{1}{x}}}$ or equivalent. You can ignore the $\mathrm{d}x$. **dM1:** Depends on the previous M1. $\pm\int \mu\frac{1}{x^2}.\frac{1}{x} \to \pm\beta x^{-2}$. **A1:** $-\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)\{+c\}$ or $=-\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\{+c\}$ or $\frac{x^{-2}}{-2}\ln x - \frac{x^{-2}}{4}\{+c\}$ or $\frac{-1-2\ln x}{4x^2}\{+c\}$ or equivalent. You can ignore subsequent working after a correct stated answer.	
(b)	**M1:** Some evidence of applying limits of 2 and 1 to their part (a) answer and subtracts the correct way round. **A1:** ***Two term exact answer*** of either $\frac{3}{16}-\frac{1}{8}\ln 2$ or $\frac{3}{16}-\ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3-2\ln 2)$ or $\frac{\ln\left(\frac{1}{4}\right)+3}{16}$ or $0.1875-0.125\ln 2$. Also allow awrt 0.1. Also note the fraction terms must be combined. **Note:** Award the final A0 in part (b) for a candidate who achieves awrt 0.1 in part (b), when their answer to part (a) is incorrect.	

2. (b) ctd	**Note:** Decimal answer is 0.100856... in part (b).	
	Alternative Solution	
	$\int \frac{1}{x^3}\ln x \,\mathrm{d}x$, $\left\{\begin{matrix} u = x^{-3} & \Rightarrow & \frac{\mathrm{d}u}{\mathrm{d}x} = -3x^{-4} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \ln x & \Rightarrow & v = x\ln x - x \end{matrix}\right\}$	
	$\int \frac{1}{x^3}\ln x \,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) - \int (x\ln x - x)\frac{-3}{x^4}\mathrm{d}x$	
	$k\int \frac{1}{x^3}\ln x \,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) \pm \int \frac{\lambda}{x^3}\mathrm{d}x$ where $k \neq 1$	M1
	$-2\int \frac{1}{x^3}\ln x \,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) - \int \frac{3}{x^3}\mathrm{d}x$ — Any one of $\frac{1}{x^3}(x\ln x - x)$ or $-\int \frac{3}{x^3}\mathrm{d}x$	A1
	$\frac{1}{x^3}(x\ln x - x) - \int \frac{3}{x^3}\mathrm{d}x$ and $k = -2$	A1
	$-2\int \frac{1}{x^3}\ln x \,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) + \frac{3}{2x^2}\ \{+c\}$ — $\pm\int \mu\frac{1}{x^3} \to \pm\beta x^{-2}$.	dM1
	$\int \frac{1}{x^3}\ln x \,\mathrm{d}x = -\frac{1}{2x^3}(x\ln x - x) - \frac{3}{4x^2}\ \{+c\}$ — $-\frac{1}{2x^3}(x\ln x - x) - \frac{3}{4x^2}$ or equivalent with/without $+c$.	A1
	$= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\ \{+c\}$	

Question Number	Scheme		Marks
3.	**Method 1: Using one identity**		
	$\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv A + \dfrac{B}{(x+2)} + \dfrac{C}{(3x-1)}$		
	$A = 3$	their constant term $= 3$	B1
	$9x^2+20x-10 \equiv A(x+2)(3x-1) + B(3x-1) + C(x+2)$	Forming a correct identity.	B1
	Either $x^2: \ 9 = 3A, \quad x: \ 20 = 5A + 3B + C$ constant: $-10 = -2A - B + 2C$ or $x = -2 \Rightarrow 36 - 40 - 10 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x = \dfrac{1}{3} \Rightarrow 1 + \dfrac{20}{3} - 10 = \dfrac{7}{3}C \Rightarrow -\dfrac{7}{3} = \dfrac{7}{3}C \Rightarrow C = -1$	Correct values for their B and their C, which are found using a correct identity.	A1
			[4]
	Method 2: Long Division		
	$\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{5x-4}{(x+2)(3x-1)}$	their constant term $= 3$	B1
	So, $\dfrac{5x-4}{(x+2)(3x-1)} \equiv \dfrac{B}{(x+2)} + \dfrac{C}{(3x-1)}$		
	$5x - 4 \equiv B(3x-1) + C(x+2)$	Forming a correct identity.	B1
	Either $x: \ 5 = 3B + C$, constant: $-4 = -B + 2C$ or $x = -2 \Rightarrow -10 - 4 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x = \dfrac{1}{3} \Rightarrow \dfrac{5}{3} - 4 = \dfrac{7}{3}C \Rightarrow -\dfrac{7}{3} = \dfrac{7}{3}C \Rightarrow C = -1$	Correct values for their B and their C, which are found using $5x - 4 \equiv B(3x-1) + C(x+2)$	A1
	So, $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{2}{(x+2)} - \dfrac{1}{(3x-1)}$		**[4]**
			4

1st B1: Their constant term must be equal to 3 for this mark.
2nd B1 (M1 on epen): Forming a correct identity. This can be implied by later working.
M1 (A1 on epen): Attempts to find the value of either one of their B or their C from their identity. This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.
A1: Correct values for their B and their C, which are found using a correct identity.
Note : $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv \dfrac{A}{(x+2)} + \dfrac{B}{(3x-1)}$, leading to $9x^2+20x-10 \equiv A(3x-1) + B(x+2)$, leading to $A = 2$ and $B = -1$ will gain a maximum of B0B0M1A0

.

3. ctd	**Note:** You can imply the 2nd B1 from either $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv \frac{A(x+2)(3x-1)+B(3x-1)+C(x+2)}{(x+2)(3x-1)}$ or $\frac{5x-4}{(x+2)(3x-1)} \equiv \frac{B(3x-1)+C(x+2)}{(x+2)(3x-1)}$ **Alternative Method 1: Initially dividing by (x + 2)** $\frac{9x^2+20x-10}{"(x+2)"(3x-1)} \equiv \frac{9x+2}{(3x-1)} - \frac{14}{(x+2)(3x-1)}$ $\equiv 3 + \frac{5}{(3x-1)} - \frac{14}{(x+2)(3x-1)}$ **B1:** their constant term $= 3$ So, $\frac{-14}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$ $-14 \equiv B(3x-1) + C(x+2)$ **B1:** Forming a correct identity. $\Rightarrow B = 2, C = -6$ **M1:** Attempts to find either one of their B or their C from their identity. So, $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{5}{(3x-1)} + \frac{2}{(x+2)} - \frac{6}{(3x-1)}$ and $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$ **A1:** Correct answer in partial fractions. **Alternative Method 2: Initially dividing by (3x - 1)** $\frac{9x^2+20x-10}{(x+2)"(3x-1)"} \equiv \frac{3x+\frac{23}{3}}{(x+2)} - \frac{\frac{7}{3}}{(x+2)(3x-1)}$ $\equiv 3 + \frac{\frac{5}{3}}{(x+2)} - \frac{\frac{7}{3}}{(x+2)(3x-1)}$ **B1:** their constant term $= 3$ So, $\frac{-\frac{7}{3}}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$ $-\frac{7}{3} \equiv B(3x-1) + C(x+2)$ **B1:** Forming a correct identity. $\Rightarrow B = \frac{1}{3}, C = -1$ **M1:** Attempts to find either one of their B or their C from their identity. So, $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{\frac{5}{3}}{(x+2)} + \frac{\frac{1}{3}}{(x+2)} - \frac{1}{(3x-1)}$ and $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$ **A1:** Correct answer in partial fractions.

Question Number	Scheme		Marks
4. (a)	1.0981		B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times 1\,;\times\underline{\left[0.5+2\left(0.8284+\text{their }1.0981\right)+1.3333\right]}$		B1; <u>M1</u>
	$=\frac{1}{2}\times 5.6863 = 2.84315 = 2.843$ (3 dp)	2.843 or awrt 2.843	A1 **[3]**
(c)	$\left\{u=1+\sqrt{x}\right\} \Rightarrow \underline{\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}x^{-\frac{1}{2}}}$ or $\underline{\frac{\mathrm{d}x}{\mathrm{d}u}=2(u-1)}$		<u>B1</u>
	$\left\{\int\frac{x}{1+\sqrt{x}}\,\mathrm{d}x=\right\}\int\frac{(u-1)^2}{u}.\,2(u-1)\,\mathrm{d}u$	$\int\frac{(u-1)^2}{u}\ldots\ldots$	M1
		$\int\frac{(u-1)^2}{u}.\,2(u-1)$	A1
	$=2\int\frac{(u-1)^3}{u}\,\mathrm{d}u \;=\{2\}\int\frac{(u^3-3u^2+3u-1)}{u}\,\mathrm{d}u$	Expands to give a "four term" cubic in u. Eg: $\pm Au^3 \pm Bu^2 \pm Cu \pm D$	M1
	$=\{2\}\int\left(u^2-3u+3-\frac{1}{u}\right)\mathrm{d}u$	An attempt to divide at least three terms in ***their cubic*** by u. See notes.	M1
	$=\{2\}\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$	$\int\frac{(u-1)^3}{u}\to\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$	A1
	$\text{Area}(R)=\left[\frac{2u^3}{3}-3u^2+6u-2\ln u\right]_2^3$		
	$=\left(\frac{2(3)^3}{3}-3(3)^2+6(3)-2\ln 3\right)-\left(\frac{2(2)^3}{3}-3(2)^2+6(2)-2\ln 2\right)$	Applies limits of 3 and 2 in u or 4 and 1 in x and subtracts either way round.	M1
	$=\frac{11}{3}+2\ln 2-2\ln 3$ or $\frac{11}{3}+2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3}-\ln\left(\frac{9}{4}\right)$, etc	Correct exact answer or equivalent.	A1 **[8]** **12**

(a) **B1:** 1.0981 correct answer only. Look for this on the table or in the candidate's working.

(b) **B1**: Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$

M1: <u>For structure of trapezium rule</u> $[\;\underline{\ldots\ldots\ldots\ldots}\;]$

A1: anything that rounds to 2.843

<u>Note:</u> Working must be seen to demonstrate the use of the trapezium rule. **<u>Note</u>**: actual area is 2.85573645…

<u>Note:</u> Award B1M1 A1 for $\frac{1}{2}(0.5+1.3333)+\left(0.8284+\text{their }1.0981\right)=2.84315$

<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly**

Award B1M0A0 for $\frac{1}{2}\times 1+0.5+2\left(0.8284+\text{their }1.0981\right)+1.3333$ (nb: answer of 6.1863).

Award B1M0A0 for $\frac{1}{2}\times 1\;(0.5+1.3333)+2\left(0.8284+\text{their }1.0981\right)$ (nb: answer of 4.76965).

4. (b) ctd

Alternative method for part (b): Adding individual trapezia

$$\text{Area} \approx 1\times\left[\frac{0.5+0.8284}{2}+\frac{0.8284+1.0981}{2}+\frac{1.0981+1.3333}{2}\right] = 2.84315$$

B1: 1 and a divisor of 2 on all terms inside brackets.
M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
A1: anything that rounds to 2.843

(c)

B1: $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}$ or $\mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x$ or $2\sqrt{x}\,\mathrm{d}u = \mathrm{d}x$ or $\mathrm{d}x = 2(u-1)\mathrm{d}u$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = 2(u-1)$ oe.

1st M1: $\frac{x}{1+\sqrt{x}}$ becoming $\frac{(u-1)^2}{u}$ (Ignore integral sign).

1st A1 (B1 on epen): $\frac{x}{1+\sqrt{x}}\,\mathrm{d}x$ becoming $\frac{(u-1)^2}{u}.\,2(u-1)\{\mathrm{d}u\}$ or $\frac{(u-1)^2}{u}.\frac{2}{(u-1)^{-1}}\{\mathrm{d}u\}$.

You can ignore the integral sign and the $\mathrm{d}u$.

2nd M1: Expands to give a "four term" cubic in u, $\pm Au^3 \pm Bu^2 \pm Cu \pm D$
where $A\neq 0$, $B\neq 0$, $C\neq 0$ and $D\neq 0$ The cubic does not need to be simplified for this mark.

3rd M1: An attempt to divide at least three terms in ***their cubic*** by u.
Ie. $\frac{(u^3-3u^2+3u-1)}{u} \to u^2-3u+3-\frac{1}{u}$

2nd A1: $\int\frac{(u-1)^3}{u}\,\mathrm{d}u \to \left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$

4th M1: Some evidence of limits of 3 and 2 in u and subtracting either way round.

3rd A1: Exact answer of $\frac{11}{3}+2\ln 2-2\ln 3$ or $\frac{11}{3}+2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3}-\ln\left(\frac{9}{4}\right)$ or $2\left(\frac{11}{6}+\ln 2-\ln 3\right)$
or $\frac{22}{6}+2\ln\left(\frac{2}{3}\right)$, etc. **Note**: that fractions must be combined to give either $\frac{11}{3}$ or $\frac{22}{6}$ or $3\frac{2}{3}$

<u>Alternative method for 2nd M1 and 3rd M1 mark</u>

$$\{2\}\int\frac{(u-1)^2}{u}.(u-1)\,\mathrm{d}u = \{2\}\int\frac{(u^2-2u+1)}{u}.(u-1)\,\mathrm{d}u$$

$$= \{2\}\int\left(u-2+\frac{1}{u}\right).(u-1)\,\mathrm{d}u = \{2\}\int\left(u^2-\ldots\right)\,\mathrm{d}u$$

An attempt to expand $(u-1)^2$, then divide the result by u and then go on to multiply by $(u-1)$. **2nd M1**

$$= \{2\}\int\left(u^2-2u+1-u+2-\frac{1}{u}\right)\mathrm{d}u$$

$$= \{2\}\int\left(u^2-3u+3-\frac{1}{u}\right)\mathrm{d}u$$

to give three out of four of $\pm Au^2$, $\pm Bu$, $\pm C$ or $\pm\frac{D}{u}$ **3rd M1**

4. (c) ctd	**Final two marks in part (c):** $u = 1 + \sqrt{x}$	
	$\text{Area}(R) = \left[\dfrac{2\left(1+\sqrt{x}\right)^3}{3} - 3\left(1+\sqrt{x}\right)^2 + 6\left(1+\sqrt{x}\right) - 2\ln\left(1+\sqrt{x}\right)\right]_1^4$	
	$= \left(\dfrac{2\left(1+\sqrt{4}\right)^3}{3} - 3\left(1+\sqrt{4}\right)^2 + 6\left(1+\sqrt{4}\right) - 2\ln\left(1+\sqrt{4}\right)\right)$ $- \left(\dfrac{2\left(1+\sqrt{1}\right)^3}{3} - 3\left(1+\sqrt{1}\right)^2 + 6\left(1+\sqrt{1}\right) - 2\ln\left(1+\sqrt{1}\right)\right)$	**M1:** Applies limits of 4 and 1 in x and subtracts either way round.
	$= (18 - 27 + 18 - 2\ln 3) - \left(\dfrac{16}{3} - 12 + 12 - 2\ln 2\right)$	
	$= \dfrac{11}{3} + 2\ln 2 - 2\ln 3$ or $\dfrac{11}{3} + 2\ln\left(\dfrac{2}{3}\right)$ or $\dfrac{11}{3} - \ln\left(\dfrac{9}{4}\right)$, etc	**A1:** Correct exact answer or equivalent.
	Alternative method for the final 5 marks in part (b)	
	$\displaystyle\int \frac{(u-1)^3}{u}\,\mathrm{d}u$, $\left\{\begin{array}{lll} "u" = u^{-1} & \Rightarrow & \dfrac{\mathrm{d}"u"}{\mathrm{d}x} = -u^{-2} \\ \dfrac{\mathrm{d}v}{\mathrm{d}x} = (u-1)^3 & \Rightarrow & v = \dfrac{(u-1)^4}{4} \end{array}\right\}$	
	$\displaystyle = \frac{(u-1)^4}{4u} - -\frac{1}{4}\int \frac{(u-1)^4}{u^2}\,\mathrm{d}u$	
	$\displaystyle = \frac{(u-1)^4}{4u} + \frac{1}{4}\int \frac{u^4 - 4u^3 + 6u^2 - 4u + 1}{u^2}\,\mathrm{d}u$	**M1:** Applies integration by parts and expands to give a five term quartic.
	$\displaystyle = \frac{(u-1)^4}{4u} + \frac{1}{4}\int u^2 - 4u + 6 - \frac{4}{u} + \frac{1}{u^2}\,\mathrm{d}u$	**M1:** Dividing at least 4 terms.
	$\displaystyle = \frac{(u-1)^4}{4u} + \frac{1}{4}\left(\frac{u^3}{3} - 2u^2 + 6u - 4\ln u - \frac{1}{u}\right)$	**A1:** Correct Integration.
	$\displaystyle \int_2^3 \frac{(u-1)^3}{u}\,\mathrm{d}u = \left[\frac{(u-1)^4}{4u} + \frac{u^3}{12} - \frac{u^2}{2} + \frac{3u}{2} - \ln u - \frac{1}{4u}\right]_2^3$	
	$= \left(\dfrac{16}{12} + \dfrac{27}{12} - \dfrac{9}{2} + \dfrac{9}{2} - \ln 3 - \dfrac{1}{12}\right) - \left(\dfrac{1}{8} + \dfrac{8}{12} - \dfrac{4}{2} + \dfrac{6}{2} - \ln 2 - \dfrac{1}{8}\right)$	**M1**
	$= (7 - \ln 3) - \left(\dfrac{5}{3} - \ln 2\right)$	
	$= \dfrac{11}{6} + \ln\dfrac{2}{3}$	
	$\displaystyle \text{Area}(R) = 2\int_2^3 \frac{(u-1)^3}{u}\,\mathrm{d}u = 2\left(\frac{11}{6} + \ln\frac{2}{3}\right)$	**A1**

Question Number	Scheme		Marks
5.	***Working parametrically:***		
	$x = 1 - \frac{1}{2}t$, $y = 2^t - 1$ or $y = e^{t\ln 2} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$	Applies $x = 0$ to obtain a value for t.	M1
	When $t = 2$, $y = 2^2 - 1 = 3$	Correct value for y.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^t - 1 \Rightarrow t = 0$	Applies $y = 0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t \ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$.	M1
	At A, $t = "2"$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $t = "2"$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	See notes.	M1 A1 oe cso
			[5]
(d)	$\text{Area}(R) = \int (2^t - 1).\left(-\frac{1}{2}\right)dt$	Complete substitution for both y and dx	M1
	$x = -1 \to t = 4$ and $x = 1 \to t = 0$		B1
	$= \left\{-\frac{1}{2}\right\}\left(\frac{2^t}{\ln 2} - t\right)$	Either $2^t \to \frac{2^t}{\ln 2}$ or $\left(2^t - 1\right) \to \frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $\left(2^t - 1\right) \to \pm\alpha(\ln 2)(2^t) - t$	M1*
		$\left(2^t - 1\right) \to \frac{2^t}{\ln 2} - t$	A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2} - t\right]_4^0\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - 4\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits in t and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6]
			15

5. (a)	**M1:** Applies $x = 0$ and obtains a value of t.
	A1: For $y = 2^2 - 1 = 3$ or $y = 4 - 1 = 3$
	Alternative Solution 1:
	M1: For substituting $t = 2$ into either x or y.
	A1: $x = 1 - \frac{1}{2}(2) = 0$ **and** $y = 2^2 - 1 = 3$
	Alternative Solution 2:
	M1: Applies $y = 3$ and obtains a value of t.
	A1: For $x = 1 - \frac{1}{2}(2) = 0$ or $x = 1 - 1 = 0$.
	Alternative Solution 3:
	M1: Applies $y = 3$ or $x = 0$ and obtains a value of t.
	A1: Shows that $t = 2$ for both $y = 3$ and $x = 0$.
(b)	**M1:** Applies $y = 0$ and obtains a value of t. Working must be seen in part (b).
	A1: For finding $x = 1$.
	Note: Award M1A1 for $x = 1$.
(c)	**B1:** Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. This mark can be implied by later working.
	M1: Their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ or their $\frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{1}{\text{their}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)}$. **Note:** their $\frac{\mathrm{d}y}{\mathrm{d}t}$ must be a function of t.
	M1: Uses their value of t found in part (a) and applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$.
	M1: $y - 3 = (\text{their normal gradient})x$ **or** $y = (\text{their normal gradient})x + 3$ or equivalent.
	A1: $y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or $y - 3 = \frac{1}{\ln 256}(x - 0)$ or $(8\ln 2)y - 24\ln 2 = x$ or $\frac{y-3}{(x-0)} = \frac{1}{8\ln 2}$. You can apply isw here.
	Working in decimals is ok for the three method marks. B1, A1 require exact values.
(d)	**M1:** Complete substitution for both y and $\mathrm{d}x$. So candidate should write down $\int (2^t - 1).\left(\text{their } \frac{\mathrm{d}x}{\mathrm{d}t}\right)$
	B1: Changes limits from $x \to t$. $x = -1 \to t = 4$ and $x = 1 \to t = 0$. Note $t = 4$ and $t = 0$ seen is B1.
	M1*: Integrates 2^t correctly to give $\frac{2^t}{\ln 2}$
	... or integrates $(2^t - 1)$ to give either $\frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $\pm\alpha(\ln 2)(2^t) - t$.
	A1: Correct integration of $(2^t - 1)$ with respect to t to give $\frac{2^t}{\ln 2} - t$.
	dM1*: **Depends upon the previous method mark.** Substitutes their limits in t and subtracts either way round.
	A1: Exact answer of $\frac{15}{2\ln 2} - 2$ or $\frac{15}{\ln 4} - 2$ or $\frac{15 - 4\ln 2}{2\ln 2}$ or $\frac{7.5}{\ln 2} - 2$ or $\frac{15}{2}\log_2 e - 2$ or equivalent.

Question Number	Scheme		Marks
5.	***Alternative: Converting to a Cartesian equation:*** $t = 2-2x \Rightarrow \quad y = 2^{2-2x} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ y = 2^2 - 1$	Applies $x = 0$ in their Cartesian equation...	M1
	$y = 3$	... to arrive at a correct answer of 3.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^{2-2x} - 1 \Rightarrow 0 = 2-2x \Rightarrow x = ...$	Applies $y = 0$ to obtain a value for x. (Must be seen in part (b)).	M1
	$x = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dy}{dx} = -2\left(2^{2-2x}\right)\ln 2$	$\pm\lambda 2^{2-2x},\ \lambda \neq 1$	M1
		$-2\left(2^{2-2x}\right)\ln 2$ or equivalent	A1
	At A, $x = 0$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $x = 0$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	***As in the original scheme.***	M1 A1 oe
			[5]
(d)	$\text{Area}(R) = \int \left(2^{2-2x} - 1\right) dx$	Form the integral of their Cartesian equation of C.	M1
	$= \int_{-1}^{1} \left(2^{2-2x} - 1\right) dx$	For $2^{2-2x} - 1$ with limits of $x = -1$ and $x = 1$. Ie. $\int_{-1}^{1}\left(2^{2-2x} - 1\right)$	B1
	$= \left(\frac{2^{2-2x}}{-2\ln 2} - x\right)$	Either $2^{2-2x} \rightarrow \frac{2^{2-2x}}{-2\ln 2}$ or $\left(2^{2-2x} - 1\right) \rightarrow \frac{2^{2-2x}}{\pm\alpha(\ln 2)} - x$ or $\left(2^{2-2x} - 1\right) \rightarrow \pm\alpha(\ln 2)(2^{2-2x}) - x$	M1*
		$\left(2^{2-2x} - 1\right) \rightarrow \frac{2^{2-2x}}{-2\ln 2} - x$	A1
	$\left\{\left[\frac{2^{2-2x}}{-2\ln 2} - x\right]_{-1}^{1}\right\} = \left(\left(\frac{1}{-2\ln 2} - 1\right) - \left(\frac{16}{-2\ln 2} + 1\right)\right)$	**Depends on the previous method mark.** Substitutes limits of -1 and their x_B and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6] **15**
(d)	**Alternative method: In Cartesian and applying *u* = 2 – 2*x***		

	$\text{Area}(R) = \int \left(2^u - 1\right)\{\mathrm{d}x\}$, where $u = 2 - 2x$ $= \int_4^0 \left(2^u - 1\right)\left(-\tfrac{1}{2}\right)\{\mathrm{d}u\}$	**M0**: Unless a candidate ***writes*** $\int \left(2^{2-2x} - 1\right)\{\mathrm{d}x\}$ Then apply the "working parametrically" mark scheme.

Question Number	Scheme		Marks
5. (d)	***Alternative method: For substitution*** $\mathbf{u = 2^t}$		
	$\text{Area}(R) = \int (2^t - 1).\left(-\frac{1}{2}\right)\mathrm{d}t$	Complete substitution for both y and $\mathrm{d}x$	M1
	where $u = 2^t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = 2^t \ln 2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = u\ln 2$		
	$x = -1 \to t = 4 \to u = 16$ and $x = 1 \to t = 0 \to u = 1$	Both correct limits in t or both correct limits in u.	B1
	So $\text{area}(R) = -\frac{1}{2}\int \frac{u-1}{u\ln 2}\mathrm{d}u$	If not awarded above, you can award M1 for this integral	
	$= -\frac{1}{2}\int \frac{1}{\ln 2} - \frac{1}{u\ln 2}\mathrm{d}u$		
	$= \left\{-\frac{1}{2}\right\}\left(\frac{u}{\ln 2} - \frac{\ln u}{\ln 2}\right)$	Either $2^t \to \frac{u}{\ln 2}$ or $(2^t - 1) \to \frac{u}{\pm\alpha(\ln 2)} - \frac{\ln u}{\ln 2}$ or $(2^t - 1) \to \pm\alpha(\ln 2)(u) - \frac{\ln u}{\ln 2}$	M1*
		$(2^t - 1) \to \frac{u}{\ln 2} - \frac{\ln u}{\ln 2}$	A1
	$\left\{-\frac{1}{2}\left[\frac{u}{\ln 2} - \frac{\ln u}{\ln 2}\right]_{16}^{1}\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - \frac{\ln 16}{\ln 2}\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits ***in u*** and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6]

Question Number	Scheme		Marks
6. (a)	$\{y=0 \Rightarrow\}\ 1-2\cos x = 0$	$1-2\cos x = 0$, seen or implied.	M1
		At least one correct value of x. (See notes).	A1
	$\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$	Both $\frac{\pi}{3}$ and $\frac{5\pi}{3}$	A1 **cso**
			[3]
(b)	$V = \pi\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1-2\cos x)^2\ \mathrm{d}x$	For $\pi\int(1-2\cos x)^2$. Ignore limits and $\mathrm{d}x$	B1
	$\left\{\int(1-2\cos x)^2\ \mathrm{d}x\right\} = \int(1-4\cos x+4\cos^2 x)\mathrm{d}x$		
	$= \int 1-4\cos x+4\left(\frac{1+\cos 2x}{2}\right)\mathrm{d}x$	$\cos 2x = 2\cos^2 x - 1$ See notes.	M1
	$= \int\left(3-4\cos x+2\cos 2x\right)\mathrm{d}x$		
	$= 3x - 4\sin x + \frac{2\sin 2x}{2}$	Attempts $\int y^2$ to give any two of $\pm A \to \pm Ax$, $\pm B\cos x \to \pm B\sin x$ or $\pm\lambda\cos 2x \to \pm\mu\sin 2x$.	M1
		Correct integration.	A1
	$V = \{\pi\}\left(\left(3\left(\frac{5\pi}{3}\right) - 4\sin\left(\frac{5\pi}{3}\right) + \frac{2\sin\left(\frac{10\pi}{3}\right)}{2}\right) - \left(3\left(\frac{\pi}{3}\right) - 4\sin\left(\frac{\pi}{3}\right) + \frac{2\sin\left(\frac{2\pi}{3}\right)}{2}\right)\right)$	Applying limits the correct way round. Ignore π.	ddM1
	$= \pi\left(\left(5\pi + 2\sqrt{3} - \frac{\sqrt{3}}{2}\right) - \left(\pi - 2\sqrt{3} + \frac{\sqrt{3}}{2}\right)\right)$		
	$= \pi\left((18.3060...) - (0.5435...)\right) = 17.7625\pi = 55.80$		
	$= \pi\left(4\pi + 3\sqrt{3}\right)$ or $4\pi^2 + 3\pi\sqrt{3}$	Two term exact answer.	A1
			[6] **9**

6. (a)	**M1:** $1-2\cos x = 0$. This can be implied by either $\cos x = \frac{1}{2}$ or any one of the correct values for x in radians or in degrees. **1st A1:** Any one of either $\frac{\pi}{3}$ or $\frac{5\pi}{3}$ or 60 or 300 or awrt 1.05 or 5.23 or awrt 5.24 . **2nd A1:** Both $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.
(b)	**B1:** (M1 on epen) For $\pi\int(1-2\cos x)^2$. Ignore limits and $\mathrm{d}x$. **1st M1:** Any correct form of $\cos 2x = 2\cos^2 x - 1$ used or written down in the same variable. This can be implied by $\cos^2 x = \frac{1+\cos 2x}{2}$ or $4\cos^2 x \to 2 + 2\cos 2x$ or $\cos 2A = 2\cos^2 A - 1$. **2nd M1:** Attempts $\int y^2$ to give any two of $\pm A \to \pm Ax$, $\pm B\cos x \to \pm B\sin x$ or $\pm\lambda\cos 2x \to \pm\mu\sin 2x$. Do not worry about the signs when integrating $\cos x$ or $\cos 2x$ for this mark. Note: $\int(1-2\cos x)^2 = \int 1 + 4\cos^2 x$ is ok for an attempt at $\int y^2$. **1st A1:** Correct integration. Eg. $3x - 4\sin x + \frac{2\sin 2x}{2}$ or $x - 4\sin x + \frac{2\sin 2x}{2} + 2x$ oe. **3rd ddM1:** Depends on both of the two previous method marks. (Ignore π). Some evidence of substituting their $x = \frac{5\pi}{3}$ and their $x = \frac{\pi}{3}$ and subtracting the correct way round. You will need to use your calculator to check for correct substitution of their limits into their integrand if a candidate does not explicitly give **some evidence.** **Note:** For correct integral and limits decimals gives: $\pi\left((18.3060...)-(0.5435...)\right) = 17.7625\pi = 55.80$ **2nd A1:** ***Two term*** exact answer of either $\pi\left(4\pi + 3\sqrt{3}\right)$ or $4\pi^2 + 3\pi\sqrt{3}$ or equivalent. **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** Decimal answer of 58.802... without correct exact answer is A0. **Note:** Applying $\int(1-2\cos x)\,\mathrm{d}x$ will usually be given no marks in this part.

Question Number	Scheme		Marks
7. (a)	$\mathbf{i}:\quad 9+\lambda = 2+2\mu \quad \mathbf{(1)}$ $\mathbf{j}:\quad 13+4\lambda = -1+\mu \quad \mathbf{(2)}$ $\mathbf{k}:\quad -3-2\lambda = 1+\mu \quad \mathbf{(3)}$	Any two equations. (Allow one slip).	M1
	Eg: $\mathbf{(2)}-\mathbf{(3)}:\ 16+6\lambda=-2$ or $\mathbf{(2)}-4\mathbf{(1)}:\ -23=-9-7\mu$	An attempt to eliminate one of the parameters.	dM1
	Leading to $\lambda=-3$ or $\mu=2$	Either $\lambda=-3$ or $\mu=2$	A1
	$l_1:\mathbf{r}=\begin{pmatrix}9\\13\\-3\end{pmatrix}-3\begin{pmatrix}1\\4\\-2\end{pmatrix}=\begin{pmatrix}6\\1\\3\end{pmatrix}$ or $l_2:\mathbf{r}=\begin{pmatrix}2\\-1\\1\end{pmatrix}+2\begin{pmatrix}2\\1\\1\end{pmatrix}=\begin{pmatrix}6\\1\\3\end{pmatrix}$	See notes	ddM1 A1
			[5]
(b)	$\mathbf{d}_1=\begin{pmatrix}1\\4\\-2\end{pmatrix},\ \mathbf{d}_2=\begin{pmatrix}2\\1\\1\end{pmatrix}\ \Rightarrow\ \begin{pmatrix}1\\4\\-2\end{pmatrix}\bullet\begin{pmatrix}2\\1\\1\end{pmatrix}$	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta=\pm\left(\dfrac{2+4-2}{\sqrt{(1)^2+(4)^2+(-2)^2}\cdot\sqrt{(2)^2+(1)^2+(1)^2}}\right)$	Correct **equation**.	A1
	$\cos\theta=\dfrac{4}{\sqrt{21}.\sqrt{6}}\Rightarrow\theta=69.1238974...=69.1\ (1\text{ dp})$	awrt 69.1	A1
			[3]
(c)	$\overrightarrow{OA}=\begin{pmatrix}4\\16\\-3\end{pmatrix},\quad \overrightarrow{OP}=\begin{pmatrix}9\\13\\-3\end{pmatrix}+\lambda\begin{pmatrix}1\\4\\-2\end{pmatrix}=\begin{pmatrix}9+\lambda\\13+4\lambda\\-3-2\lambda\end{pmatrix}$		
	$\overrightarrow{AP}=\begin{pmatrix}9+\lambda\\13+4\lambda\\-3-2\lambda\end{pmatrix}-\begin{pmatrix}4\\16\\-3\end{pmatrix}=\begin{pmatrix}\lambda+5\\4\lambda-3\\-2\lambda\end{pmatrix}$		M1 A1
	$\overrightarrow{AP}\bullet\mathbf{d}_1=0\Rightarrow\begin{pmatrix}\lambda+5\\4\lambda-3\\-2\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\4\\-2\end{pmatrix}=\lambda+5+16\lambda-12+4\lambda=0$		dM1
	leading to $\{21\lambda-7=0\ \Rightarrow\}\ \lambda=\frac{1}{3}$	$\lambda=\frac{1}{3}$	A1
	Position vector $\overrightarrow{OP}=\begin{pmatrix}9\\13\\-3\end{pmatrix}+\frac{1}{3}\begin{pmatrix}1\\4\\-2\end{pmatrix}=\begin{pmatrix}9\frac{1}{3}\\14\frac{1}{3}\\-3\frac{2}{3}\end{pmatrix}$ or $\begin{pmatrix}\frac{28}{3}\\\frac{43}{3}\\-\frac{11}{3}\end{pmatrix}$		ddM1 A1
			[6] **14**

7. (a)

M1: Writes down any two equations. Allow one slip.

dM1: Attempts to eliminate either λ or μ to form an equation in one parameter only.

A1: For either $\lambda = -3$ or $\mu = 2$. **Note**: candidates only need to find one of the parameters.

ddM1: For either substituting their value of λ into l_1 or their μ into l_2.

2nd A1: For either $\begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$ or $6\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $(6 \quad 1 \quad 3)$.

Note: Each of the method marks in this part are dependent upon the previous method marks.

(b)

M1: Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$. Allow one slip in $\mathbf{d}_1 = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

A1: Correct application of the dot product formula $\mathbf{d}_1 \bullet \mathbf{d}_2 = \pm |\mathbf{d}_1||\mathbf{d}_2|\cos\theta$ or $\cos\theta = \pm\left(\dfrac{\mathbf{d}_1 \bullet \mathbf{d}_2}{|\mathbf{d}_1||\mathbf{d}_2|}\right)$

The dot product must be correctly applied and the square roots although they can be un-simplified must be correctly applied.

A1: awrt 69.1 . This can be also be achieved by $180 - 110.876 = \text{awrt } 69.1$. $\theta = 1.2064...^c$ is A0.

Common response: $\cos\theta = \left(\dfrac{-12-24+12}{\sqrt{(-3)^2+(-12)^2+(6)^2}.\sqrt{(4)^2+(2)^2+(2)^2}}\right) = \dfrac{-24}{\sqrt{189}.\sqrt{24}}$ is M1A1...

<u>Alternative Method: Vector Cross Product</u>

Only apply this scheme if it is clear that a candidate is applying a vector cross product method.

$$\underline{\mathbf{d}_1 \times \mathbf{d}_2 = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & -2 \\ 2 & 1 & 1 \end{vmatrix} = 6\mathbf{i} - 5\mathbf{j} - 7\mathbf{k} \right\}$$

<u>M1</u>: Realisation that the vector cross product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$. Allow one slip in $\mathbf{d}_1 = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

$$\sin\theta = \frac{\sqrt{(6)^2+(5)^2+(-7)^2}}{\sqrt{(1)^2+(4)^2+(-2)^2}.\sqrt{(2)^2+(1)^2+(1)^2}}$$

A1: Correct applied **equation**.

$$\sin\theta = \frac{\sqrt{110}}{\sqrt{21}.\sqrt{6}} \Rightarrow \theta = 69.1238974... = 69.1 \text{ (1 dp)}$$

A1: awrt 69.1

(c)

M1: Attempts to find $\overrightarrow{AP}$ in terms of the parameter by subtracting the components of $\overrightarrow{OP}$ from l_1 and $\overrightarrow{OA}$. Ignore the direction of subtraction and ignore any confusion between $\overrightarrow{OP}$ and $\overrightarrow{PO}$ or between $\overrightarrow{OA}$ and $\overrightarrow{AO}$. The correct subtraction of two components is enough to establish that subtraction is intended. The coordinates or position vector of P must be given in terms of a parameter. Taking $P:(x, y, z)$ gains no marks although this can be recovered later. See **Additional Solutions**.

A1: (M1 on epen) A correct expression for $\overrightarrow{AP}$. Again accept the reverse direction.

dM1: Depends on the previous M. Taking the scalar product of their expression for $\overrightarrow{AP}$ with $\mathbf{d}_1$ or a multiple of $\mathbf{d}_1$ and equating to 0 and obtaining an equation for λ. The equation must derive from an expression of the form $x_1x_2 + y_1y_2 + z_1z_2 = 0$. Differentiation can be used. See **Additional Solutions**.

A1: Solving to find $\lambda = \frac{1}{3}$.

ddM1: Depends on both previous Ms. Substitutes their value of the parameter into their expression for $\overrightarrow{OP}$. Substituting into $\overrightarrow{AP}$ is a common error which loses the mark.

Note: Needs 2 correct co-ordinates if $\lambda = \frac{1}{3}$ found and then P stated without method to gain ddM1.

	A1: $9\frac{1}{3}\mathbf{i}+14\frac{1}{3}\mathbf{j}-3\frac{2}{3}\mathbf{k}$. Accept vector notation or coordinates. ***Must be exact.***

7. (c)

Additional Solution 1:

Taking $\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, in itself, can gain no marks but this may be converted to a parameter at a later stage in the solution and, at that stage, any relevant marks can be awarded.

For example, $\overrightarrow{AP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix} = \begin{pmatrix} x-4 \\ y-16 \\ z+3 \end{pmatrix}$

leading to: $\begin{pmatrix} x-4 \\ y-16 \\ z+3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = x - 4 + 4y - 64 - 2z - 6 = 0$ No marks gained at this stage.

Using, $\overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9+\lambda \\ 13+4\lambda \\ -3-2\lambda \end{pmatrix}$ on $x + 4y - 2z = 74$

which gives: $9 + \lambda + 4(13+4\lambda) - 2(-3-2\lambda) = 74$ At this stage award **M1A1** and **dM1** (which is implied by an equation)

$\Rightarrow \quad 21\lambda + 67 = 74 \Rightarrow \underline{\lambda = \frac{1}{3}}$ A1: Solving to find $\lambda = \frac{1}{3}$.

Position vector

$\overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9\frac{1}{3} \\ 14\frac{1}{3} \\ -3\frac{2}{3} \end{pmatrix}$ or $\begin{pmatrix} \frac{28}{3} \\ \frac{43}{3} \\ -\frac{11}{3} \end{pmatrix}$ ddM1 A1

Additional Solution 2: Using Differentiation

$\overrightarrow{AP} = \begin{pmatrix} 9+\lambda \\ 13+4\lambda \\ -3-2\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix} = \begin{pmatrix} \lambda+5 \\ 4\lambda-3 \\ -2\lambda \end{pmatrix}$ **M1A1:** As main scheme

$AP^2 = (\lambda+5)^2 + (4\lambda-3)^2 + (-2\lambda)^2 = \{21\lambda^2 - 14\lambda + 34\}$

$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(AP^2\right) = 42\lambda - 14 = 0$ **M1**

leading to $\lambda = \frac{1}{3}$ **A1:** Solving to find $\lambda = \frac{1}{3}$.

... then apply the main scheme.

Question Number	Scheme	Marks
8. (a)	$\left\{\dfrac{d\theta}{dt} = \dfrac{(3-\theta)}{125}\right\} \Rightarrow \int \dfrac{1}{3-\theta}\,d\theta = \int \dfrac{1}{125}\,dt$ or $\int \dfrac{125}{3-\theta}\,d\theta = \int dt$	B1
	$-\ln(\theta-3) = \dfrac{1}{125}t\ \{+c\}$ or $-\ln(3-\theta) = \dfrac{1}{125}t\ \{+c\}$ See notes.	M1 A1
	$\ln(\theta-3) = -\dfrac{1}{125}t + c$	
	$\theta - 3 = e^{-\frac{1}{125}t + c}$ or $e^{-\frac{1}{125}t}e^{c}$	
	$\theta = Ae^{-0.008t} + 3$ * ***Correct*** completion to $\theta = Ae^{-0.008t} + 3$.	A1
		[4]
(b)	$\{t = 0,\ \theta = 16 \Rightarrow\}$ $16 = Ae^{-0.008(0)} + 3$; $\Rightarrow \underline{A = 13}$ ***See notes.***	M1; A1
	$10 = 13e^{-0.008t} + 3$ Substitutes $\theta = 10$ into an equation **of the form** $\theta = Ae^{-0.008t} + 3$, or equivalent. ***See notes.***	M1
	$e^{-0.008t} = \dfrac{7}{13} \Rightarrow -0.008t = \ln\left(\dfrac{7}{13}\right)$ Correct algebra to $-0.008t = \ln k$, where k is a positive value. ***See notes.***	M1
	$\left\{t = \dfrac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute) awrt 77	A1
		[5] **9**

8. (a)

B1: (M1 on epen) Separates variables as shown. $d\theta$ and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
M1: Both $\pm\lambda\ln(3-\theta)$ or $\pm\lambda\ln(\theta-3)$ **and** $\pm\mu t$ where λ and μ are constants.

A1: For $-\ln(\theta-3) = \dfrac{1}{125}t$ or $-\ln(3-\theta) = \dfrac{1}{125}t$ or $-125\ln(\theta-3) = t$ or $-125\ln(3-\theta) = t$

Note: $+c$ is not needed for this mark.

A1: Correct completion to $\theta = Ae^{-0.008t} + 3$. **Note:** $+c$ is needed for this mark.

Note: $\ln(\theta-3) = -\dfrac{1}{125}t + c$ leading to $\theta - 3 = e^{-\frac{1}{125}t} + e^{c}$ or $\theta - 3 = e^{-\frac{1}{125}t} + A$, would be final A0.

Note: From $-\ln(\theta-3) = \dfrac{1}{125}t + c$, then $\ln(\theta-3) = -\dfrac{1}{125}t + c$

$\Rightarrow \theta - 3 = e^{-\frac{1}{125}t + c}$ **or** $\theta - 3 = e^{-\frac{1}{125}t}e^{c} \Rightarrow \theta = Ae^{-0.008t} + 3$ is required for A1.

Note: From $-\ln(3-\theta) = \dfrac{1}{125}t + c$, then $\ln(3-\theta) = -\dfrac{1}{125}t + c$

$\Rightarrow 3 - \theta = e^{-\frac{1}{125}t + c}$ **or** $3 - \theta = e^{-\frac{1}{125}t}e^{c} \Rightarrow \theta = Ae^{-0.008t} + 3$ is sufficient for A1.

Note: The jump from $3 - \theta = Ae^{-\frac{1}{125}t}$ to $\theta = Ae^{-0.008t} + 3$ is fine.

	Note: $\ln(\theta - 3) = -\frac{1}{125}t + c \Rightarrow \theta - 3 = Ae^{-\frac{1}{125}t}$, where candidate writes $A = \mathrm{e}^{c}$ is also acceptable.

8. (b)

M1: (B1 on epen) Substitutes $\theta = 16,\ t = 0$, into either their equation containing an unknown constant or the printed equation. **Note:** You can imply this method mark.

A1: (M1 on epen) $A = 13$. **Note:** $\theta = 13e^{-0.008t} + 3$ without any working implies the first two marks, M1A1.

M1: Substitutes $\theta = 10$ into an equation **of the form** $\theta = Ae^{-0.008t} + 3$, or equivalent. where A is a positive or negative numerical value and A can be equal to 1 or -1.

M1: Uses correct algebra to rearrange **their equation** into the form $-0.008t = \ln k$, where k is ***a positive numerical value***.

A1: awrt 77 or awrt 1 hour 17 minutes.

<u>Alternative Method 1 for part (b)</u>

$$\int \frac{1}{3-\theta}\,d\theta = \int \frac{1}{125}\,dt \Rightarrow -\ln(\theta - 3) = \frac{1}{125}t + c$$

$$\{t = 0,\ \theta = 16 \Rightarrow\}\ -\ln(16-3) = \frac{1}{125}(0) + c \Rightarrow c = -\ln 13$$

M1: Substitutes $t = 0, \theta = 16$, into $-\ln(\theta - 3) = \frac{1}{125}t + c$

A1: $c = -\ln 13$

$$-\ln(\theta - 3) = \frac{1}{125}t - \ln 13 \quad \text{or} \quad \ln(\theta - 3) = -\frac{1}{125}t + \ln 13$$

$$-\ln(10-3) = \frac{1}{125}t - \ln 13$$

M1: Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(\theta - 3) = \pm\frac{1}{125}t \pm \mu$ where λ, μ are numerical values.

$$\ln 13 - \ln 7 = \frac{1}{125}t$$

M1: Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***.

$$t = 77.3799... = 77 \text{ (nearest minute)}$$

A1: awrt 77.

<u>Alternative Method 2 for part (b)</u>

$$\int \frac{1}{3-\theta}\,d\theta = \int \frac{1}{125}\,dt \Rightarrow -\ln|3 - \theta| = \frac{1}{125}t + c$$

$$\{t = 0,\ \theta = 16 \Rightarrow\}\ -\ln|3-16| = \frac{1}{125}(0) + c \Rightarrow c = -\ln 13$$

M1: Substitutes $t = 0, \theta = 16$, into $-\ln(3 - \theta) = \frac{1}{125}t + c$

A1: $c = -\ln 13$

$$-\ln|3 - \theta| = \frac{1}{125}t - \ln 13 \quad \text{or} \quad \ln|3 - \theta| = -\frac{1}{125}t + \ln 13$$

$$-\ln(3-10) = \frac{1}{125}t - \ln 13$$

M1: Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(3 - \theta) = \pm\frac{1}{125}t \pm \mu$ where λ, μ are numerical values.

$$\ln 13 - \ln 7 = \frac{1}{125}t$$

M1: Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$,

	$t = 77.3799... = 77$ (nearest minute)	where C, D are ***positive numerical values***. **A1:** awrt 77.

8. (b)	***Alternative Method 3 for part (b)*** $\int_{16}^{10} \frac{1}{3-\theta}\,\mathrm{d}\theta = \int_{0}^{t} \frac{1}{125}\,\mathrm{d}t$ $= \left[-\ln\lvert 3-\theta\rvert\right]_{16}^{10} = \left[\frac{1}{125}t\right]_{0}^{t}$ $-\ln 7 - -\ln 13 = \frac{1}{125}t$ $t = 77.3799... = 77$ (nearest minute)	**M1A1:** $\ln 13$ **M1:** Substitutes limit of $\theta = 10$ correctly. **M1:** Uses correct algebra to rearrange **their own equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***. **A1:** awrt 77.
	Alternative Method 4 for part (b) $\{\theta = 16 \Rightarrow\}\quad 16 = Ae^{-0.008t} + 3$ $\{\theta = 10 \Rightarrow\}\quad 10 = Ae^{-0.008t} + 3$ $-0.008t = \ln\left(\frac{13}{A}\right)$ or $-0.008t = \ln\left(\frac{7}{A}\right)$ $t_{(1)} = \frac{\ln\left(\frac{13}{A}\right)}{-0.008}$ and $t_{(2)} = \frac{\ln\left(\frac{7}{A}\right)}{-0.008}$ $t = t_{(1)} - t_{(2)} = \frac{\ln\left(\frac{13}{A}\right)}{-0.008} - \frac{\ln\left(\frac{7}{A}\right)}{-0.008}$ $\left\{t = \frac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute)	**M1*:** Writes down a pair of equations in A and t, for $\theta = 16$ and $\theta = 10$ with either A unknown or A being a positive or negative value. **A1:** Two equations with an unknown A. **M1:** Uses ***correct algebra*** to solve both of **their equations** leading to answers of the form $-0.008t = \ln k$, where k is ***a positive numerical value***. **M1:** Finds difference between the two times. (either way round). **A1:** awrt 77. Correct solution only.