June 2006
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1.	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{=}\right\}$ $6x - 4y\frac{dy}{dx} + 2 - 3\frac{dy}{dx} = 0$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.) Correct equation.	M1 A1
	$\left\{\frac{dy}{dx} = \frac{6x+2}{4y+3}\right\}$	*not necessarily required.*	
	At (0, 1), $\frac{dy}{dx} = \frac{0+2}{4+3} = \frac{2}{7}$	Substituting x = 0 & y = 1 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$	dM1; A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$ oe.
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{7}{2}x + 1$	$y - 1 = m(x - 0)$ with 'their tangent or normal gradient'; or uses $y = mx + 1$ with 'their tangent or normal gradient';	M1;
	N: 7x + 2y – 2 = 0	Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers.	A1 oe **cso**
			[7]
			7 marks

Beware: $\frac{dy}{dx} = \frac{2}{7}$ does not necessarily imply the award of all the first four marks in this question. So please ensure that you check candidates' initial differentiation before awarding the first A1 mark.

Beware: The final accuracy mark is for completely correct solutions. If a candidate flukes the final line then they must be awarded A0.

Beware: A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) = ∞, but obtains M0 if they write $y - 1 = \infty(x - 0)$. If they write, however, **N**: x = 0, then can score M1.

Beware: A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0, and also obtains M1 if they write $y - 1 = 0(x - 0)$ or y = 1.

Beware: The final **cso** refers to the whole question.

Question Number	Scheme		Marks
Aliter			
1.	$\left\{\frac{\cancel{d}\cancel{x}}{\cancel{d}\cancel{y}}\cancel{\times}\right\}$ $6x\frac{dx}{dy} - 4y + 2\frac{dx}{dy} - 3 = 0$	Differentiates implicitly to include either $\pm kx\frac{dx}{dy}$ or $\pm 2\frac{dx}{dy}$. (Ignore $\left(\frac{dx}{dy} = \right)$.) Correct equation.	M1 A1
Way 2	$\left\{\frac{dx}{dy} = \frac{4y+3}{6x+2}\right\}$	*not necessarily required.*	
	At (0, 1), $\frac{dx}{dy} = \frac{4+3}{0+2} = \frac{7}{2}$	Substituting x = 0 & y = 1 into an *equation* involving $\frac{dx}{dy}$; to give $\frac{7}{2}$	dM1; A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$	Uses m(**T**) or $\frac{dx}{dy}$ to 'correctly' find m(**N**). Can be ft using "$-1.\frac{dx}{dy}$".	A1$\sqrt{}$ oe.
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{7}{2}x + 1$	$y - 1 = m(x - 0)$ with 'their tangent, $\frac{dx}{dy}$ or normal gradient'; or uses $y = mx + 1$ with 'their tangent, $\frac{dx}{dy}$ or normal gradient';	M1;
	N: 7x + 2y – 2 = 0	Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers.	A1 oe **cso**
			7 marks

Question Number	Scheme		Marks
Aliter **1. Way 3**	$2y^2 + 3y - 3x^2 - 2x - 5 = 0$		
	$\left(y + \frac{3}{4}\right)^2 - \frac{9}{16} = \frac{3x^2}{2} + x + \frac{5}{2}$		
	$y = \sqrt{\left(\frac{3x^2}{2} + x + \frac{49}{16}\right)} - \frac{3}{4}$		
	$\frac{dy}{dx} = \frac{1}{2}\left(\frac{3x^2}{2} + x + \frac{49}{16}\right)^{-\frac{1}{2}}(3x+1)$	Differentiates using the chain rule; Correct expression for $\frac{dy}{dx}$.	M1; A1 oe
	At (0, 1), $\frac{dy}{dx} = \frac{1}{2}\left(\frac{49}{16}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{4}{7}\right) = \frac{2}{7}$	Substituting x = 0 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$	dM1 A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{2}{7}x + 1$	$y - 1 = m(x - 0)$ with 'their tangent or normal gradient'; or uses $y = mx + 1$ with 'their tangent or normal gradient'	M1
	N: 7x + 2y – 2 = 0	Correct equation in the form $'ax + by + c = 0'$, where a, b and c are integers.	A1 oe
			[7]
			7 marks

Question Number	Scheme		Marks
2. (a)	$3x-1 \equiv A(1-2x)+B$	Considers this identity and either substitutes $x=\frac{1}{2}$, equates coefficients or solves simultaneous equations	*complete* M1
	Let $x=\frac{1}{2}$; $\frac{3}{2}-1=B \Rightarrow B=\frac{1}{2}$		
	Equate x terms; $3=-2A \Rightarrow A=-\frac{3}{2}$	$A=-\frac{3}{2}$; $B=\frac{1}{2}$	A1;A1
	(**No working seen**, but A and B correctly stated $\Rightarrow$ award all three marks. If one of A or B correctly stated give two out of the three marks available for this part.)		**[3]**
(b)	$f(x)=-\frac{3}{2}(1-2x)^{-1}+\frac{1}{2}(1-2x)^{-2}$	Moving powers to top on any one of the two expressions	M1
	$=-\frac{3}{2}\left\{\underline{1+(-1)(-2x);+\frac{(-1)(-2)}{2!}(-2x)^2+\frac{(-1)(-2)(-3)}{3!}(-2x)^3+\ldots}\right\}$	Either $1\pm 2x$ or $1\pm 4x$ from either first or second expansions respectively	dM1;
	$+\frac{1}{2}\left\{\underline{1+(-2)(-2x);+\frac{(-2)(-3)}{2!}(-2x)^2+\frac{(-2)(-3)(-4)}{3!}(-2x)^3+\ldots}\right\}$	Ignoring $-\frac{3}{2}$ and $\frac{1}{2}$, any one correct $\{\underline{\ldots\ldots\ldots\ldots}\}$ expansion. Both $\{\underline{\ldots\ldots\ldots\ldots}\}$ correct.	A1 A1
	$=-\frac{3}{2}\{1+2x+4x^2+8x^3+\ldots\}+\frac{1}{2}\{1+4x+12x^2+32x^3+\ldots\}$		
	$=-1-x\ ;+0x^2+4x^3$	$-1-x$; $(0x^2)+4x^3$	A1; A1 **[6]**
			9 marks

Question Number	Scheme		Marks
Aliter **2.** (b) **Way 2**	$f(x) = (3x-1)(1-2x)^{-2}$	Moving power to top	M1
	$= (3x-1) \times \left(1 + (-2)(-2x)\,;\, + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots\right)$	$1 \pm 4x$; Ignoring $(3x-1)$, correct $(\ldots\ldots\ldots\ldots)$ expansion	dM1; A1
	$= (3x-1)(1 + 4x + 12x^2 + 32x^3 + \ldots)$		
	$= \underline{3x + 12x^2 + 36x^3 - 1 - 4x - 12x^2 - 32x^3} + \ldots$	Correct expansion	A1
	$= -1 - x\,; + 0x^2 + 4x^3$	$-1-x$; $(0x^2) + 4x^3$	A1; A1 **[6]**
Aliter **2.** (b) **Way 3**	Maclaurin expansion		
	$f(x) = -\tfrac{3}{2}(1-2x)^{-1} + \tfrac{1}{2}(1-2x)^{-2}$	Bringing both powers to top	M1
	$f'(x) = -3(1-2x)^{-2} + 2(1-2x)^{-3}$	Differentiates to give $a(1-2x)^{-2} \pm b(1-2x)^{-3}$; $-3(1-2x)^{-2} + 2(1-2x)^{-3}$	M1; A1 oe
	$f''(x) = -12(1-2x)^{-3} + 12(1-2x)^{-4}$		
	$f'''(x) = -72(1-2x)^{-4} + 96(1-2x)^{-5}$	Correct $f''(x)$ and $f'''(x)$	A1
	$\therefore f(0) = -1$, $f'(0) = -1$, $f''(0) = 0$ and $f'''(0) = 24$		
	gives $f(x) = -1 - x; + 0x^2 + 4x^3 + \ldots$	$-1-x$; $(0x^2) + 4x^3$	A1; A1 **[6]**

Question Number	Scheme		Marks
Aliter			
2. (b)	$f(x) = -3(2-4x)^{-1} + \frac{1}{2}(1-2x)^{-2}$	Moving powers to top on any one of the two expressions	M1
Way 4			
	$= -3\left\{\begin{array}{l}(2)^{-1} + (-1)(2)^{-2}(-4x); + \frac{(-1)(-2)}{2!}(2)^{-3}(-4x)^2 \\ + \frac{(-1)(-2)(-3)}{3!}(2)^{-4}(-4x)^3 + \ldots\end{array}\right\}$	Either $\frac{1}{2} \pm x$ or $1 \pm 4x$ from either first or second expansions respectively	dM1;
	$+\frac{1}{2}\left\{1 + (-2)(-2x); + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots\right\}$	Ignoring -3 and $\frac{1}{2}$, any one correct $\{\ldots\ldots\ldots.\}$ expansion.	A1
		Both $\{\ldots\ldots\ldots.\}$ correct.	A1
	$= -3\left\{\frac{1}{2} + x + 2x^2 + 4x^3 + \ldots\right\} + \frac{1}{2}\left\{1 + 4x + 12x^2 + 32x^3 + \ldots\right\}$		
	$= -1 - x\ ; + 0x^2 + 4x^3$	$-1 - x\ ;\quad (0x^2) + 4x^3$	A1; A1
			[6]

Question Number	Scheme		Marks
3. (a)	Area Shaded $= \int_0^{2\pi} 3\sin\left(\frac{x}{2}\right)\,dx$		
	$= \left[\dfrac{-3\cos\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{2\pi}$	Integrating $3\sin\left(\frac{x}{2}\right)$ to give $k\cos\left(\frac{x}{2}\right)$ with $k \neq 1$. Ignore limits.	M1
	$= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$	$-6\cos\left(\frac{x}{2}\right)$ or $\frac{-3}{\frac{1}{2}}\cos\left(\frac{x}{2}\right)$	A1 oe.
	$= [-6(-1)] - [-6(1)] = 6 + 6 = \underline{12}$	$\underline{12}$	A1 cao
			[3]
	(Answer of 12 with no working scores M0A0A0.)		
(b)	Volume $= \underline{\pi \int_0^{2\pi} \left(3\sin\left(\frac{x}{2}\right)\right)^2}\,dx = 9\pi \int_0^{2\pi} \sin^2\left(\frac{x}{2}\right)\,dx$	**Use** of $\underline{V = \pi \int y^2}\,dx$. Can be implied. Ignore limits.	M1
	$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ gives } \sin^2 x = \frac{1-\cos 2x}{2}\right]$ $\left[\text{NB}: \underline{\cos x = \pm 1 \pm 2\sin^2\left(\frac{x}{2}\right)} \text{ gives } \sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}\right]$	Consideration of the Half Angle Formula for $\sin^2\left(\frac{x}{2}\right)$ or the Double Angle Formula for $\sin^2 x$	M1 *
	$\therefore$ Volume $= 9(\pi) \int_0^{2\pi} \left(\dfrac{1-\cos x}{2}\right) dx$	Correct expression for Volume Ignore limits and π.	A1
	$= \dfrac{9(\pi)}{2} \int_0^{2\pi} \underline{(1-\cos x)}\,dx$		
	$= \dfrac{9(\pi)}{2} \left[\underline{x - \sin x}\right]_0^{2\pi}$	$\underline{\text{Integrating to give } \pm ax \pm b\sin x}$; $\underline{\text{Correct integration}}$ $\underline{k - k\cos x \to kx - k\sin x}$	depM1 *; A1
	$= \dfrac{9\pi}{2}\left[(2\pi - 0) - (0 - 0)\right]$		
	$= \dfrac{9\pi}{2}(2\pi) = \underline{9\pi^2}$ or $\underline{88.8264\ldots}$	Use of limits to give either $9\pi^2$ or awrt 88.8 Solution must be completely correct. No flukes allowed.	A1 **cso** **[6]**
			9 marks

Question Number	Scheme		Marks
4. (a)	$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right)$		
	$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$	Attempt to differentiate both x and y wrt t to give two terms in cos Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	M1 A1
	When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58$	Divides in correct way and substitutes for t to give any of the four underlined oe: Ignore the double negative if candidate has differentiated $\sin \to -\cos$	A1
	When $t = \frac{\pi}{6}, \quad x = \frac{1}{2}, \; y = \frac{\sqrt{3}}{2}$	The point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(\frac{1}{2}, \text{awrt } 0.87\right)$	B1
	T: $y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$. Correct EXACT equation of tangent oe.	dM1 A1 oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}\right]$		
			[6]
(b)	$y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$	Use of compound angle formula for sine.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$		
	$\therefore x = \sin t$ gives $\cos t = \sqrt{(1 - x^2)}$	Use of trig identity to find $\cos t$ in terms of x or $\cos^2 t$ in terms of x.	M1
	$\therefore y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}$ **AG**	Substitutes for $\sin t$, $\cos\frac{\pi}{6}$, $\cos t$ and $\sin\frac{\pi}{6}$ to give y in terms of x.	A1 cso
			[3]
			9 marks

Question Number	Scheme		Marks
Aliter **4.** (a) **Way 2**	$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$	(Do not give this for part (b)) Attempt to differentiate x and y wrt t to give $\frac{dx}{dt}$ in terms of cos and $\frac{dy}{dt}$ in the form $\pm a\cos t \pm b\sin t$	M1
	$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos t \cos\frac{\pi}{6} - \sin t \sin\frac{\pi}{6}$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\frac{\cos\frac{\pi}{6}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\sin\frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}}$ $= \underline{\frac{\frac{3}{4} - \frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$	Divides in correct way and substitutes for t to give any of the four underlined oe:	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + \text{"c"}$.	dM1
		Correct EXACT equation of <u>tangent</u> oe.	<u>A1</u> oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		**[6]**

Question Number	Scheme		Marks
Aliter **4.** (a) **Way 3**	$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$		
	$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$	Attempt to differentiate two terms using the chain rule for the second term. Correct $\frac{dy}{dx}$	M1 A1
	$\frac{dy}{dx} = \underline{\frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-(0.5)^2)^{-\frac{1}{2}}(-2(0.5)) = \frac{1}{\sqrt{3}}}$	Correct substitution of $x = \frac{1}{2}$ into a correct $\frac{dy}{dx}$	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$. Correct <u>EXACT</u> equation of <u>tangent</u> oe.	dM1 <u>A1</u> oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		**[6]**
Aliter **4.** (b) **Way 2**	$x = \sin t$ gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{(1-\sin^2 t)}$	Substitutes $x = \sin t$ into the equation give in y.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$		
	$\cos t = \sqrt{(1-\sin^2 t)}$	Use of trig identity to deduce that $\cos t = \sqrt{(1-\sin^2 t)}$.	M1
	gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	Hence $y = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6} = \sin\left(t + \frac{\pi}{6}\right)$	Using the compound angle formula to prove $y = \sin\left(t + \frac{\pi}{6}\right)$	A1 cso **[3]**
			9 marks

Question Number	Scheme		Marks
5. (a)	Equating **i** ; $0 = 6 + \lambda \Rightarrow \lambda = -6$	$\underline{\lambda = -6}$ Can be implied	B1 $\Rightarrow$ d
	Using $\lambda = -6$ and		
	equating **j** ; $a = 19 + 4(-6) = -5$	For inserting **their stated** λ into either a correct **j** or **k** component Can be implied.	M1 $\Rightarrow$ d
	equating **k** ; $b = -1 - 2(-6) = 11$	$a = -5$ and $b = 11$	A1
			[3]
	With no working… … only one of a or b stated correctly gains the first 2 marks. … both a and b stated correctly gains 3 marks.		
(b)	$\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$		
	direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$		
	$\overrightarrow{OP} \perp l_1 \Rightarrow \underline{\overrightarrow{OP} \bullet \mathbf{d} = 0}$	Allow <u>this statement</u> for M1 if $\overrightarrow{OP}$ and $\mathbf{d}$ are defined as above.	
	ie. $\underline{\begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 0}$ (or $\underline{x + 4y - 2z = 0}$)	Allow either of these two <u>underlined statements</u>	M1
	$\therefore\ 6 + \lambda + 4(19 + 4\lambda) - 2(-1 - 2\lambda) = 0$	Correct equation	A1 oe
	$6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0$	Attempt to solve the equation in λ	dM1
	$21\lambda + 84 = 0 \Rightarrow \lambda = -4$	$\lambda = -4$	A1
	$\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$	Substitutes their λ into an expression for $\overrightarrow{OP}$	M1
	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7)	A1
			[6]

Question Number	Scheme		Marks
Aliter (b) **Way 2**	$\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$		
	$\overrightarrow{AP} = (6+\lambda-0)\mathbf{i} + (19+4\lambda+5)\mathbf{j} + (-1-2\lambda-11)\mathbf{k}$		
	direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$		
	$\overrightarrow{AP} \perp \overrightarrow{OP} \Rightarrow \underline{\overrightarrow{AP} \bullet \overrightarrow{OP} = 0}$	Allow <u>this statement</u> for M1 if $\overrightarrow{AP}$ and $\overrightarrow{OP}$ are defined as above.	
	ie. $\underline{\begin{pmatrix} 6+\lambda \\ 24+4\lambda \\ -12-2\lambda \end{pmatrix} \bullet \begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} = 0}$	<u>underlined statement</u>	M1
	$\therefore (6+\lambda)(6+\lambda) + (24+4\lambda)(19+4\lambda) + (-12-2\lambda)(-1-2\lambda) = 0$	Correct equation	A1 oe
	$36+12\lambda+\lambda^2+456+96\lambda+76\lambda+16\lambda^2+12+24\lambda+2\lambda+4\lambda^2 = 0$	Attempt to solve the equation in λ	dM1
	$21\lambda^2+210\lambda+504=0$		
	$\lambda^2+10\lambda+24=0 \Rightarrow (\lambda=-6)\ \underline{\lambda=-4}$	$\lambda = -4$	A1
	$\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$	Substitutes their λ into an expression for $\overrightarrow{OP}$	M1
	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7)	A1
			[6]

Question Number	Scheme		Marks
5. (c)	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$		
	$\overrightarrow{OA} = 0\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}$ and $\overrightarrow{OB} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$		
	$\overrightarrow{AP} = \pm(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k})$, $\overrightarrow{PB} = \pm(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})$ $\overrightarrow{AB} = \pm(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k})$	Subtracting vectors to find any two of $\overrightarrow{AP}$, $\overrightarrow{PB}$ or $\overrightarrow{AB}$; and both are correctly ft using candidate's $\overrightarrow{OA}$ and $\overrightarrow{OP}$ found in parts (a) and (b) respectively.	M1; A1 $\sqrt{}$ ±
	As $\overrightarrow{AP} = \frac{2}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{2}{3}\overrightarrow{PB}$ or $\overrightarrow{AB} = \frac{5}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{5}{2}\overrightarrow{AP}$ or $\overrightarrow{AB} = \frac{5}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{5}{3}\overrightarrow{PB}$ or $\overrightarrow{PB} = \frac{3}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{3}{2}\overrightarrow{AP}$ or $\overrightarrow{AP} = \frac{2}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{2}{5}\overrightarrow{AB}$ or $\overrightarrow{PB} = \frac{3}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{3}{5}\overrightarrow{AB}$ etc…	$\overrightarrow{AP} = \frac{2}{3}\overrightarrow{PB}$ or $\overrightarrow{AB} = \frac{5}{2}\overrightarrow{AP}$ or $\overrightarrow{AB} = \frac{5}{3}\overrightarrow{PB}$ or $\overrightarrow{PB} = \frac{3}{2}\overrightarrow{AP}$ or $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$ or $\overrightarrow{PB} = \frac{3}{5}\overrightarrow{AB}$	
	alternatively candidates could say for example that $\overrightarrow{AP} = 2(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$ $\overrightarrow{PB} = 3(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$		
	then <u>the points A, P and B are collinear</u>.	<u>A, P and B are collinear</u> Completely correct proof.	A1
	$\therefore \overrightarrow{AP} : \overrightarrow{PB} = 2:3$	2:3 or 1: $\frac{3}{2}$ or $\sqrt{84} : \sqrt{189}$ aef allow SC $\frac{2}{3}$	B1 oe **[4]**
Aliter **5.** (c) Way 2	At B; <u>$5 = 6 + \lambda$</u>, <u>$15 = 19 + 4\lambda$</u> or <u>$1 = -1 - 2\lambda$</u> or at B; $\lambda = -1$	Writing down any of the three <u>underlined equations</u>.	M1
	gives $\lambda = -1$ for all three equations. or when $\lambda = -1$, this gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$	$\lambda = -1$ for all three equations or $\lambda = -1$ gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$	A1
	<u>Hence B lies on l_1</u>. As stated in the question both A and P lie on l_1. $\therefore$ <u>A, P and B are collinear</u>.	<u>Must state B lies on l_1</u> $\Rightarrow$ A, P and B are collinear	A1
	$\therefore \overrightarrow{AP} : \overrightarrow{PB} = 2:3$	2:3 or aef	B1 oe
			[4]
			13 marks

Question Number	Scheme		Marks
6. (a)			

x	1	1.5	2	2.5	3
y	0	0.5 ln 1.5	ln 2	1.5 ln 2.5	2 ln 3
or y	0	0.2027325541 …	ln2	1.374436098 …	2 ln 3

Question Number	Scheme		Marks
		Either 0.5 ln 1.5 and 1.5 ln 2.5 or awrt 0.20 and 1.37 (or mixture of decimals and ln's)	B1 [1]
(b)(i)	$I_1 \approx \frac{1}{2} \times 1 \times \underline{\{0 + 2(\ln 2) + 2\ln 3\}}$	For structure of trapezium rule $\{............\}$;	M1;
	$= \frac{1}{2} \times 3.583518938... = 1.791759... = 1.792$ (4sf)	1.792	A1 cao
(ii)	$I_2 \approx \frac{1}{2} \times 0.5 \; ; \times \underline{\{0 + 2(0.5\ln 1.5 + \ln 2 + 1.5\ln 2.5) + 2\ln 3\}}$	Outside brackets $\frac{1}{2} \times 0.5$	B1;
		For structure of trapezium rule $\{............\}$;	M1 $\sqrt{}$
	$= \frac{1}{4} \times 6.737856242... = 1.684464...$	awrt 1.684	A1 [5]
(c)	With increasing ordinates, the line segments at the top of the trapezia are closer to the curve.	Reason or an appropriate diagram elaborating the correct reason.	B1 [1]

Question Number	Scheme		Marks
6. (d)	$\left\{\begin{matrix} u = \ln x & \Rightarrow & \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x - 1 & \Rightarrow & v = \frac{x^2}{2} - x \end{matrix}\right\}$	Use of 'integration by parts' formula in the correct direction	M1
	$I = \left(\frac{x^2}{2} - x\right)\ln x - \int \frac{1}{x}\left(\frac{x^2}{2} - x\right) dx$	Correct expression	A1
	$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\int\left(\frac{x}{2} - 1\right) dx}$	An attempt to multiply at least one term through by $\frac{1}{x}$ and an attempt to ...	
	$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\left(\frac{x^2}{4} - x\right)}$ (+c)	… integrate; correct integration	M1; A1
	$\therefore I = \left[\left(\frac{x^2}{2} - x\right)\ln x - \frac{x^2}{4} + x\right]_1^3$		
	$= \left(\frac{3}{2}\ln 3 - \frac{9}{4} + 3\right) - \left(-\frac{1}{2}\ln 1 - \frac{1}{4} + 1\right)$	Substitutes limits of 3 and 1 and subtracts.	ddM1
	$= \frac{3}{2}\ln 3 + \frac{3}{4} + 0 - \frac{3}{4} \quad = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]
Aliter **6.** (d) **Way 2**	$\int (x-1)\ln x\, dx = \int x\ln x\, dx - \int \ln x\, dx$		
	$\int x\ln x\, dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}.\left(\frac{1}{x}\right) dx$	Correct application of 'by parts'	M1
	$= \frac{x^2}{2}\ln x - \frac{x^2}{4}$ (+ c)	Correct integration	A1
	$\int \ln x\, dx = x\ln x - \int x.\left(\frac{1}{x}\right) dx$	Correct application of 'by parts'	M1
	$= x\ln x - x$ (+ c)	Correct integration	A1
	$\therefore \int_1^3 (x-1)\ln x\, dx = \left(\frac{9}{2}\ln 3 - 2\right) - \left(3\ln 3 - 2\right) = \frac{3}{2}\ln 3$ **AG**	Substitutes limits of 3 and 1 into both integrands and subtracts. $\frac{3}{2}\ln 3$	ddM1 A1 cso
			[6]

Question Number	Scheme		Marks
Aliter **6.** (d) **Way 3**	$\left\{\begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = (x-1) \Rightarrow v = \frac{(x-1)^2}{2} \end{array}\right\}$	Use of 'integration by parts' formula in the correct direction	M1
	$I = \frac{(x-1)^2}{2}\ln x - \int \frac{(x-1)^2}{2x}\,dx$	Correct expression	A1
	$= \frac{(x-1)^2}{2}\ln x - \int \frac{x^2-2x+1}{2x}\,dx$	Candidate multiplies out numerator to obtain three terms…	
	$= \frac{(x-1)^2}{2}\ln x - \int \underline{\left(\frac{1}{2}x - 1 + \frac{1}{2x}\right)}\,dx$	… multiplies at least one term through by $\frac{1}{x}$ and then attempts to ...	
	$= \frac{(x-1)^2}{2}\ln x - \underline{\left(\frac{x^2}{4} - x + \frac{1}{2}\ln x\right)}$ (+c)	… integrate the result; <u>correct integration</u>	M1; A1
	$\therefore I = \left[\frac{(x-1)^2}{2}\ln x - \frac{x^2}{4} + x - \frac{1}{2}\ln x\right]_1^3$		
	$= \left(2\ln 3 - \frac{9}{4} + 3 - \frac{1}{2}\ln 3\right) - \left(0 - \frac{1}{4} + 1 - 0\right)$	Substitutes limits of 3 and 1 and subtracts.	ddM1
	$= 2\ln 3 - \frac{1}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 \quad = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]

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Question Number	Scheme		Marks
Aliter **6.** (d) **Way 4**	By substitution $u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$		
	$I = \int (e^u - 1).ue^u \, du$	Correct expression	
	$= \int u(e^{2u} - e^u)du$	Use of 'integration by parts' formula in the correct direction	M1
	$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \int \underline{\left(\frac{1}{2}e^{2u} - e^u\right)} dx$	Correct expression	A1
	$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \underline{\left(\frac{1}{4}e^{2u} - e^u\right)}$ (+c)	Attempt <u>to integrate</u>; <u>correct integration</u>	M1; A1
	$\therefore I = \left[\frac{1}{2}ue^{2u} - ue^u - \frac{1}{4}e^{2u} + e^u\right]_{\ln 1}^{\ln 3}$		
	$= \left(\frac{9}{2}\ln 3 - 3\ln 3 - \frac{9}{4} + 3\right) - \left(0 - 0 - \frac{1}{4} + 1\right)$	Substitutes limits of ln3 and ln1 and subtracts.	ddM1
	$= \frac{3}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]
			13 marks

Question Number	Scheme		Marks
7. (a)	From question, $\frac{dS}{dt} = 8$	$\frac{dS}{dt} = 8$	B1
	$S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$	$\frac{dS}{dx} = 12x$	B1
	$\frac{dx}{dt} = \frac{dS}{dt} \div \frac{dS}{dx} = \underline{\frac{8}{12x}}\,; = \frac{\frac{2}{3}}{x} \Rightarrow \left(k = \frac{2}{3}\right)$	Candidate's $\frac{dS}{dt} \div \frac{dS}{dx}\,;\ \underline{\frac{8}{12x}}$	M1; A1oe
			[4]
(b)	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3x^2.\left(\frac{2}{3x}\right);\ = 2x$	Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}\,;\ \lambda x$	M1; A1 $\sqrt{\ }$
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG**	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	A1
			[4]
(c)	$\int \frac{dV}{V^{\frac{1}{3}}} = \int 2\ dt$	Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2\ dt$ on the other side. integral signs not necessary.	B1
	$\int V^{-\frac{1}{3}}\ dV = \int 2\ dt$		
	$\frac{3}{2}V^{\frac{2}{3}} = 2t\ \ (+c)$	Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and 2t; Correct equation with/without + c.	M1; A1
	$\frac{3}{2}(8)^{\frac{2}{3}} = 2(0) + c \Rightarrow c = 6$	Use of V = 8 and t = 0 in a changed equation containing c ; $c = 6$	M1 ∗; A1
	Hence: $\frac{3}{2}V^{\frac{2}{3}} = 2t + 6$		
	$\frac{3}{2}\left(16\sqrt{2}\right)^{\frac{2}{3}} = 2t + 6 \Rightarrow 12 = 2t + 6$	Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c".	depM1 ∗
	giving t = 3.	t = 3	A1 cao **[7]**
			15 marks

Question Number	Scheme		Marks
Aliter **7.** (b) **Way 2**	$x = V^{\frac{1}{3}}$ & $S = 6x^2 \Rightarrow S = 6V^{\frac{2}{3}}$	$S = 6V^{\frac{2}{3}}$	B1 $\sqrt{}$
	$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$	$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$	B1
	$\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS} = 8.\left(\frac{1}{4V^{-\frac{1}{3}}}\right); = \frac{2}{V^{-\frac{1}{3}}} = 2V^{\frac{1}{3}}$ **AG**	Candidate's $\frac{dS}{dt} \times \frac{dV}{dS}$; $2V^{\frac{1}{3}}$	M1; A1
		In ePEN, award Marks for Way 2 in the order they appear on this mark scheme.	
			[4]
Aliter **7.** (c) **Way 2**	$\int \frac{dV}{2V^{\frac{1}{3}}} = \int 1\, dt$	Separates the variables with $\int \frac{dV}{2V^{\frac{1}{3}}}$ or $\int \frac{1}{2}V^{-\frac{1}{3}}dV$ oe on one side and $\int 1\, dt$ on the other side. integral signs not necessary.	B1
	$\frac{1}{2}\int V^{-\frac{1}{3}}\, dV = \int 1\, dt$		
	$\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)V^{\frac{2}{3}} = t \quad (+c)$	Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and t; Correct equation with/without + c.	M1; A1
	$\frac{3}{4}(8)^{\frac{2}{3}} = (0) + c \Rightarrow c = 3$	Use of V = 8 and t = 0 in a changed equation containing c ; $c = 3$	M1∗; A1
	Hence: $\frac{3}{4}V^{\frac{2}{3}} = t + 3$		
	$\frac{3}{4}\left(16\sqrt{2}\right)^{\frac{2}{3}} = t + 3 \quad \Rightarrow \quad 6 = t + 3$	Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c".	depM1 ∗
	giving t = 3.	t = 3	A1 cao **[7]**

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Question Number	Scheme		Marks
Aliter	*similar to way 1.*		
(b)	$V = x^3 \;\Rightarrow\; \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$	B1
Way 3			
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS} \;=\; 3x^2.8.\left(\frac{1}{12x}\right); = \;2x$	Candidate's $\frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS}$; λx	M1; A1 $\sqrt{}$
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG**	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	A1
			[4]
Aliter			
(c)	$\int \frac{dV}{V^{\frac{1}{3}}} = \int 2\ dt$	Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2\ dt$ on the other side. integral signs not necessary.	B1
Way 3			
	$\int V^{-\frac{1}{3}}\ dV = \int 2\ dt$		
	$V^{\frac{2}{3}} = \frac{4}{3}t\ \ (+c)$	Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and $\frac{4}{3}t$; Correct equation with/without + c.	M1; A1
	$(8)^{\frac{2}{3}} = \frac{4}{3}(0) + c \;\Rightarrow\; c = 4$	Use of V = 8 and t = 0 in a changed equation containing c ; $c = 4$	M1∗; A1
	Hence: $V^{\frac{2}{3}} = \frac{4}{3}t + 4$		
	$\left(16\sqrt{2}\right)^{\frac{2}{3}} = \frac{4}{3}t + 6 \quad\Rightarrow\quad 8 = \frac{4}{3}t + 4$	Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c".	depM1 ∗
	giving t = 3.	t = 3	A1 cao **[7]**