

June 2006
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1.	<p style="text-align: right;">Differentiates implicitly to include either $\pm ky \frac{dy}{dx}$ or $\pm 3 \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)</p> $\left\{ \frac{dy}{dx} \right\} \times \left\{ 6x - 4y \frac{dy}{dx} + 2 - 3 \frac{dy}{dx} = 0 \right.$ <p style="text-align: right;">Correct equation.</p> $\left\{ \frac{dy}{dx} = \frac{6x+2}{4y+3} \right\}$ <p style="text-align: right;"><i>not necessarily required.</i></p> <p style="text-align: right;">Substituting $x = 0$ & $y = 1$ into an <i>equation</i> involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $-\frac{2}{7}$</p> <p>At $(0, 1)$, $\frac{dy}{dx} = \frac{0+2}{4+3} = \frac{2}{7}$</p> <p>Hence $m(\mathbf{N}) = -\frac{7}{2}$ or $-\frac{1}{\frac{2}{7}}$</p> <p style="text-align: right;">Uses $m(\mathbf{T})$ to 'correctly' find $m(\mathbf{N})$. Can be ft from "their tangent gradient".</p> <p style="text-align: right;">$y - 1 = m(x - 0)$ with 'their tangent or normal gradient'; or uses $y = mx + 1$ with 'their tangent or normal gradient' ;</p> <p>Either \mathbf{N}: $y - 1 = -\frac{7}{2}(x - 0)$</p> <p>or \mathbf{N}: $y = -\frac{7}{2}x + 1$</p> <p style="text-align: right;">Correct equation in the form 'ax + by + c = 0', where a, b and c are integers.</p> <p>\mathbf{N}: $7x + 2y - 2 = 0$</p>	<p>M1 A1</p> <p>dM1; A1 cs0</p> <p>A1√ oe.</p> <p>M1;</p> <p>A1 oe cs0</p> <p>[7]</p>
		7 marks

Beware: $\frac{dy}{dx} = \frac{2}{7}$ does not necessarily imply the award of all the first four marks in this question.

So please ensure that you check candidates' initial differentiation before awarding the first A1 mark.

Beware: The final accuracy mark is for completely correct solutions. If a candidate flukes the final line then they must be awarded A0.

Beware: A candidate finding an $m(\mathbf{T}) = 0$ can obtain A1ft for $m(\mathbf{N}) = \infty$, but obtains M0 if they write $y - 1 = \infty(x - 0)$. If they write, however, \mathbf{N} : $x = 0$, then can score M1.

Beware: A candidate finding an $m(\mathbf{T}) = \infty$ can obtain A1ft for $m(\mathbf{N}) = 0$, and also obtains M1 if they write $y - 1 = 0(x - 0)$ or $y = 1$.

Beware: The final **cs0** refers to the whole question.

Question Number	Scheme	Marks
<p>Aliter</p> <p>1.</p> <p>Way 3</p>	$2y^2 + 3y - 3x^2 - 2x - 5 = 0$ $\left(y + \frac{3}{4}\right)^2 - \frac{9}{16} = \frac{3x^2}{2} + x + \frac{5}{2}$ $y = \sqrt{\left(\frac{3x^2}{2} + x + \frac{49}{16}\right)} - \frac{3}{4}$ $\frac{dy}{dx} = \frac{1}{2} \left(\frac{3x^2}{2} + x + \frac{49}{16}\right)^{-\frac{1}{2}} (3x + 1)$ <p>At (0, 1),</p> $\frac{dy}{dx} = \frac{1}{2} \left(\frac{49}{16}\right)^{-\frac{1}{2}} = \frac{1}{2} \left(\frac{4}{7}\right) = \frac{2}{7}$ <p>Hence $m(\mathbf{N}) = -\frac{7}{2}$</p> <p>Either $\mathbf{N}: y - 1 = -\frac{7}{2}(x - 0)$</p> <p>or $\mathbf{N}: y = -\frac{7}{2}x + 1$</p> <p>$\mathbf{N}: 7x + 2y - 2 = 0$</p>	<p>M1;</p> <p>A1 oe</p> <p>dM1</p> <p>A1 cso</p> <p>A1√</p> <p>M1</p> <p>A1 oe</p> <p>[7]</p> <p>7 marks</p>

Question Number	Scheme	Marks
2. (a)	$3x - 1 \equiv A(1 - 2x) + B$ <p>Let $x = \frac{1}{2}$; $\frac{3}{2} - 1 = B \Rightarrow B = \frac{1}{2}$</p> <p>Equate x terms; $3 = -2A \Rightarrow A = -\frac{3}{2}$</p> <p>(No working seen, but A and B correctly stated \Rightarrow award all three marks. If one of A or B correctly stated give two out of the three marks available for this part.)</p>	<p>Considers this identity and either substitutes $x = \frac{1}{2}$, equates coefficients or solves simultaneous equations</p> <p><i>complete</i></p> <p>M1</p> <p>A1;A1</p> <p>[3]</p>
(b)	$f(x) = -\frac{3}{2}(1 - 2x)^{-1} + \frac{1}{2}(1 - 2x)^{-2}$ $= -\frac{3}{2} \left\{ 1 + (-1)(-2x); + \frac{(-1)(-2)}{2!}(-2x)^2 + \frac{(-1)(-2)(-3)}{3!}(-2x)^3 + \dots \right\}$ $+ \frac{1}{2} \left\{ 1 + (-2)(-2x); + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \dots \right\}$ $= -\frac{3}{2} \{1 + 2x + 4x^2 + 8x^3 + \dots\} + \frac{1}{2} \{1 + 4x + 12x^2 + 32x^3 + \dots\}$ $= -1 - x; + 0x^2 + 4x^3$	<p>Moving powers to top on any one of the two expressions</p> <p>Either $1 \pm 2x$ or $1 \pm 4x$ from either first or second expansions respectively</p> <p>Ignoring $-\frac{3}{2}$ and $\frac{1}{2}$, any one correct {.....} expansion.</p> <p>Both {.....} correct.</p> <p>M1</p> <p>dM1;</p> <p>A1</p> <p>A1</p> <p>A1; A1</p> <p>[6]</p>
		9 marks

Question Number	Scheme	Marks
<p>Aliter 2. (b) Way 2</p>	$f(x) = (3x - 1)(1 - 2x)^{-2}$ $= (3x - 1) \times \left(1 + (-2)(-2x) + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \dots \right)$ $= (3x - 1)(1 + 4x + 12x^2 + 32x^3 + \dots)$ $= \underline{3x + 12x^2 + 36x^3 - 1 - 4x - 12x^2 - 32x^3 + \dots}$ $= -1 - x + 0x^2 + 4x^3$	<p>Moving power to top $1 \pm 4x$; Ignoring $(3x - 1)$, correct (.....) expansion <u>Correct expansion</u> $-1 - x$; $(0x^2) + 4x^3$</p> <p>M1 dM1; A1 A1 A1; A1 [6]</p>
<p>Aliter 2. (b) Way 3</p>	<p>Maclaurin expansion</p> $f(x) = -\frac{3}{2}(1 - 2x)^{-1} + \frac{1}{2}(1 - 2x)^{-2}$ $f'(x) = -3(1 - 2x)^{-2} + 2(1 - 2x)^{-3}$ $f''(x) = -12(1 - 2x)^{-3} + 12(1 - 2x)^{-4}$ $f'''(x) = -72(1 - 2x)^{-4} + 96(1 - 2x)^{-5}$ <p>$\therefore f(0) = -1, f'(0) = -1, f''(0) = 0$ and $f'''(0) = 24$</p> <p>gives $f(x) = -1 - x + 0x^2 + 4x^3 + \dots$</p>	<p>Bringing both powers to top Differentiates to give $a(1 - 2x)^{-2} \pm b(1 - 2x)^{-3}$; $-3(1 - 2x)^{-2} + 2(1 - 2x)^{-3}$ Correct $f''(x)$ and $f'''(x)$ $-1 - x$; $(0x^2) + 4x^3$</p> <p>M1 M1; A1 oe A1 A1; A1 [6]</p>

Question Number	Scheme	Marks
<p>Aliter</p> <p>2. (b)</p> <p>Way 4</p>	$f(x) = -3(2 - 4x)^{-1} + \frac{1}{2}(1 - 2x)^{-2}$ $= -3 \left\{ (2)^{-1} + (-1)(2)^{-2}(-4x) + \frac{(-1)(-2)}{2!} (2)^{-3}(-4x)^2 + \frac{(-1)(-2)(-3)}{3!} (2)^{-4}(-4x)^3 + \dots \right\}$ $+ \frac{1}{2} \left\{ 1 + (-2)(-2x) + \frac{(-2)(-3)}{2!} (-2x)^2 + \frac{(-2)(-3)(-4)}{3!} (-2x)^3 + \dots \right\}$ $= -3 \left\{ \frac{1}{2} + x + 2x^2 + 4x^3 + \dots \right\} + \frac{1}{2} \left\{ 1 + 4x + 12x^2 + 32x^3 + \dots \right\}$ $= -1 - x ; +0x^2 + 4x^3$	<p>Moving powers to top on any one of the two expressions</p> <p>M1</p> <p>Either $\frac{1}{2} \pm x$ or $1 \pm 4x$ from either first or second expansions respectively</p> <p>dM1;</p> <p>Ignoring -3 and $\frac{1}{2}$, any one correct {.....} expansion.</p> <p>A1</p> <p>Both {.....} correct.</p> <p>A1</p> <p>$-1 - x ; (0x^2) + 4x^3$</p> <p>A1; A1</p> <p>[6]</p>

Question Number	Scheme	Marks
<p>3. (a)</p>	<p>Area Shaded = $\int_0^{2\pi} 3 \sin\left(\frac{x}{2}\right) dx$</p> <p>$= \left[\frac{-3 \cos\left(\frac{x}{2}\right)}{\frac{1}{2}} \right]_0^{2\pi}$</p> <p>$= \left[-6 \cos\left(\frac{x}{2}\right) \right]_0^{2\pi}$</p> <p>$= [-6(-1)] - [-6(1)] = 6 + 6 = \underline{12}$</p> <p>(Answer of 12 with no working scores M0A0A0.)</p>	<p>Integrating $3 \sin\left(\frac{x}{2}\right)$ to give $k \cos\left(\frac{x}{2}\right)$ with $k \neq 1$. Ignore limits. A1 oe. A1 cao [3]</p>
<p>(b)</p>	<p>Volume = $\pi \int_0^{2\pi} \left(3 \sin\left(\frac{x}{2}\right)\right)^2 dx = 9\pi \int_0^{2\pi} \sin^2\left(\frac{x}{2}\right) dx$</p> <p>[NB: $\cos 2x = \pm 1 \pm 2 \sin^2 x$ gives $\sin^2 x = \frac{1 - \cos 2x}{2}$] [NB: $\cos x = \pm 1 \pm 2 \sin^2\left(\frac{x}{2}\right)$ gives $\sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos x}{2}$]</p> <p>$\therefore$ Volume = $9(\pi) \int_0^{2\pi} \left(\frac{1 - \cos x}{2}\right) dx$</p> <p>$= \frac{9(\pi)}{2} \int_0^{2\pi} (1 - \cos x) dx$</p> <p>$= \frac{9(\pi)}{2} [x - \sin x]_0^{2\pi}$</p> <p>$= \frac{9\pi}{2} [(2\pi - 0) - (0 - 0)]$</p> <p>$= \frac{9\pi}{2} (2\pi) = \underline{9\pi^2}$ or <u>88.8264...</u></p>	<p>Use of $V = \pi \int y^2 dx$. Can be implied. Ignore limits. M1 Consideration of the Half Angle Formula for $\sin^2\left(\frac{x}{2}\right)$ or the Double Angle Formula for $\sin^2 x$ M1* Correct expression for Volume Ignore limits and π. A1 Integrating to give $\pm ax \pm b \sin x$; Correct integration <u>$k - k \cos x \rightarrow kx - k \sin x$</u> depM1* ; A1 Use of limits to give either $9\pi^2$ or awrt 88.8 Solution must be completely correct. No flukes allowed. A1 cso [6]</p>
		<p>9 marks</p>

Question Number	Scheme	Marks
4. (a)	<p>$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right)$</p> <p>$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$</p> <p>When $t = \frac{\pi}{6}$,</p> <p>$\frac{dy}{dx} = \frac{\cos\left(\frac{\pi}{6} + \frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58$</p> <p>When $t = \frac{\pi}{6}, \quad x = \frac{1}{2}, \quad y = \frac{\sqrt{3}}{2}$</p> <p><u>T: $y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)$</u></p> <p>or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$</p> <p>or T: $\left[y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3} \right]$</p>	<p>Attempt to differentiate both x and y wrt t to give two terms in cos Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>M1 A1</p> <p>Divides in correct way and substitutes for t to give any of the four underlined oe: Ignore the double negative if candidate has differentiated $\sin \rightarrow -\cos$</p> <p>A1</p> <p>The point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(\frac{1}{2}, \text{awrt } 0.87\right)$</p> <p>B1</p> <p>Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$. Correct <u>EXACT</u> equation of <u>tangent</u> oe.</p> <p>dM1 A1 oe</p> <p>[6]</p>
(b)	<p>$y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6}$</p> <p>Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$</p> <p>$\therefore x = \sin t$ gives $\cos t = \sqrt{1 - x^2}$</p> <p>$\therefore y = \frac{\sqrt{3}}{2} \sin t + \frac{1}{2} \cos t$</p> <p>gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1 - x^2}$ AG</p>	<p>Use of compound angle formula for sine.</p> <p>M1</p> <p>Use of trig identity to find $\cos t$ in terms of x or $\cos^2 t$ in terms of x.</p> <p>M1</p> <p>Substitutes for $\sin t, \cos \frac{\pi}{6}, \cos t$ and $\sin \frac{\pi}{6}$ to give y in terms of x.</p> <p>A1 cso</p> <p>[3]</p>
		9 marks

Question Number	Scheme	Marks
<p>Aliter 4. (a) Way 2</p>	<p>$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6}$</p> <p>$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos t \cos \frac{\pi}{6} - \sin t \sin \frac{\pi}{6}$</p> <p>When $t = \frac{\pi}{6}, \quad \frac{dy}{dx} = \frac{\cos \frac{\pi}{6} \cos \frac{\pi}{6} - \sin \frac{\pi}{6} \sin \frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}$</p> <p>$= \frac{\frac{3}{4} - \frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58$</p> <p>When $t = \frac{\pi}{6}, \quad x = \frac{1}{2}, \quad y = \frac{\sqrt{3}}{2}$</p> <p>T: $y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)$</p> <p>or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$</p> <p>or T: $\left[y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3} \right]$</p>	<p>(Do not give this for part (b)) Attempt to differentiate x and y wrt t to give $\frac{dx}{dt}$ in terms of cos and $\frac{dy}{dt}$ in the form $\pm a \cos t \pm b \sin t$</p> <p>M1</p> <p>Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>A1</p> <p>Divides in correct way and substitutes for t to give any of the four underlined oe:</p> <p>A1</p> <p>The point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(\frac{1}{2}, \text{awrt } 0.87\right)$</p> <p>B1</p> <p>Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$.</p> <p>dM1</p> <p>Correct EXACT equation of <u>tangent</u> oe.</p> <p>A1 oe</p> <p>[6]</p>

Question Number	Scheme	Marks
<p>Aliter</p> <p>4. (a)</p> <p>Way 3</p>	$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}$ $\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$ $\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-(0.5)^2)^{-\frac{1}{2}}(-2(0.5)) = \frac{1}{\sqrt{3}}$ <p>When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$</p> <p>T: $y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)$</p> <p>or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$</p> <p>or T: $\left[y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}\right]$</p>	<p>Attempt to differentiate two terms using the chain rule for the second term. M1</p> <p>Correct $\frac{dy}{dx}$ A1</p> <p>Correct substitution of $x = \frac{1}{2}$ into a correct $\frac{dy}{dx}$ A1</p> <p>The point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(\frac{1}{2}, \text{awrt } 0.87\right)$ B1</p> <p>Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$. dM1</p> <p>Correct EXACT equation of tangent oe. A1 oe</p> <p style="text-align: right;">[6]</p>
<p>Aliter</p> <p>4. (b)</p> <p>Way 2</p>	<p>$x = \sin t$ gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{1-\sin^2 t}$</p> <p>Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$</p> <p>$\cos t = \sqrt{1-\sin^2 t}$</p> <p>gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$</p> <p>Hence $y = \sin t \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6} = \sin\left(t + \frac{\pi}{6}\right)$</p>	<p>Substitutes $x = \sin t$ into the equation give in y. M1</p> <p>Use of trig identity to deduce that $\cos t = \sqrt{1-\sin^2 t}$. M1</p> <p>Using the compound angle formula to prove $y = \sin\left(t + \frac{\pi}{6}\right)$ A1 cso</p> <p style="text-align: right;">[3]</p>
		<p>9 marks</p>

Question Number	Scheme	Marks
5. (a)	<p>Equating i; $0 = 6 + \lambda \Rightarrow \lambda = -6$</p> <p>Using $\lambda = -6$ and</p> <p>equating j; $a = 19 + 4(-6) = -5$</p> <p>equating k; $b = -1 - 2(-6) = 11$</p> <p>With no working... ... only one of a or b stated correctly gains the first 2 marks. ... both a and b stated correctly gains 3 marks.</p>	<p>$\lambda = -6$ \Rightarrow d B1</p> <p>Can be implied</p> <p>For inserting their stated λ into either a correct j or k component Can be implied. M1 \Rightarrow d</p> <p>a = -5 and b = 11 A1</p> <p>[3]</p>
(b)	<p>$\overline{OP} = (6 + \lambda)\mathbf{i} + (19 + 4\lambda)\mathbf{j} + (-1 - 2\lambda)\mathbf{k}$</p> <p>direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$</p> <p>$\overline{OP} \perp l_1 \Rightarrow \overline{OP} \cdot \mathbf{d} = 0$</p> <p>ie. $\begin{pmatrix} 6 + \lambda \\ 19 + 4\lambda \\ -1 - 2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 0$ (or <u>$x + 4y - 2z = 0$</u>)</p> <p>$\therefore 6 + \lambda + 4(19 + 4\lambda) - 2(-1 - 2\lambda) = 0$</p> <p>$6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0$</p> <p>$21\lambda + 84 = 0 \Rightarrow \lambda = -4$</p> <p>$\overline{OP} = (6 - 4)\mathbf{i} + (19 + 4(-4))\mathbf{j} + (-1 - 2(-4))\mathbf{k}$</p> <p>$\overline{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$</p>	<p>Allow <u>this statement</u> for M1 if \overline{OP} and \mathbf{d} are defined as above.</p> <p>Allow either of these two <u>underlined statements</u> M1</p> <p>Correct equation A1 oe</p> <p>Attempt to solve the equation in λ dM1</p> <p>$\lambda = -4$ A1</p> <p>Substitutes their λ into an expression for \overline{OP} M1</p> <p>$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7) A1</p> <p>[6]</p>

Question Number	Scheme	Marks
<p>Aliter (b) Way 2</p>	$\overline{OP} = (6 + \lambda)\mathbf{i} + (19 + 4\lambda)\mathbf{j} + (-1 - 2\lambda)\mathbf{k}$ $\overline{AP} = (6 + \lambda - 0)\mathbf{i} + (19 + 4\lambda + 5)\mathbf{j} + (-1 - 2\lambda - 11)\mathbf{k}$ <p>direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$</p> $\overline{AP} \perp \overline{OP} \Rightarrow \underline{\overline{AP} \cdot \overline{OP} = 0}$ $\text{ie. } \begin{pmatrix} 6 + \lambda \\ 24 + 4\lambda \\ -12 - 2\lambda \end{pmatrix} \cdot \begin{pmatrix} 6 + \lambda \\ 19 + 4\lambda \\ -1 - 2\lambda \end{pmatrix} = 0$ $\therefore (6 + \lambda)(6 + \lambda) + (24 + 4\lambda)(19 + 4\lambda) + (-12 - 2\lambda)(-1 - 2\lambda) = 0$ $36 + 12\lambda + \lambda^2 + 456 + 96\lambda + 76\lambda + 16\lambda^2 + 12 + 24\lambda + 2\lambda + 4\lambda^2 = 0$ $21\lambda^2 + 210\lambda + 504 = 0$ $\lambda^2 + 10\lambda + 24 = 0 \Rightarrow (\lambda = -6) \quad \underline{\lambda = -4}$ $\overline{OP} = (6 - 4)\mathbf{i} + (19 + 4(-4))\mathbf{j} + (-1 - 2(-4))\mathbf{k}$ $\overline{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	<p>Allow <u>this statement</u> for M1 if \overline{AP} and \overline{OP} are defined as above.</p> <p><u>underlined statement</u> M1</p> <p>Correct equation A1 oe</p> <p>Attempt to solve the equation in λ dM1</p> <p>$\lambda = -4$ A1</p> <p>Substitutes their λ into an expression for \overline{OP} M1</p> <p>$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7) A1</p> <p>[6]</p>

Question Number	Scheme	Marks
<p>5. (c)</p>	<p>$\vec{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$</p> <p>$\vec{OA} = 0\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}$ and $\vec{OB} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$</p> <p>$\vec{AP} = \pm(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k})$, $\vec{PB} = \pm(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})$ $\vec{AB} = \pm(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k})$</p> <p>As $\vec{AP} = \frac{2}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{2}{3}\vec{PB}$ or $\vec{AB} = \frac{5}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{5}{2}\vec{AP}$ or $\vec{AB} = \frac{5}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{5}{3}\vec{PB}$ or $\vec{PB} = \frac{3}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{3}{2}\vec{AP}$ or $\vec{AP} = \frac{2}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{2}{5}\vec{AB}$ or $\vec{PB} = \frac{3}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{3}{5}\vec{AB}$ etc...</p> <p>alternatively candidates could say for example that $\vec{AP} = 2(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$ $\vec{PB} = 3(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$</p> <p>then <u>the points A, P and B are collinear.</u></p> <p>$\therefore \vec{AP} : \vec{PB} = 2 : 3$</p>	<p>Subtracting vectors to find any two of \vec{AP}, \vec{PB} or \vec{AB}; and both are correctly ft using candidate's \vec{OA} and \vec{OP} found in parts (a) and (b) respectively.</p> <p>M1; A1 $\sqrt{\pm}$</p> <p>A, P and B are collinear Completely correct proof. A1</p> <p>2:3 or $1 : \frac{3}{2}$ or $\sqrt{84} : \sqrt{189}$ aef B1 oe allow SC $\frac{2}{3}$ [4]</p>
<p>Aliter</p> <p>5. (c)</p> <p>Way 2</p>	<p>At B; $5 = 6 + \lambda$, $15 = 19 + 4\lambda$ or $1 = -1 - 2\lambda$ or at B; $\lambda = -1$</p> <p>gives $\lambda = -1$ for all three equations. or when $\lambda = -1$, this gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$</p> <p><u>Hence B lies on l_1.</u> As stated in the question both A and P lie on l_1. \therefore <u>A, P and B are collinear.</u></p> <p>$\therefore \vec{AP} : \vec{PB} = 2 : 3$</p>	<p>Writing down any of the three <u>underlined equations.</u> M1</p> <p>$\lambda = -1$ for all three equations or $\lambda = -1$ gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$ A1</p> <p><u>Must state B lies on l_1</u> \Rightarrow A, P and B are collinear A1</p> <p>2:3 or aef B1 oe [4]</p>
		<p>13 marks</p>

Question Number	Scheme	Marks																								
6. (a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">1.5</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">2.5</td> <td style="padding: 5px;">3</td> </tr> <tr> <td style="padding: 5px;">y</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0.5 ln 1.5</td> <td style="padding: 5px;">ln 2</td> <td style="padding: 5px;">1.5 ln 2.5</td> <td style="padding: 5px;">2 ln 3</td> </tr> <tr> <td style="padding: 5px;">or y</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0.2027325541</td> <td style="padding: 5px;">ln2</td> <td style="padding: 5px;">1.374436098</td> <td style="padding: 5px;">2 ln 3</td> </tr> <tr> <td></td> <td></td> <td style="padding: 5px;">...</td> <td></td> <td style="padding: 5px;">...</td> <td></td> </tr> </table> <p style="text-align: right; margin-right: 20px;">Either 0.5 ln 1.5 and 1.5 ln 2.5 or awrt 0.20 and 1.37 (or mixture of decimals and ln's)</p>	x	1	1.5	2	2.5	3	y	0	0.5 ln 1.5	ln 2	1.5 ln 2.5	2 ln 3	or y	0	0.2027325541	ln2	1.374436098	2 ln 3				B1 [1]
x	1	1.5	2	2.5	3																					
y	0	0.5 ln 1.5	ln 2	1.5 ln 2.5	2 ln 3																					
or y	0	0.2027325541	ln2	1.374436098	2 ln 3																					
																							
(b)(i)	$I_1 \approx \frac{1}{2} \times 1 \times \{0 + 2(\ln 2) + 2\ln 3\}$ $= \frac{1}{2} \times 3.583518938... = 1.791759... = 1.792 \text{ (4sf)}$	<p style="text-align: right;">For structure of trapezium rule {.....};</p> <p style="text-align: right;">1.792</p>	M1; A1 cao																							
(ii)	$I_2 \approx \frac{1}{2} \times 0.5 \times \{0 + 2(0.5\ln 1.5 + \ln 2 + 1.5\ln 2.5) + 2\ln 3\}$ $= \frac{1}{4} \times 6.737856242... = 1.684464...$	<p style="text-align: right;">Outside brackets $\frac{1}{2} \times 0.5$</p> <p style="text-align: right;">For structure of trapezium rule {.....};</p> <p style="text-align: right;">awrt 1.684</p>	B1; M1 $\sqrt{\quad}$ A1 [5]																							
(c)	<p>With increasing ordinates, <u>the line segments at the top of the trapezia are closer to the curve.</u></p>	<p style="text-align: right;"><u>Reason</u> or an appropriate diagram elaborating the correct reason.</p>	B1 [1]																							

Question Number	Scheme	Marks
<p>6. (d)</p>	$\left\{ \begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x - 1 \Rightarrow v = \frac{x^2}{2} - x \end{array} \right\}$ <p>Use of 'integration by parts' formula in the correct direction</p> $I = \left(\frac{x^2}{2} - x \right) \ln x - \int \frac{1}{x} \left(\frac{x^2}{2} - x \right) dx$ <p>Correct expression</p> $= \left(\frac{x^2}{2} - x \right) \ln x - \int \left(\frac{x}{2} - 1 \right) dx$ <p>An attempt to multiply at least one term through by $\frac{1}{x}$ and an attempt to ...</p> $= \left(\frac{x^2}{2} - x \right) \ln x - \left(\frac{x^2}{4} - x \right) (+c)$ <p>... integrate; correct integration</p> $\therefore I = \left[\left(\frac{x^2}{2} - x \right) \ln x - \frac{x^2}{4} + x \right]_1^3$ <p>Substitutes limits of 3 and 1 and subtracts.</p> $= \left(\frac{3}{2} \ln 3 - \frac{9}{4} + 3 \right) - \left(-\frac{1}{2} \ln 1 - \frac{1}{4} + 1 \right)$ $= \frac{3}{2} \ln 3 + \frac{3}{4} + 0 - \frac{3}{4} = \frac{3}{2} \ln 3 \quad \mathbf{AG}$	<p>M1</p> <p>A1</p> <p>M1;</p> <p>A1</p> <p>ddM1</p> <p>A1 cso</p> <p>[6]</p>
<p>Aliter</p> <p>6. (d)</p> <p>Way 2</p>	$\int (x - 1) \ln x \, dx = \int x \ln x \, dx - \int \ln x \, dx$ $\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \left(\frac{1}{x} \right) dx$ <p>Correct application of 'by parts'</p> $= \frac{x^2}{2} \ln x - \frac{x^2}{4} (+c)$ <p>Correct integration</p> $\int \ln x \, dx = x \ln x - \int x \cdot \left(\frac{1}{x} \right) dx$ <p>Correct application of 'by parts'</p> $= x \ln x - x (+c)$ <p>Correct integration</p> $\therefore \int_1^3 (x - 1) \ln x \, dx = \left(\frac{9}{2} \ln 3 - 2 \right) - (3 \ln 3 - 2) = \frac{3}{2} \ln 3 \quad \mathbf{AG}$ <p>Substitutes limits of 3 and 1 into both integrands and subtracts.</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>ddM1</p> <p>A1 cso</p> <p>[6]</p>

Question Number	Scheme	Marks
Aliter		
6. (d)	$\left\{ \begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = (x-1) \Rightarrow v = \frac{(x-1)^2}{2} \end{array} \right\}$	Use of 'integration by parts' formula in the correct direction M1
Way 3		
	$I = \frac{(x-1)^2}{2} \ln x - \int \frac{(x-1)^2}{2x} dx$	Correct expression A1
	$= \frac{(x-1)^2}{2} \ln x - \int \frac{x^2 - 2x + 1}{2x} dx$	Candidate multiplies out numerator to obtain three terms... ... multiplies at least one term through by $\frac{1}{x}$ and then attempts to integrate the result; <u>correct integration</u>
	$= \frac{(x-1)^2}{2} \ln x - \int \left(\frac{1}{2}x - 1 + \frac{1}{2x} \right) dx$	
	$= \frac{(x-1)^2}{2} \ln x - \left(\frac{x^2}{4} - x + \frac{1}{2} \ln x \right) (+c)$	
	$\therefore I = \left[\frac{(x-1)^2}{2} \ln x - \frac{x^2}{4} + x - \frac{1}{2} \ln x \right]_1^3$	
	$= \left(2 \ln 3 - \frac{9}{4} + 3 - \frac{1}{2} \ln 3 \right) - \left(0 - \frac{1}{4} + 1 - 0 \right)$	Substitutes limits of 3 and 1 and subtracts. ddM1
	$= 2 \ln 3 - \frac{1}{2} \ln 3 + \frac{3}{4} + \frac{1}{4} - 1 = \underline{\underline{\frac{3}{2} \ln 3}} \quad \mathbf{AG}$	$\frac{3}{2} \ln 3$ A1 cso
		[6]

Question Number	Scheme	Marks
<p>Aliter 6. (d) Way 4</p>	<p>By substitution $u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$</p> <p>$I = \int (e^u - 1).ue^u du$</p> <p>$= \int u(e^{2u} - e^u) du$</p> <p>$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \int \left(\frac{1}{2}e^{2u} - e^u\right) dx$</p> <p>$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \left(\frac{1}{4}e^{2u} - e^u\right) (+c)$</p> <p>$\therefore I = \left[\frac{1}{2}ue^{2u} - ue^u - \frac{1}{4}e^{2u} + e^u \right]_{\ln 1}^{\ln 3}$</p> <p>$= \left(\frac{9}{2}\ln 3 - 3\ln 3 - \frac{9}{4} + 3\right) - \left(0 - 0 - \frac{1}{4} + 1\right)$</p> <p>$= \frac{3}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 = \frac{3}{2}\ln 3$ AG</p>	<p>Correct expression</p> <p>Use of 'integration by parts' formula in the correct direction M1</p> <p>Correct expression A1</p> <p>Attempt to <u>integrate</u>; M1; <u>correct integration</u> A1</p> <p>Substitutes limits of $\ln 3$ and $\ln 1$ and subtracts. ddM1</p> <p>$\frac{3}{2}\ln 3$ A1 cso</p> <p>[6]</p> <p>13 marks</p>

Question Number	Scheme	Marks
7. (a)	<p>From question, $\frac{dS}{dt} = 8$</p> <p>$S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$</p> <p>$\frac{dx}{dt} = \frac{dS}{dt} \div \frac{dS}{dx} = \frac{8}{12x}; = \frac{2}{3x} \Rightarrow (k = \frac{2}{3})$</p>	<p>$\frac{dS}{dt} = 8$ B1</p> <p>$\frac{dS}{dx} = 12x$ B1</p> <p>Candidate's $\frac{dS}{dt} \div \frac{dS}{dx}; \frac{8}{12x}$ M1; A1oe</p> <p>[4]</p>
(b)	<p>$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$</p> <p>$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3x^2 \cdot \left(\frac{2}{3x}\right); = 2x$</p> <p>As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ AG</p>	<p>$\frac{dV}{dx} = 3x^2$ B1</p> <p>Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}; \lambda x$ M1; A1✓</p> <p>Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ A1</p> <p>[4]</p>
(c)	<p>$\int \frac{dV}{V^{\frac{1}{3}}} = \int 2 dt$</p> <p>$\int V^{-\frac{1}{3}} dV = \int 2 dt$</p> <p>$\frac{3}{2} V^{\frac{2}{3}} = 2t (+c)$</p> <p>$\frac{3}{2} (8)^{\frac{2}{3}} = 2(0) + c \Rightarrow c = 6$</p> <p>Hence: $\frac{3}{2} V^{\frac{2}{3}} = 2t + 6$</p> <p>$\frac{3}{2} (16\sqrt{2})^{\frac{2}{3}} = 2t + 6 \Rightarrow 12 = 2t + 6$</p> <p>giving $t = 3$.</p>	<p>Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2 dt$ on the other side. integral signs not necessary. B1</p> <p>Attempts to integrate and must see $V^{\frac{2}{3}}$ and $2t$; Correct equation with/without $+c$. M1; A1</p> <p>Use of $V = 8$ and $t = 0$ in a changed equation containing c; $c = 6$ M1*; A1</p> <p>Having found their "c" candidate substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c". depM1*</p> <p>$t = 3$ A1 cao</p> <p>[7]</p>
		15 marks

Question Number	Scheme	Marks
<p>Aliter</p> <p>7. (b)</p> <p>Way 2</p>	<p>$x = V^{\frac{1}{3}}$ & $S = 6x^2 \Rightarrow S = 6V^{\frac{2}{3}}$</p> <p>$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$</p> <p>$\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS} = 8 \cdot \left(\frac{1}{4V^{-\frac{1}{3}}} \right); = \frac{2}{V^{-\frac{1}{3}}} = 2V^{\frac{1}{3}}$ AG</p>	<p>$S = 6V^{\frac{2}{3}}$ B1 $\sqrt{\quad}$</p> <p>B1</p> <p>M1; A1</p> <p>In ePEN, award Marks for Way 2 in the order they appear on this mark scheme.</p> <p>[4]</p>
<p>Aliter</p> <p>7. (c)</p> <p>Way 2</p>	<p>$\int \frac{dV}{2V^{\frac{1}{3}}} = \int 1 dt$</p> <p>$\frac{1}{2} \int V^{-\frac{1}{3}} dV = \int 1 dt$</p> <p>$\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)V^{\frac{2}{3}} = t (+c)$</p> <p>$\frac{3}{4}(8)^{\frac{2}{3}} = (0) + c \Rightarrow c = 3$</p> <p>Hence: $\frac{3}{4}V^{\frac{2}{3}} = t + 3$</p> <p>$\frac{3}{4}(16\sqrt{2})^{\frac{2}{3}} = t + 3 \Rightarrow 6 = t + 3$</p> <p>giving $t = 3$.</p>	<p>Separates the variables with $\int \frac{dV}{2V^{\frac{1}{3}}}$ or $\int \frac{1}{2}V^{-\frac{1}{3}}dV$ oe on one side and $\int 1 dt$ on the other side. integral signs not necessary.</p> <p>Attempts to integrate and must see $V^{\frac{2}{3}}$ and t; Correct equation with/without + c. M1; A1</p> <p>Use of $V = 8$ and $t = 0$ in a changed equation containing c ; c = 3 M1 *; A1</p> <p>Having found their "c" candidate substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c". depM1 *</p> <p>t = 3 A1 cao [7]</p>

Question Number	Scheme	Marks
Aliter	<i>similar to way 1.</i>	
(b)	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$ B1
Way 3	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS} = 3x^2 \cdot 8 \cdot \left(\frac{1}{12x}\right); = 2x$	Candidate's $\frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS}; \lambda x$ M1; A1 $\sqrt{\quad}$
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ AG	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ A1
Aliter		[4]
(c)	$\int \frac{dV}{V^{\frac{1}{3}}} = \int 2 dt$	Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2 dt$ on the other side. B1
Way 3	$\int V^{-\frac{1}{3}} dV = \int 2 dt$	integral signs not necessary.
	$V^{\frac{2}{3}} = \frac{4}{3}t + c$	Attempts to integrate and must see $V^{\frac{2}{3}}$ and $\frac{4}{3}t$; M1; A1
	$(8)^{\frac{2}{3}} = \frac{4}{3}(0) + c \Rightarrow c = 4$	Correct equation with/without + c. Use of $V = 8$ and $t = 0$ in a changed equation containing c ; $c = 4$ M1*; A1
	Hence: $V^{\frac{2}{3}} = \frac{4}{3}t + 4$	
	$(16\sqrt{2})^{\frac{2}{3}} = \frac{4}{3}t + 6 \Rightarrow 8 = \frac{4}{3}t + 4$	Having found their "c" candidate substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c". depM1*
	giving $t = 3$.	t = 3 A1 cao
		[7]