

Mark Scheme (Results)

Summer 2008

GCE

GCE Mathematics (6666/01)

June 2008
6666 Core Mathematics C4
Mark Scheme

| Question | Scheme | Marks | | | | | | | | | | | | | | | | | | | | | |
|------------------------|--|---|-------------------|-------------------|-------------------|----------------|-----|---|---|----------------|-------------------|-------------------|-------------------|-------------------|----------------|------|---|----------------|------------|------------|------------|------------|---------------|
| 1. (a) | <table><tr><td>x</td><td>0</td><td>0.4</td><td>0.8</td><td>1.2</td><td>1.6</td><td>2</td></tr><tr><td>y</td><td>e⁰</td><td>e^{0.08}</td><td>e^{0.32}</td><td>e^{0.72}</td><td>e^{1.28}</td><td>e²</td></tr><tr><td>or y</td><td>1</td><td>1.08329 ...</td><td>1.37713...</td><td>2.05443...</td><td>3.59664...</td><td>7.38906...</td></tr></table> <p>Either e^{0.32} and e^{1.28} or awrt 1.38 and 3.60 (or a mixture of e's and decimals)</p> | x | 0 | 0.4 | 0.8 | 1.2 | 1.6 | 2 | y | e ⁰ | e ^{0.08} | e ^{0.32} | e ^{0.72} | e ^{1.28} | e ² | or y | 1 | 1.08329 ... | 1.37713... | 2.05443... | 3.59664... | 7.38906... | B1 [1] |
| x | 0 | 0.4 | 0.8 | 1.2 | 1.6 | 2 | | | | | | | | | | | | | | | | | |
| y | e ⁰ | e ^{0.08} | e ^{0.32} | e ^{0.72} | e ^{1.28} | e ² | | | | | | | | | | | | | | | | | |
| or y | 1 | 1.08329 ... | 1.37713... | 2.05443... | 3.59664... | 7.38906... | | | | | | | | | | | | | | | | | |
| (b) Way 1 | <p>Area $\approx \frac{1}{2} \times 0.4 \times [e^0 + 2(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}) + e^2]$</p> <p>= 0.2 \times 24.61203164... = 4.922406... = <u>4.922</u> (4sf) <u>4.922</u></p> | Outside brackets $\frac{1}{2} \times 0.4$ or 0.2 For structure of <u>trapezium</u> <u>rule</u> [.....] ; <u>M1</u> $\sqrt{}$ A1 cao [3] | | | | | | | | | | | | | | | | | | | | | |
| Aliter (b) Way 2 | <p>Area $\approx 0.4 \times \left[\frac{e^0 + e^{0.08}}{2} + \frac{e^{0.08} + e^{0.32}}{2} + \frac{e^{0.32} + e^{0.72}}{2} + \frac{e^{0.72} + e^{1.28}}{2} + \frac{e^{1.28} + e^2}{2} \right]$</p> <p>which is equivalent to:</p> <p>Area $\approx \frac{1}{2} \times 0.4 \times [e^0 + 2(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}) + e^2]$</p> <p>= 0.2 \times 24.61203164... = 4.922406... = <u>4.922</u> (4sf) <u>4.922</u></p> | 0.4 and a divisor of 2 on all terms inside brackets. One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. <u>M1</u> $\sqrt{}$ A1 cao [3] | | | | | | | | | | | | | | | | | | | | | |
| | | 4 marks | | | | | | | | | | | | | | | | | | | | | |

Note an expression like Area $\approx \frac{1}{2} \times 0.4 + e^0 + 2(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}) + e^2$ would score B1M1A0

Allow one term missing (slip!) in the () brackets for

The M1 mark for structure is for the material found in the curly brackets ie
[first y ordinate + 2(intermediate ft y ordinate) + final y ordinate]

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|-----------------|--|--|
| 2. (a) | $\left\{ \begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$ | |
| | $\int x e^x dx = x e^x - \int e^x \cdot 1 dx$ <p>Use of 'integration by parts' formula in the correct direction. (See note.)</p> <p>Correct expression. (Ignore dx)</p> $= x e^x - \int e^x dx$ $= x e^x - e^x (+ c)$ <p>Correct integration with/without + c</p> | <p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p> |
| (b) | $\left\{ \begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$ | |
| | $\int x^2 e^x dx = x^2 e^x - \int e^x \cdot 2x dx$ <p>Use of 'integration by parts' formula in the correct direction.</p> <p>Correct expression. (Ignore dx)</p> $= x^2 e^x - 2 \int x e^x dx$ $= x^2 e^x - 2(x e^x - e^x) + c$ <p>Correct expression including + c. (seen at any stage! in part (b))</p> <p>You can ignore subsequent working.</p> $\left\{ \begin{array}{l} = x^2 e^x - 2x e^x + 2e^x + c \\ = e^x (x^2 - 2x + 2) + c \end{array} \right\}$ <p>Ignore subsequent working</p> | <p>M1</p> <p>A1</p> <p>A1 ISW</p> <p>[3]</p> |
| | | 6 marks |

Note integration by parts in the **correct direction** means that u and $\frac{dv}{dx}$ must be assigned/used as $u = x$ and $\frac{dv}{dx} = e^x$ in part (a) for example

+ c is not required in part (a).

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|-----------------|--|---|
| 3. (a) | From question, $\frac{dA}{dt} = 0.032$ | $\frac{dA}{dt} = 0.032$ seen or implied from working. B1 |
| | $\left\{ A = \pi x^2 \Rightarrow \frac{dA}{dx} = \right\} 2\pi x$ | $2\pi x$ by itself seen or implied from working B1 |
| | $\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032) \frac{1}{2\pi x}; \left\{ = \frac{0.016}{\pi x} \right\}$ | $0.032 \div \text{Candidate's } \frac{dA}{dx};$ M1; |
| | When $x = 2 \text{ cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$ | |
| | Hence, $\frac{dx}{dt} = 0.002546479... \text{ (cm s}^{-1}\text{)}$ | awrt 0.00255 A1 cso |
| | | [4] |
| (b) | $V = \pi x^2(5x) = 5\pi x^3$ | $V = \pi x^2(5x)$ or $5\pi x^3$ B1 |
| | $\frac{dV}{dx} = 15\pi x^2$ | $\frac{dV}{dx} = 15\pi x^2$ or ft from candidate's V in one variable B1 ✓ |
| | $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2 \cdot \left(\frac{0.016}{\pi x} \right); \{ = 0.24x \}$ | Candidate's $\frac{dV}{dx} \times \frac{dx}{dt};$ M1 ✓ |
| | When $x = 2 \text{ cm}$, $\frac{dV}{dt} = 0.24(2) = 0.48 \text{ (cm}^3 \text{ s}^{-1}\text{)}$ | 0.48 or awrt 0.48 A1 cso |
| | | [4] |
| | | 8 marks |

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|-----------------|---|---|
| 4. (a) | $3x^2 - y^2 + xy = 4 \quad (\text{eqn *})$ <p>Differentiates implicitly to include either $\pm ky \frac{dy}{dx}$ or $x \frac{dy}{dx}$. (Ignore $(\frac{dy}{dx} =)$)</p> $\left\{ \frac{\cancel{dy}}{\cancel{dx}} \right\} \times \left(6x - 2y \frac{dy}{dx} + \left(y + x \frac{dy}{dx} \right) \right) = 0$ <p>Correct application () of product rule</p> $(3x^2 - y^2) \rightarrow \left(6x - 2y \frac{dy}{dx} \right) \text{ and } (4 \rightarrow 0)$ <p><i>not necessarily required.</i></p> $\left\{ \frac{dy}{dx} = \frac{-6x - y}{x - 2y} \right\} \text{ or } \left\{ \frac{dy}{dx} = \frac{6x + y}{2y - x} \right\}$ <p>Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation.</p> $\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$ <p>giving $-18x - 3y = 8x - 16y$</p> <p>giving $13y = 26x$</p> <p>Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$</p> <p>simplifying to give $\underline{y - 2x = 0}$ AG</p> | <p>M1</p> <p>B1</p> <p>A1</p> <p>M1 *</p> <p>dM1 *</p> <p>A1 cso</p> <p>[6]</p> |
| (b) | <p>At P & Q, $y = 2x$. Substituting into eqn *</p> <p>gives $3x^2 - (2x)^2 + x(2x) = 4$</p> <p>Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$</p> <p>$y = 2x \Rightarrow y = \pm 4$</p> <p>Hence coordinates are $\underline{(2, 4)}$ and $\underline{(-2, -4)}$</p> <p>Both $\underline{(2, 4)}$ and $\underline{(-2, -4)}$</p> | <p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p> |
| | | 9 marks |

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|-----------------|---|--|
| 5. (a) | <p>** represents a constant (which must be consistent for first accuracy mark)</p> $\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)}^{-\frac{1}{2}} \left(1 - \frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\underline{2}} \left(1 - \frac{3x}{4}\right)^{-\frac{1}{2}} \quad \underline{(4)}^{-\frac{1}{2}} \text{ or } \underline{\underline{2}} \text{ outside brackets}$ | B1 |
| | <p>Expands $(1+**x)^{-\frac{1}{2}}$ to give a simplified or an un-simplified $1 + (-\frac{1}{2})(**x)$;</p> <p>A correct simplified or an un-simplified [.....] expansion with candidate's followed through $(**x)$</p> | M1; |
| | $= \frac{1}{2} \left[1 + (-\frac{1}{2})(**x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (**x)^2 + \dots \right]$ <p>with $** \neq 1$</p> | A1 $\sqrt{}$ |
| | $= \frac{1}{2} \left[1 + (-\frac{1}{2})(-\frac{3x}{4}) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (-\frac{3x}{4})^2 + \dots \right]$ | <p>Award SC M1 if you see $(-\frac{1}{2})(**x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (**x)^2$</p> |
| | $= \frac{1}{2} \left[1 + \frac{3}{8}x + \frac{27}{128}x^2 + \dots \right]$ | A1 isw |
| (b) | $\left\{ = \frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \dots \right\}$ | A1 isw |
| | <p>Ignore subsequent working</p> | |
| | <p>Writing $(x+8)$ multiplied by candidate's part (a) expansion.</p> | [5] |
| | <p>Multiply out brackets to find a constant term, two x terms and two x^2 terms.</p> | M1 |
| | <p>Anything that cancels to $4 + 2x; \frac{33}{32}x^2$</p> | <p>↓ ↓</p> <p>A1; A1</p> |
| | | [4] |
| | | 9 marks |

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|-----------------|--|---|
| 6. (a) | <p>Lines meet where:</p> $\begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ <p> i: $-9 + 2\lambda = 3 + 3\mu$ (1) Any two of j: $\lambda = 1 - \mu$ (2) k: $10 - \lambda = 17 + 5\mu$ (3) </p> <p>(1) - 2(2) gives: $-9 = 1 + 5\mu \Rightarrow \mu = -2$</p> <p>(2) gives: $\lambda = 1 - (-2) = 3$</p> $\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} - 2 \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ <p>Intersect at $\mathbf{r} = \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix}$ or $\mathbf{r} = \underline{-3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}}$</p> <p>Either check k: $\lambda = 3$: LHS = $10 - \lambda = 10 - 3 = 7$ $\mu = -2$: RHS = $17 + 5\mu = 17 - 10 = 7$</p> <p>(As LHS = RHS then the lines intersect.)</p> | <p>Need any two of these correct equations seen anywhere in part (a). M1</p> <p>Attempts to solve simultaneous equations to find one of either λ or μ dM1</p> <p>Both $\underline{\lambda = 3}$ & $\underline{\mu = -2}$ A1</p> <p>Substitutes their value of either λ or μ into the line l_1 or l_2 respectively. This mark can be implied by any two correct components of $(-3, 3, 7)$. ddM1</p> <p>$\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix}$ or $\underline{-3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}}$ A1</p> <p>or $(-3, 3, 7)$</p> <p>Either check that $\lambda = 3, \mu = -2$ in a third equation or check that $\lambda = 3, \mu = -2$ give the same coordinates on the other line. B1</p> <p>Conclusion not needed.</p> <p>[6]</p> |
| (b) | <p>$\mathbf{d}_1 = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{d}_2 = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$</p> $\text{As } \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = \underline{(2 \times 3) + (1 \times -1) + (-1 \times 5)} = 0$ <p>Then l_1 is perpendicular to l_2.</p> | <p>Dot product calculation between the two direction vectors: $\underline{(2 \times 3) + (1 \times -1) + (-1 \times 5)}$ or $\underline{6 - 1 - 5}$ M1</p> <p>Result '$=0$' and appropriate conclusion A1</p> <p>[2]</p> |

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| 6. (c) | <p>Equating \mathbf{i} ; $-9 + 2\lambda = 5 \Rightarrow \lambda = 7$</p> $\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 7 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$ <p>(= \overrightarrow{OA}. Hence the point A lies on l_1.)</p> | <p>B1</p> <p>[1]</p> |
| (d) | <p>Let $\overrightarrow{OX} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ be point of intersection</p> $\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} = \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$ $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$ $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2 \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$ <p>Hence, $\overrightarrow{OB} = \begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$</p> | <p>Finding the difference between their \overrightarrow{OX} (can be implied) and \overrightarrow{OA}.</p> $\overrightarrow{AX} = \pm \left(\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} \right)$ <p>M1 $\sqrt{\pm}$</p> <p>dM1 $\sqrt{\pm}$</p> <p>A1</p> <p>[3]</p> |
| | | 12 marks |

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|-----------------|---|--|
| 7. (a) | $\frac{2}{4-y^2} \equiv \frac{2}{(2-y)(2+y)} \equiv \frac{A}{(2-y)} + \frac{B}{(2+y)}$ <p>Forming this identity. NB: A & B are not assigned in this question</p> $2 \equiv A(2+y) + B(2-y)$ <p>Let $y = -2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$</p> <p>Let $y = 2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$</p> <p>giving $\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}$</p> <p>Either one of $A = \frac{1}{2}$ or $B = \frac{1}{2}$</p> <p>$\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}$, aef</p> <p>(If no working seen, but candidate writes down correct partial fraction then award all three marks. If no working is seen but one of A or B is incorrect then M0A0A0.)</p> | <p>M1</p> <p>A1</p> <p>A1 cao</p> <p>[3]</p> |

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|-----------------|--|--|
| 7. (b) | $\int \frac{2}{4-y^2} dy = \int \frac{1}{\cot x} dx$ <p>Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.</p> $\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)} dy = \int \tan x dx$ <p>$\ln(\sec x)$ or $-\ln(\cos x)$</p> <p>Either $\pm a \ln(\lambda - y)$ or $\pm b \ln(\lambda + y)$</p> <p>their $\int \frac{1}{\cot x} dx = \text{LHS correct with ft for their A and B and no error with the "2" with or without } + c$</p> <p>$\therefore -\frac{1}{2} \ln(2-y) + \frac{1}{2} \ln(2+y) = \ln(\sec x) + (c)$</p> <p>Use of $y=0$ and $x=\frac{\pi}{3}$ in an integrated equation containing c ;</p> <p>$y=0, x=\frac{\pi}{3} \Rightarrow -\frac{1}{2} \ln 2 + \frac{1}{2} \ln 2 = \ln\left(\frac{1}{\cos(\frac{\pi}{3})}\right) + c$</p> <p>$\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\}$</p> <p>$-\frac{1}{2} \ln(2-y) + \frac{1}{2} \ln(2+y) = \ln(\sec x) - \ln 2$</p> <p>Using either the quotient (or product) or power laws for logarithms CORRECTLY.</p> <p>$\frac{1}{2} \ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$</p> <p>$\ln\left(\frac{2+y}{2-y}\right) = 2 \ln\left(\frac{\sec x}{2}\right)$</p> <p>Using the log laws correctly to obtain a single log term on both sides of the equation.</p> <p>$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec^2 x}{2}\right)$</p> <p>$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$</p> <p>Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$</p> <p>$\underline{\sec^2 x = \frac{8+4y}{2-y}}$</p> | <p>B1</p> <p>B1</p> <p>M1;</p> <p>A1 $\sqrt{\quad}$</p> <p>M1*</p> <p>M1</p> <p>dM1*</p> <p>A1 aef</p> <p>[8]</p> |
| | | |
| | | |
| | | |
| | | 11 marks |

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|-----------------|---|---|
| 8. (a) | <p>At $P(4, 2\sqrt{3})$ either $4 = 8\cos t$ or $2\sqrt{3} = 4\sin 2t$ $4 = 8\cos t$ or $2\sqrt{3} = 4\sin 2t$</p> <p>\Rightarrow only solution is $t = \frac{\pi}{3}$ where $0 \leq t \leq \frac{\pi}{2}$ $t = \frac{\pi}{3}$ or awrt 1.05 (radians) only stated in the range $0 \leq t \leq \frac{\pi}{2}$</p> | <p>M1</p> <p>A1</p> <p>[2]</p> |
| (b) | <p>$x = 8\cos t$, $y = 4\sin 2t$</p> <p>$\frac{dx}{dt} = -8\sin t$, $\frac{dy}{dt} = 8\cos 2t$</p> <p>At P, $\frac{dy}{dx} = \frac{8\cos(\frac{2\pi}{3})}{-8\sin(\frac{\pi}{3})}$</p> <p>$\left\{ = \frac{8(-\frac{1}{2})}{(-8)(\frac{\sqrt{3}}{2})} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$</p> <p>Hence $m(N) = -\sqrt{3}$ or $-\frac{1}{\sqrt{3}}$</p> <p>N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$</p> <p>N: $y = -\sqrt{3}x + 6\sqrt{3}$ AG</p> <p>or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$</p> <p>so N: $y = -\sqrt{3}x + 6\sqrt{3}$</p> | <p>Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and $\pm q\cos 2t$ respectively</p> <p>Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.</p> <p>M1</p> <p>A1</p> <p>M1*</p> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>You may need to check candidate's substitutions for M1*</p> <p>Note the next two method marks are dependent on M1*</p> </div> <p>Uses $m(N) = -\frac{1}{\text{their } m(T)}$.</p> <p>Uses $y - 2\sqrt{3} = (\text{their } m_N)(x - 4)$ or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses $y = (\text{their } m_N)x + "c"$.</p> <p>$y = -\sqrt{3}x + 6\sqrt{3}$</p> <p>dM1*</p> <p>dM1*</p> <p>A1 cso</p> <p>AG</p> <p>[6]</p> |

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| 8. (c) | $A = \int_0^4 y \, dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4 \sin 2t \cdot (-8 \sin t) \, dt$ | <p>attempt at $A = \int y \frac{dx}{dt} \, dt$ M1 correct expression (ignore limits and dt) A1</p> |
| | $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32 \sin 2t \cdot \sin t \, dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32 (2 \sin t \cos t) \cdot \sin t \, dt$ | <p>Seeing $\sin 2t = 2 \sin t \cos t$ anywhere in PART (c). M1</p> |
| (d) | $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64 \cdot \sin^2 t \cos t \, dt$ | <p>Correct proof. Appreciation of how the negative sign affects the limits. Note that the answer is given in the question. A1 AG</p> |
| | $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 64 \cdot \sin^2 t \cos t \, dt$ | |
| (d) | <p>{Using substitution $u = \sin t \Rightarrow \frac{du}{dt} = \cos t$ } {change limits: when $t = \frac{\pi}{3}$, $u = \frac{\sqrt{3}}{2}$ & when $t = \frac{\pi}{2}$, $u = 1$ }</p> | <p>$k \sin^3 t$ or ku^3 with $u = \sin t$ M1 Correct integration ignoring limits. A1</p> |
| | $A = 64 \left[\frac{\sin^3 t}{3} \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \quad \text{or} \quad A = 64 \left[\frac{u^3}{3} \right]_{\frac{\sqrt{3}}{2}}^1$ | |
| (d) | $A = 64 \left[\frac{1}{3} - \left(\frac{1}{3} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \right) \right]$ | <p>Substitutes limits of either $(t = \frac{\pi}{2} \text{ and } t = \frac{\pi}{3})$ or $(u = 1 \text{ and } u = \frac{\sqrt{3}}{2})$ and subtracts the correct way round. dM1</p> |
| | $A = 64 \left(\frac{1}{3} - \frac{1}{8} \sqrt{3} \right) = \frac{64}{3} - 8\sqrt{3}$ | |
| (d) | <p>(Note that $a = \frac{64}{3}$, $b = -8$)</p> | <p>$\frac{64}{3} - 8\sqrt{3}$ Aef in the form $a + b\sqrt{3}$, with awrt 21.3 and anything that cancels to $a = \frac{64}{3}$ and $b = -8$. A1 aef isw</p> |
| | | <p>[4]</p> |
| | | 16 marks |