Mark Scheme (Results)

Summer 2012

GCE Core Mathematics C4 (6666) Paper 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q), \text{ where} |pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where} |pq| = |c| \text{ and } |mn| = |a|$, leading to x = ...

2. Formula

Attempt to use correct formula (with values for a, b and c), leading to x = ...

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

June 2012
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $1 = A(3x-1)^2 + Bx(3x-1) + Cx$	B1
	$x \to 0$ $\quad (1 = A)$	M1
	$x \to \frac{1}{3}$ $\quad 1 = \frac{1}{3}C \Rightarrow C = 3$ any two constants correct	A1
	Coefficients of x^2	
	$0 = 9A + 3B \Rightarrow B = -3$ all three constants correct	A1 **(4)**
	(b)(i) $\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) dx$	
	$= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$	M1 A1ft A1ft
	$\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$	
	(ii) $\int_1^2 f(x)\,dx = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$	
	$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$	M1
	$= \ln \frac{2 \times 2}{5} + \ldots$	M1
	$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$	A1 **(6)**
		[10]

Question Number	Scheme	Marks
2.	(a) $V = x^3 \Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$ ∗ cso	B1 **(1)**
	(b) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\mathrm{d}x}{\mathrm{d}V} \times \dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{0.048}{3x^2}$	M1
	At $x = 8$	
	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{0.048}{3(8^2)} = 0.00025 \quad \left(\mathrm{cm\,s^{-1}}\right)$ 2.5×10^{-4}	A1 **(2)**
	(c) $S = 6x^2 \Rightarrow \dfrac{\mathrm{d}S}{\mathrm{d}x} = 12x$	B1
	$\dfrac{\mathrm{d}S}{\mathrm{d}t} = \dfrac{\mathrm{d}S}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}t} = 12x\left(\dfrac{0.048}{3x^2}\right)$	M1
	At $x = 8$	
	$\dfrac{\mathrm{d}S}{\mathrm{d}t} = 0.024 \quad \left(\mathrm{cm^2\,s^{-1}}\right)$	A1 **(3)**
		[6]

Question Number	Scheme	Marks
3.	(a) $\mathrm{f}(x) = \ \dots\ (\dots\ - \dots\ x)^{-\frac{1}{2}}$	M1
	$= 6 \times 9^{-\frac{1}{2}}(\ \dots\ )$ $\quad \dfrac{6}{9^{\frac{1}{2}}}$, $\dfrac{6}{3}$, 2 or equivalent	B1
	$= \ \dots\ \left(1 + \left(-\tfrac{1}{2}\right)(kx); + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(kx)^2 + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(kx)^3 + \ \dots\right)$	M1; A1ft
	$= 2\left(1 + \dfrac{2}{9}x + \ \dots\right)$ $\quad$ or $2 + \dfrac{4}{9}x$	A1
	$= 2 + \dfrac{4}{9}x + \dfrac{4}{27}x^2 + \dfrac{40}{729}x^3 + \ \dots$	A1 **(6)**
	(b) $\mathrm{g}(x) = 2 - \dfrac{4}{9}x + \dfrac{4}{27}x^2 - \dfrac{40}{729}x^3 + \ \dots$	B1ft **(1)**
	(c) $\mathrm{h}(x) = 2 + \dfrac{4}{9}(2x) + \dfrac{4}{27}(2x)^2 + \dfrac{40}{729}(2x)^3 + \ \dots$	M1 A1 **(2)**
	$\left(= 2 + \dfrac{8}{9}x + \dfrac{16}{27}x^2 + \dfrac{320}{729}x^3 + \ \dots\right)$	**[9]**

Question Number	Scheme		Marks	
4.	$\int y\,\mathrm{d}y = \int \frac{3}{\cos^2 x}\,\mathrm{d}x$ $= \int 3\sec^2 x\,\mathrm{d}x$	Can be implied. Ignore integral signs	B1	
	$\frac{1}{2}y^2 = 3\tan x \quad (+C)$		M1 A1	
	$y = 2,\ x = \frac{\pi}{4}$ $\frac{1}{2}2^2 = 3\tan\frac{\pi}{4} + C$		M1	
	Leading to $C = -1$ $\frac{1}{2}y^2 = 3\tan x - 1$	or equivalent	A1	**(5)**
				[5]

Question Number	Scheme	Marks
5.	(a) Differentiating implicitly to obtain $\pm ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$ and/or $\pm bx^2\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$48y^2\frac{\mathrm{d}y}{\mathrm{d}x}+\ \dots\ -54\ \dots$	A1
	$9x^2y \to 9x^2\frac{\mathrm{d}y}{\mathrm{d}x}+18xy$ or equivalent	B1
	$\left(48y^2+9x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}+18xy-54=0$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{54-18xy}{48y^2+9x^2}\quad\left(=\frac{18-6xy}{16y^2+3x^2}\right)$	A1 **(5)**
	(b) $18-6xy=0$	M1
	Using $x=\frac{3}{y}$ or $y=\frac{3}{x}$	
	$16y^3+9\left(\frac{3}{y}\right)^2y-54\left(\frac{3}{y}\right)=0$ or $16\left(\frac{3}{x}\right)^3+9x^2\left(\frac{3}{x}\right)-54x=0$	M1
	Leading to	
	$16y^4+81-162=0$ or $16+x^4-2x^4=0$	M1
	$y^4=\frac{81}{16}$ or $x^4=16$	
	$y=\frac{3}{2},-\frac{3}{2}$ or $x=2,-2$	A1 A1
	Substituting either of their values into $xy=3$ to obtain a value of the other variable.	M1
	$\left(2,\frac{3}{2}\right),\ \left(-2,-\frac{3}{2}\right)$ both	A1 **(7)**
		[12]

Question Number	Scheme	Marks
6.	(a) $\frac{dx}{dt} = 2\sqrt{3}\cos 2t$	B1
	$\frac{dy}{dt} = -8\cos t \sin t$	M1 A1
	$\frac{dy}{dx} = \frac{-8\cos t \sin t}{2\sqrt{3}\cos 2t}$	M1
	$= -\frac{4\sin 2t}{2\sqrt{3}\cos 2t}$	
	$\frac{dy}{dx} = -\frac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\frac{2}{3}\right)$	A1 **(5)**
	(b) When $t = \frac{\pi}{3}$ $\qquad x = \frac{3}{2},\ y = 1$ $\qquad$ can be implied	B1
	$m = -\frac{2}{3}\sqrt{3}\tan\left(\frac{2\pi}{3}\right) \quad (= 2)$	M1
	$y - 1 = 2\left(x - \frac{3}{2}\right)$	M1
	$y = 2x - 2$	A1 **(4)**
	(c) $x = \sqrt{3}\sin 2t = \sqrt{3} \times 2\sin t \cos t$	M1
	$x^2 = 12\sin^2 t \cos^2 t = 12\left(1 - \cos^2 t\right)\cos^2 t$	
	$x^2 = 12\left(1 - \frac{y}{4}\right)\frac{y}{4}$ $\qquad$ or equivalent	M1 A1 **(3)**
		[12]
	Alternative to (c)	
	$y = 2\cos 2t + 2$	M1
	$\sin^2 2t + \cos^2 2t = 1$	
	$\frac{x^2}{3} + \frac{(y-2)^2}{4} = 1$	M1 A1 **(3)**

Question Number	Scheme		Marks
7.	(a)		
		x	1	2	3	4	
		y	ln2	$\sqrt{2}\ln 4$	$\sqrt{3}\ln 6$	2ln8	
			0.6931	1.9605	3.1034	4.1589	M1
	Area $=\frac{1}{2}\times 1(\ \ldots)$		B1
	$\approx\ \ldots\ \left(0.6931+2\left(1.9605+3.1034\right)+4.1589\right)$		M1
	$\approx \frac{1}{2}\times 14.97989\ \ldots\ \approx 7.49$	7.49 cao	A1 **(4)**
	(b) $\int x^{\frac{1}{2}}\ln 2x\,dx=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\int \frac{2}{3}x^{\frac{3}{2}}\times\frac{1}{x}\,dx$		M1 A1
	$=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\int \frac{2}{3}x^{\frac{1}{2}}\,dx$		
	$=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\frac{4}{9}x^{\frac{3}{2}}\quad (+C)$		M1 A1 **(4)**
	(c) $\left[\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\frac{4}{9}x^{\frac{3}{2}}\right]_1^4=\left(\frac{2}{3}4^{\frac{3}{2}}\ln 8-\frac{4}{9}4^{\frac{3}{2}}\right)-\left(\frac{2}{3}\ln 2-\frac{4}{9}\right)$		M1
	$=\left(16\ln 2-\ \ldots\right)-\ \ldots$	Using or implying $\ln 2^n=n\ln 2$	M1
	$=\frac{46}{3}\ln 2-\frac{28}{9}$		A1 **(3)**
			[11]

Question Number	Scheme	Marks
8.	(a) $\overrightarrow{AB} = \begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$	M1 A1 **(2)**
	(b) $\mathbf{r} = \begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ $\qquad$ $\mathbf{r} = \begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$	M1 A1ft **(2)**
	(c) $\overrightarrow{CP} = \begin{pmatrix} 10-2t \\ 2+t \\ 3+t \end{pmatrix} - \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 7-2t \\ t-10 \\ t \end{pmatrix}$	M1 A1
	$\begin{pmatrix} 7-2t \\ t-10 \\ t \end{pmatrix} . \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -14+4t+t-10+t=0$	M1
	Leading to $\qquad t=4$	A1
	Position vector of P is $\begin{pmatrix} 10-8 \\ 2+4 \\ 3+4 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$	M1 A1 **(6)**
		[10]
	Alternative working for (c)	
	$\overrightarrow{CP} = \begin{pmatrix} 8-2t \\ 3+t \\ 4+t \end{pmatrix} - \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 5-2t \\ t-9 \\ t+1 \end{pmatrix}$	M1 A1
	$\begin{pmatrix} 5-2t \\ t-9 \\ t+1 \end{pmatrix} . \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -10+4t+t-9+t+1=0$	M1
	Leading to $\qquad t=3$	A1
	Position vector of P is $\begin{pmatrix} 8-6 \\ 3+3 \\ 4+3 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$	M1 A1 **(6)**

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