

# Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Core Mathematics 4  
(6666/01)

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### General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
  - dM1 denotes a method mark which is dependent upon the award of the previous method mark.
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Core Mathematics Marking

*(But note that specific mark schemes may sometimes override these general principles).*

### **Method mark for solving 3 term quadratic:**

#### **1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$ , where  $|pq| = |c|$ , leading to  $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = \dots$

#### **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

#### **3. Completing the square**

Solving  $x^2 + bx + c = 0$ :  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$ ,  $q \neq 0$ , leading to  $x = \dots$

### **Method marks for differentiation and integration:**

#### **1. Differentiation**

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

#### **2. Integration**

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

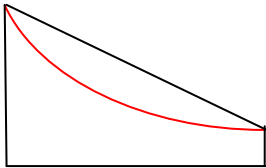
Question Number	Scheme		Marks
1.	$x^3 + 2xy - x - y^3 - 20 = 0$		
(a)	<div><math display="block">\left\{ \begin{array}{l} \cancel{\frac{dx}{dy}} \times \end{array} \right\} \underline{\underline{3x^2 + \left( 2y + 2x \frac{dy}{dx} \right) - 1 - 3y^2 \frac{dy}{dx} = 0}}</math><math display="block">3x^2 + 2y - 1 + \left( 2x - 3y^2 \right) \frac{dy}{dx} = 0</math><math display="block">\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x} \quad \text{or} \quad \frac{1 - 3x^2 - 2y}{2x - 3y^2}</math></div>		M1 <u>A1</u> <u>B1</u>  dM1  A1 <b>cso</b>  <b>[5]</b>
(b)	<div>At <math>P(3, -2)</math>, <math>m(\mathbf{T}) = \frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}; = \frac{22}{6}</math> or <math>\frac{11}{3}</math>  <b>and either T:</b> <math>y - -2 = \frac{11}{3} (x - 3)</math>  <b>or</b> <math>(-2) = \left( \frac{11}{3} \right) (3) + c \Rightarrow c = \dots,</math></div>		<div>see notes M1</div>
	<b>T:</b> $11x - 3y - 39 = 0$ or $K(11x - 3y - 39) = 0$		A1 <b>cso</b>
			<b>[2]</b> 7
(a)	<div><b><u>Alternative method for part (a)</u></b>  <math display="block">\left\{ \begin{array}{l} \cancel{\frac{dx}{dy}} \times \end{array} \right\} \underline{\underline{3x^2 \frac{dx}{dy} + \left( 2y \frac{dx}{dy} + 2x \right) - \frac{dx}{dy} - 3y^2 = 0}}</math><math display="block">2x - 3y^2 + \left( 3x^2 + 2y - 1 \right) \frac{dx}{dy} = 0</math><math display="block">\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x} \quad \text{or} \quad \frac{1 - 3x^2 - 2y}{2x - 3y^2}</math></div>		M1 <u>A1</u> <u>B1</u>  dM1  A1 <b>cso</b>  <b>[5]</b>
<b>Question 1 Notes</b>			
(a) <b>General</b>	<b>Note</b>	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$ from <b>no working</b> is full marks.	
	<b>Note</b>	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{2x - 3y^2}$ or $\frac{1 - 3x^2 - 2y}{3y^2 - 2x}$ from <b>no working</b> is M1A0B0M1A0	
	<b>Note</b>	Few candidates will write $3x^2 + 2y + 2x dy - 1 - 3y^2 dy = 0$ leading to $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ , o.e.  This should get full marks.	
1. (a)	<b>M1</b>	Differentiates implicitly to include either $2x \frac{dy}{dx}$ or $-y^3 \rightarrow \pm k y^2 \frac{dy}{dx}$ . (Ignore $\left( \frac{dy}{dx} = \right)$ ).	
	<b>A1</b>	$x^3 \rightarrow 3x^2$ <b>and</b> $-x - y^3 - 20 = 0 \rightarrow -1 - 3y^2 \frac{dy}{dx} = 0$	
	<b>B1</b>	$2xy \rightarrow 2y + 2x \frac{dy}{dx}$	
	<b>Note</b>	If an extra term appears then award 1 <sup>st</sup> A0.	



1. (a) ctd	<p><b>Note</b></p> <p><b>dM1</b></p> <p><b>Note</b></p> <p><b>A1</b></p>	<p><math>3x^2 + 2y + 2x \frac{dy}{dx} - 1 - 3y^2 \frac{dy}{dx} \rightarrow 3x^2 + 2y - 1 = 3y^2 \frac{dy}{dx} - 2x \frac{dy}{dx}</math></p> <p>will get 1<sup>st</sup> A1 (implied) as the "= 0" can be implied by rearrangement of their equation.</p> <p><b>dependent on the first method mark being awarded.</b></p> <p>An attempt to factorise out <b>all the terms in</b> <math>\frac{dy}{dx}</math> as long as there are <b>at least two terms</b> in <math>\frac{dy}{dx}</math>.</p> <p>ie. <math>\dots + (2x - 3y^2) \frac{dy}{dx} = \dots</math></p> <p>Placing an extra <math>\frac{dy}{dx}</math> at the beginning and then including it in their factorisation is fine for dM1.</p> <p>For <math>\frac{1 - 2y - 3x^2}{2x - 3y^2}</math> or equivalent. Eg: <math>\frac{3x^2 + 2y - 1}{3y^2 - 2x}</math></p> <p><b>cs0:</b> If the candidate's solution is not completely correct, then do not give this mark.</p> <p><b>isw:</b> You can, however, ignore subsequent working following on from correct solution.</p>
1. (b)	<p><b>M1</b></p> <p><b>Note</b></p> <p><b>A1</b></p> <p><b>cs0</b></p> <p><b>isw</b></p>	<p><b>Some</b> attempt to substitute <b>both</b> <math>x = 3</math> <b>and</b> <math>y = -2</math> into their <math>\frac{dy}{dx}</math> which contains both <math>x</math> and <math>y</math> to find <math>m_T</math> <b>and</b></p> <ul style="list-style-type: none"> <li><b>either</b> applies <math>y - -2 = (\text{their } m_T)(x - 3)</math>, where <math>m_T</math> is a numerical value.</li> <li><b>or</b> finds <math>c</math> by solving <math>(-2) = (\text{their } m_T)(3) + c</math>, where <math>m_T</math> is a numerical value.</li> </ul> <p>Using a changed gradient (i.e. applying <math>\frac{-1}{\text{their } \frac{dy}{dx}}</math> or <math>\frac{1}{\text{their } \frac{dy}{dx}}</math> is M0).</p> <p>Accept any integer multiple of <math>11x - 3y - 39 = 0</math> or <math>11x - 39 - 3y = 0</math> or <math>-11x + 3y + 39 = 0</math>, where their tangent equation is equal to 0.</p> <p>A correct solution is required from a correct <math>\frac{dy}{dx}</math>.</p> <p>You can ignore subsequent working following a correct solution.</p>
1. (a)	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>dM1</b></p> <p><b>A1</b></p>	<p><b><u>Alternative method for part (a): Differentiating with respect to y</u></b></p> <p>Differentiates implicitly to include either <math>2y \frac{dx}{dy}</math> or <math>x^3 \rightarrow \pm kx^2 \frac{dx}{dy}</math> or <math>-x \rightarrow -\frac{dx}{dy}</math></p> <p>(Ignore <math>\left(\frac{dx}{dy} = \right)</math>).</p> <p><math>x^3 \rightarrow 3x^2 \frac{dx}{dy}</math> <b>and</b> <math>-x - y^3 - 20 = 0 \rightarrow -\frac{dx}{dy} - 3y^2 = 0</math></p> <p><math>2xy \rightarrow 2y \frac{dx}{dy} + 2x</math></p> <p><b>dependent on the first method mark being awarded.</b></p> <p>An attempt to factorise out <b>all the terms in</b> <math>\frac{dx}{dy}</math> as long as there are <b>at least two terms</b> in <math>\frac{dx}{dy}</math>.</p> <p>For <math>\frac{1 - 2y - 3x^2}{2x - 3y^2}</math> or equivalent. Eg: <math>\frac{3x^2 + 2y - 1}{3y^2 - 2x}</math></p> <p><b>cs0:</b> If the candidate's solution is not completely correct, then do not give this mark.</p>

Question Number	Scheme	Marks
2.	$\left\{ (1 + kx)^{-4} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + \dots \right\}$	
(a)	<p><b>Either</b> <math>(-4)k = -6</math> <b>or</b> <math>(1 + kx)^{-4} = 1 + (-4)(kx)</math> <b>see notes</b></p> <p>leading to <math>k = \frac{3}{2}</math> <span style="float: right;"><math>k = \frac{3}{2}</math> or 1.5 or <math>\frac{6}{4}</math></span></p>	<p>M1</p> <p>A1</p> <p>[2]</p>
(b)	<p><math>\frac{(-4)(-5)}{2}(k)^2</math> <b>Either</b> <math>\frac{(-4)(-5)}{2!}</math> <b>or</b> <math>(k)^2</math> <b>or</b> <math>(kx)^2</math></p> <p><b>Either</b> <math>\frac{(-4)(-5)}{2!}(k)^2</math> <b>or</b> <math>\frac{(-4)(-5)}{2!}(kx)^2</math></p> <p><math>\left\{ A = \frac{(-4)(-5)}{2!} \left( \frac{3}{2} \right)^2 \right\} \Rightarrow A = \frac{45}{2}</math> <span style="float: right;"><math>\frac{45}{2}</math> or 22.5</span></p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>5</p>
<b>Question 2 Notes</b>		
<b>Note</b>	<b>In this question ignore part labelling and mark part (a) and part (b) together.</b>	
<b>Note</b>	Writing down $\left\{ (1 + kx)^{-4} \right\} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + \dots$ gets all the method marks in Q2. i.e. (a) M1 and (b) M1M1	
(a)	<p><b>M1</b> Award M1 for</p> <ul style="list-style-type: none"> <li><b>either</b> writing down <math>(-4)k = -6</math> or <math>4k = 6</math></li> <li><b>or</b> expanding <math>(1 + kx)^{-4}</math> to give <math>1 + (-4)(kx)</math></li> <li><b>or</b> writing down <math>(-4)kx = -6</math> or <math>(-4k) = -6x</math> or <math>-4kx = -6x</math></li> </ul> <p><b>A1</b> <math>k = \frac{3}{2}</math> or 1.5 or <math>\frac{6}{4}</math> <b>from no incorrect sign errors.</b></p>	
<b>Note</b>	The M1 mark can be implied by a candidate writing down the correct value of $k$ .	
<b>Note</b>	Award M1 for writing down $4k = 6$ and then A1 for $k = 1.5$ (or equivalent).	
<b>Note</b>	Award M0 for $4k = -6$ (if there is no evidence that $(1 + kx)^{-4}$ expands to give $1 + (-4)(kx) + \dots$ )	
<b>Note</b>	$1 + (-4)(kx)$ leading to $(-4)k = 6$ leading to $k = \frac{3}{2}$ is M1A0.	
(b)	<p><b>M1</b> For <b>either</b> <math>\frac{(-4)(-4-1)}{2!}</math> <b>or</b> <math>\frac{(-4)(-5)}{2!}</math> <b>or</b> 10 <b>or</b> <math>(k)^2</math> <b>or</b> <math>(kx)^2</math></p> <p><b>M1</b> <b>Either</b> <math>\frac{(-4)(-4-1)}{2!}(k)^2</math> <b>or</b> <math>\frac{(-4)(-5)}{2!}(k)^2</math> <b>or</b> <math>\frac{(-4)(-5)}{2!}(kx)^2</math> <b>or</b> <math>\frac{(-4)(-5)}{2!}(\text{their } k)^2</math> <b>or</b> <math>10k^2</math></p>	
<b>Note</b>	Candidates are allowed to use 2 instead of 2!	
<b>A1</b>	Uses $k = 1.5$ to give $A = \frac{45}{2}$ or 22.5	
<b>Note</b>	$A = \frac{90}{4}$ which has not been simplified is A0.	
<b>Note</b>	Award A0 for $A = \frac{45}{2}x^2$ .	
<b>Note</b>	Allow A1 for $A = \frac{45}{2}x^2$ followed by $A = \frac{45}{2}$	
<b>Note</b>	$k = -1.5$ leading to $A = \frac{45}{2}$ or 22.5 is A0.	

Question Number	Scheme					Marks
3.	$\frac{x}{y}$	1	2	3	4	$y = \frac{10}{2x + 5\sqrt{x}}$
(a)	{ At $x = 3,$ } $y = 0.68212$ (5 dp)					0.68212 B1 cao [1]
(b)	$\frac{1}{2} \times 1 \times [1.42857 + 0.55556 + 2(0.90326 + \text{their } 0.68212)]$					Outside brackets $\frac{1}{2} \times 1$ or $\frac{1}{2}$ B1 aef For structure of [.....] M1
(c)	{ = $\frac{1}{2}(5.15489)$ } = 2.577445 = 2.5774 (4 dp)					anything that rounds to 2.5774 A1 [3]
(d)	<ul style="list-style-type: none"><li>Overestimate</li><li>and a reason such as<ul style="list-style-type: none"><li>{ top of } <u>trapezia lie above the curve</u></li><li>a diagram which gives reference to the extra area</li><li>concave or convex</li><li><math>\frac{d^2y}{dx^2} &gt; 0</math> (can be implied)</li><li>bends inwards</li><li>curves downwards</li></ul></li></ul>					B1 [1]
	$\{u = \sqrt{x} \Rightarrow \} \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \text{ or } \frac{dx}{du} = 2u$ $\int \frac{10}{2u^2 + 5u} \cdot 2u \, du$ Either $\left\{ \int \right\} \frac{\pm k u}{\alpha u^2 \pm \beta u} \{du\}$ or $\left\{ \int \right\} \frac{\pm k}{u(\alpha u^2 \pm \beta u)} \{du\}$					M1
	$\left\{ = \int \frac{20}{2u + 5} \, du \right\} = \frac{20}{2} \ln(2u + 5)$ $\pm \lambda \ln(2u + 5)$ or $\pm \lambda \ln\left(u + \frac{5}{2}\right), \lambda \neq 0$ <b>with no other terms.</b> $\frac{20}{2u + 5} \rightarrow \frac{20}{2} \ln(2u + 5)$ or $10 \ln\left(u + \frac{5}{2}\right)$					M1 A1 cso
	$\left\{ \left[ \frac{20}{2} \ln(2u + 5) \right]_1^2 \right\} = 10 \ln(2(2) + 5) - 10 \ln(2(1) + 5)$ $10 \ln 9 - 10 \ln 7$ or $10 \ln\left(\frac{9}{7}\right)$ or $20 \ln 3 - 10 \ln 7$					Substitutes limits of 2 and 1 in $u$ (or 4 and 1 in $x$ ) and subtracts the correct way round. M1 A1 oe cso [6] 11
Question 3 Notes						
3. (a)	B1	0.68212 correct answer only. Look for this on the table or in the candidate's working.				
(b)	B1	Outside brackets $\frac{1}{2} \times 1$ or $\frac{1}{2}$ or equivalent.				
	M1	For structure of trapezium rule [ ..... ]				
	Note A1	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate]. anything that rounds to 2.5774				
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 2.51314428...)				

<p>3. (b) contd</p>	<p><b>Note</b> Award B1M1A1 for <math>\frac{1}{2}(1.42857 + 0.55556) + (0.90326 + \text{their } 0.68212) = 2.577445</math></p> <p><b>Bracketing mistake:</b> Unless the final answer implies that the calculation has been done correctly award B1M0A0 for <math>\frac{1}{2} \times 1 + 1.42857 + 2(0.90326 + \text{their } 0.68212) + 0.55556</math> (nb: answer of 5.65489).</p> <p>award B1M0A0 for <math>\frac{1}{2} \times 1 (1.42857 + 0.55556) + 2(0.90326 + \text{their } 0.68212)</math> (nb: answer of 4.162825).</p> <p><b>Alternative method: Adding individual trapezia</b></p> $\text{Area} \approx 1 \times \left[ \frac{1.42857 + 0.90326}{2} + \frac{0.90326 + "0.68212"}{2} + \frac{"0.68212" + 0.55556}{2} \right] = 2.577445$ <p><b>B1</b> B1: 1 and a divisor of 2 on all terms inside brackets.</p> <p><b>M1</b> M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.</p> <p><b>A1</b> A1: anything that rounds to 2.5774</p>
<p>(c)</p>	<p><b>B1</b> Overestimate <b>and</b> either trapezia lie above curve <b>or</b> a diagram that gives reference to the extra area</p> <p><b>eg.</b> This diagram is sufficient. It must show the top of a trapezium lying above the curve.</p>  <p><b>or</b> concave or convex <b>or</b> <math>\frac{d^2y}{dx^2} &gt; 0</math> (can be implied) <b>or</b> bends inwards <b>or</b> curves downwards.</p> <p><b>Note</b> Reason of "gradient is negative" by itself is B0.</p>
<p>(d)</p>	<p><b>B1</b> <math>\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}</math> or <math>du = \frac{1}{2\sqrt{x}} dx</math> or <math>2\sqrt{x} du = dx</math> or <math>dx = 2u du</math> or <math>\frac{dx}{du} = 2u</math> o.e.</p> <p><b>M1</b> Applying the substitution and achieving <math>\left\{ \int \right\} \frac{\pm k u}{\alpha u^2 \pm \beta u} \{du\}</math> or <math>\left\{ \int \right\} \frac{\pm k}{u(\alpha u^2 \pm \beta u)} \{du\}</math>, <math>k, \alpha, \beta \neq 0</math>. Integral sign and <math>du</math> not required for this mark.</p> <p><b>M1</b> Cancelling <math>u</math> and integrates to achieve <math>\pm \lambda \ln(2u + 5)</math> or <math>\pm \lambda \ln\left(u + \frac{5}{2}\right)</math>, <math>\lambda \neq 0</math> <b>with no other terms.</b></p> <p><b>A1</b> <b>cso.</b> Integrates <math>\frac{20}{2u + 5}</math> to give <math>\frac{20}{2} \ln(2u + 5)</math> or <math>10 \ln\left(u + \frac{5}{2}\right)</math>, un-simplified or simplified.</p> <p><b>Note</b> BE CAREFUL! Candidates must be integrating <math>\frac{20}{2u + 5}</math> or equivalent.</p> <p>So <math>\int \frac{10}{2u + 5} du = 10 \ln(2u + 5)</math> WOULD BE A0 and final A0.</p> <p><b>M1</b> Applies limits of 2 and 1 in <math>u</math> or 4 and 1 in <math>x</math> in their (i.e. any) changed function and subtracts the correct way round.</p> <p><b>A1</b> Exact answers of either <math>10 \ln 9 - 10 \ln 7</math> or <math>10 \ln\left(\frac{9}{7}\right)</math> or <math>20 \ln 3 - 10 \ln 7</math> or <math>20 \ln\left(\frac{3}{\sqrt{7}}\right)</math> or <math>\ln\left(\frac{9^{10}}{7^{10}}\right)</math> or equivalent. <b>Correct solution only.</b></p> <p><b>Note</b> You can ignore subsequent working which follows from a correct answer.</p> <p><b>Note</b> A decimal answer of 2.513144283... (without a correct <b>exact</b> answer) is A0.</p>

Question Number	Scheme	Marks
4.	$\frac{dV}{dt} = 80\pi$ , $V = 4\pi h(h + 4) = 4\pi h^2 + 16\pi h$ , $\frac{dV}{dh} = 8\pi h + 16\pi$	$\pm \alpha h \pm \beta$ , $\alpha \neq 0$ , $\beta \neq 0$ $8\pi h + 16\pi$ M1 A1
	$\left\{ \frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow \right\} (8\pi h + 16\pi) \frac{dh}{dt} = 80\pi$ $\left( \text{Candidate's } \frac{dV}{dh} \right) \times \frac{dh}{dt} = 80\pi$ $\left\{ \frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow \right\} \frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$	M1 oe }
	When $h = 6$ , $\left\{ \frac{dh}{dt} = \right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi \left\{ = \frac{80\pi}{64\pi} \right\}$ <b>dependent on the previous M1 see notes</b> $\frac{dh}{dt} = \underline{1.25} \text{ (cms}^{-1}\text{)}$	dM1 A1 oe <b>[5]</b> <b>5</b>
	<b><u>Alternative Method for the first M1A1</u></b> Product rule: $\begin{cases} u = 4\pi h & v = h + 4 \\ \frac{du}{dh} = 4\pi & \frac{dv}{dh} = 1 \end{cases}$ $\frac{dV}{dh} = 4\pi(h + 4) + 4\pi h$	$\pm \alpha h \pm \beta$ , $\alpha \neq 0$ , $\beta \neq 0$ $4\pi(h + 4) + 4\pi h$ M1 A1
<b>Question 4 Notes</b>		
	<b>M1</b>	An expression of the form $\pm \alpha h \pm \beta$ , $\alpha \neq 0$ , $\beta \neq 0$ . <b>Can be simplified or un-simplified.</b>
	<b>A1</b>	Correct simplified or un-simplified differentiation of $V$ . eg. $8\pi h + 16\pi$ or $4\pi(h + 4) + 4\pi h$ or $8\pi(h + 2)$ or equivalent.
	<b>Note</b>	Some candidates will use the product rule to differentiate $V$ with respect to $h$ . <b>(See Alt Method 1).</b>
	<b>Note</b>	$\frac{dV}{dh}$ does not have to be explicitly stated, but it should be clear that they are differentiating their $V$ .
	<b>M1</b>	$\left( \text{Candidate's } \frac{dV}{dh} \right) \times \frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$
	<b>Note</b>	Also allow 2 <sup>nd</sup> M1 for $\left( \text{Candidate's } \frac{dV}{dh} \right) \times \frac{dh}{dt} = \mathbf{80}$ or $\mathbf{80} \div \text{Candidate's } \frac{dV}{dh}$
	<b>Note</b>	Give 2 <sup>nd</sup> M0 for $\left( \text{Candidate's } \frac{dV}{dh} \right) \times \frac{dh}{dt} = \mathbf{80\pi t}$ or $\mathbf{80k}$ or $\mathbf{80\pi t}$ or $\mathbf{80k} \div \text{Candidate's } \frac{dV}{dh}$
	<b>dM1</b>	<b>which is dependent on the previous M1 mark.</b> Substitutes $h = 6$ into an expression which is a result of a quotient of their $\frac{dV}{dh}$ and $80\pi$ (or 80)
	<b>A1</b>	$1.25$ or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).
	<b>Note</b>	$\frac{80\pi}{64\pi}$ as a final answer is A0.
	<b>Note</b>	Substituting $h = 6$ into a correct $\frac{dV}{dh}$ gives $64\pi$ but the final M1 mark can only be awarded if this is used as a quotient with $80\pi$ (or 80)

Question Number	Scheme	Marks
5.	$x = 4 \cos\left(t + \frac{\pi}{6}\right), \quad y = 2 \sin t$	
(a)	<p><b>Main Scheme</b></p> $x = 4 \left( \cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right) \right) \qquad \cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ <p>So, <math>\{x + y\} = 4 \left( \cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right) \right) + 2 \sin t</math> <span style="float: right;">Adds their expanded <math>x</math> (which is in terms of <math>t</math>) to <math>2 \sin t</math></span></p> $= 4 \left( \left( \frac{\sqrt{3}}{2} \right) \cos t - \left( \frac{1}{2} \right) \sin t \right) + 2 \sin t$ $= 2\sqrt{3} \cos t \quad *$ <p style="text-align: right;">Correct proof</p>	<p>M1 oe</p> <p>dM1</p> <p>A1 * <b>[3]</b></p>
(a)	<p><b>Alternative Method 1</b></p> $x = 4 \left( \cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right) \right) \qquad \cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ $= 4 \left( \left( \frac{\sqrt{3}}{2} \right) \cos t - \left( \frac{1}{2} \right) \sin t \right) = 2\sqrt{3} \cos t - 2 \sin t$ <p>So, <math>x = 2\sqrt{3} \cos t - y</math> <span style="float: right;">Forms an equation in <math>x, y</math> and <math>t</math>.</span></p> $x + y = 2\sqrt{3} \cos t \quad *$ <p style="text-align: right;">Correct proof</p>	<p>M1 oe</p> <p>dM1</p> <p>A1 * <b>[3]</b></p>
(b)	<p><b>Main Scheme</b></p> $\left( \frac{x+y}{2\sqrt{3}} \right)^2 + \left( \frac{y}{2} \right)^2 = 1$ $\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$ $\Rightarrow (x+y)^2 + 3y^2 = 12$ <p style="text-align: right;">Applies <math>\cos^2 t + \sin^2 t = 1</math> to achieve an equation containing <b>only</b> <math>x</math>'s and <math>y</math>'s.</p> <p style="text-align: right;"><math>(x+y)^2 + 3y^2 = 12</math> <math>\{a=3, b=12\}</math></p>	<p>M1</p> <p>A1 <b>[2]</b></p>
(b)	<p><b>Alternative Method 1</b></p> $(x+y)^2 = 12 \cos^2 t = 12(1 - \sin^2 t) = 12 - 12 \sin^2 t$ <p>So, <math>(x+y)^2 = 12 - 3y^2</math> <span style="float: right;">Applies <math>\cos^2 t + \sin^2 t = 1</math> to achieve an equation containing <b>only</b> <math>x</math>'s and <math>y</math>'s.</span></p> $\Rightarrow (x+y)^2 + 3y^2 = 12$ <p style="text-align: right;"><math>(x+y)^2 + 3y^2 = 12</math></p>	<p>M1</p> <p>A1 <b>[2]</b></p>
(b)	<p><b>Alternative Method 2</b></p> $(x+y)^2 = 12 \cos^2 t$ <p>As <math>12 \cos^2 t + 12 \sin^2 t = 12</math></p> <p>then <math>(x+y)^2 + 3y^2 = 12</math></p>	<p>M1, A1 <b>[2]</b></p>
		<b>5</b>

Question 5 Notes		
5. (a)	M1	$\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$
	Note	If a candidate states $\cos(A + B) = \cos A \cos B \pm \sin A \sin B$ , but there is an error <i>in its application</i> then give M1.  <b><u>Awarding the dM1 mark which is dependent on the first method mark</u></b>
Main	dM1	Adds their expanded $x$ (which is in terms of $t$ ) to $2\sin t$
	Note	Writing $x + y = \dots$ is not needed in the <b>Main Scheme</b> method.
Alt 1	dM1	Forms an equation in $x$ , $y$ and $t$ .
(b)	A1*	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x + y\} = 4\cos\left(t + \frac{\pi}{6}\right) + 2\sin t$ , by itself is M0M0A0.
	M1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing <b>only</b> $x$ 's and $y$ 's.
	A1	leading $(x + y)^2 + 3y^2 = 12$
	SC	Award <b>Special Case B1B0</b> for a candidate who writes down <b>either</b> <ul style="list-style-type: none"> <li><math>(x + y)^2 + 3y^2 = 12</math> from no working</li> <li><math>a = 3, b = 12</math>, but <b><u>does not provide a correct proof.</u></b></li> </ul>
	Note	Alternative method 2 is fine for M1 A1
	Note	Writing $(x + y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b \Rightarrow a = 3, b = 12$ is SC: B1B0
	Note	Writing $(x + y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b$ <ul style="list-style-type: none"> <li>states <math>a = 3, b = 12</math></li> <li>and refers to either <math>\cos^2 t + \sin^2 t = 1</math> or <math>12\cos^2 t + 12\sin^2 t = 12</math></li> <li>and there is no incorrect working</li> </ul> would get M1A1

Question Number	Scheme	Marks
6. (i)	$\int x e^{4x} dx = \frac{1}{4} x e^{4x} - \int \frac{1}{4} e^{4x} \{dx\}$ $= \frac{1}{4} x e^{4x} - \frac{1}{16} e^{4x} \{+ c\}$	<div> <math>\pm \alpha x e^{4x} - \int \beta e^{4x} \{dx\}, \alpha \neq 0, \beta &gt; 0</math> M1                 </div> <div> <math>\frac{1}{4} x e^{4x} - \int \frac{1}{4} e^{4x} \{dx\}</math> A1                 </div> <div> <math>\frac{1}{4} x e^{4x} - \frac{1}{16} e^{4x}</math> A1                 </div>
(ii)	$\int \frac{8}{(2x-1)^3} dx = \frac{8(2x-1)^{-2}}{(2)(-2)} \{+ c\}$ $\{-2(2x-1)^{-2} \{+ c\}\}$	<div> <math>\pm \lambda (2x-1)^{-2}</math> M1                 </div> <div> <math>\frac{8(2x-1)^{-2}}{(2)(-2)}</math> or equivalent. A1                 </div> <div> <i>{Ignore subsequent working}.</i> [2]                 </div>
(iii)	$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y \quad y = \frac{\pi}{6} \text{ at } x = 0$	
	<p><b>Main Scheme</b></p> $\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} dy = \int e^x dx \quad \text{or} \quad \int \sin 2y \sin y dy = \int e^x dx$ $\int 2 \sin y \cos y \sin y dy = \int e^x dx$ $\frac{2}{3} \sin^3 y = e^x \{+ c\}$ $\frac{2}{3} \sin^3 \left( \frac{\pi}{6} \right) = e^0 + c \quad \text{or} \quad \frac{2}{3} \left( \frac{1}{8} \right) - 1 = c$ $\left\{ \Rightarrow c = -\frac{11}{12} \right\} \quad \text{giving} \quad \frac{2}{3} \sin^3 y = e^x - \frac{11}{12}$	<div>B1 oe</div> <div>M1</div> <div>M1</div> <div>A1</div> <div>B1</div> <div>M1</div> <div>A1</div>
	<p><b>Alternative Method 1</b></p> $\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} dy = \int e^x dx \quad \text{or} \quad \int \sin 2y \sin y dy = \int e^x dx$ $\int -\frac{1}{2} (\cos 3y - \cos y) dy = \int e^x dx$ $-\frac{1}{2} \left( \frac{1}{3} \sin 3y - \sin y \right) = e^x \{+ c\}$ $-\frac{1}{2} \left( \frac{1}{3} \sin \left( \frac{3\pi}{6} \right) - \sin \left( \frac{\pi}{6} \right) \right) = e^0 + c \quad \text{or} \quad -\frac{1}{2} \left( \frac{1}{3} - \frac{1}{2} \right) - 1 = c$ $\left\{ \Rightarrow c = -\frac{11}{12} \right\} \quad \text{giving} \quad -\frac{1}{6} \sin 3y + \frac{1}{2} \sin y = e^x - \frac{11}{12}$	<div>B1 oe</div> <div>M1</div> <div>M1</div> <div>A1</div> <div>B1</div> <div>M1</div> <div>A1</div>
		<div>[7]</div> <div>12</div>



Question 6 Notes		
6. (i)	M1	Integration by parts is applied in the form $\pm \alpha x e^{4x} - \int \beta e^{4x} \{dx\}$ , where $\alpha \neq 0$ , $\beta > 0$ . (must be in this form).
	A1	$\frac{1}{4} x e^{4x} - \int \frac{1}{4} e^{4x} \{dx\}$ or equivalent.
	A1	$\frac{1}{4} x e^{4x} - \frac{1}{16} e^{4x}$ with/without + c. Can be un-simplified.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	<b>SPECIAL CASE:</b> A candidate who uses $u = x$ , $\frac{dv}{dx} = e^{4x}$ , writes down the correct “by parts” formula, but makes only one error when applying it can be awarded Special Case M1.
(ii)	M1	$\pm \lambda (2x - 1)^{-2}$ , $\lambda \neq 0$ . <b>Note</b> that $\lambda$ can be 1.
	A1	$\frac{8(2x - 1)^{-2}}{(2)(-2)}$ or $-2(2x - 1)^{-2}$ or $\frac{-2}{(2x - 1)^2}$ with/without + c. Can be un-simplified.
	Note	You can ignore subsequent working which follows from a correct answer.
(iii)	B1	Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	<b>Allow B1 for</b> $\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} = \int e^x$ or $\int \sin 2y \sin y = \int e^x$
	M1	$\frac{1}{\operatorname{cosec} 2y} \rightarrow 2 \sin y \cos y$ or $\sin 2y \rightarrow 2 \sin y \cos y$ or $\sin 2y \sin y \rightarrow \pm \lambda \cos 3y \pm \lambda \cos y$ seen anywhere in the candidate’s working to (iii).
	M1	Integrates to give $\pm \mu \sin^3 y$ , $\mu \neq 0$ or $\pm \alpha \sin 3y \pm \beta \sin y$ , $\alpha \neq 0$ , $\beta \neq 0$
	A1	$2 \sin^2 y \cos y \rightarrow \frac{2}{3} \sin^3 y$ (with no extra terms) or integrates to give $-\frac{1}{2} \left( \frac{1}{3} \sin 3y - \sin y \right)$
	B1	Evidence that $e^x$ has been integrated to give $e^x$ <b>as part of solving their DE.</b>
	M1	Some evidence of using both $y = \frac{\pi}{6}$ and $x = 0$ in an integrated or changed equation containing c.
	Note	that is mark can be implied by the correct value of c.
	A1	$\frac{2}{3} \sin^3 y = e^x - \frac{11}{12}$ or $-\frac{1}{6} \sin 3y + \frac{1}{2} \sin y = e^x - \frac{11}{12}$ or any equivalent correct answer.
	Note	You can ignore subsequent working which follows from a correct answer.
<b>Alternative Method 2 (Using integration by parts twice)</b>		
$\int \sin 2y \sin y \, dy = \int e^x \, dx$		B1 oe
		Applies integration by parts <b>twice</b> to give $\pm \alpha \cos y \sin 2y \pm \beta \sin y \cos 2y$ <b>M2</b>
$\frac{1}{3} \cos y \sin 2y - \frac{2}{3} \sin y \cos 2y = e^x \{ + c \}$		$\frac{1}{3} \cos y \sin 2y - \frac{2}{3} \sin y \cos 2y$ (simplified or un-simplified) A1
		$e^x \rightarrow e^x$ <b>as part of solving their DE.</b> B1
		as in the main scheme M1
$\frac{1}{3} \cos y \sin 2y - \frac{2}{3} \sin y \cos 2y = e^x - \frac{11}{12}$		$-\frac{1}{6} \sin 3y + \frac{1}{2} \sin y = e^x - \frac{11}{12}$ A1
<b>[7]</b>		

Question Number	Scheme	Marks
7.	$x = 3 \tan \theta$ , $y = 4 \cos^2 \theta$ or $y = 2 + 2 \cos 2\theta$ , $0 \leq \theta < \frac{\pi}{2}$ .	
(a)	$\frac{dx}{d\theta} = 3 \sec^2 \theta$ , $\frac{dy}{d\theta} = -8 \cos \theta \sin \theta$ or $\frac{dy}{d\theta} = -4 \sin 2\theta$	
	$\frac{dy}{dx} = \frac{-8 \cos \theta \sin \theta}{3 \sec^2 \theta} \left\{ = -\frac{8}{3} \cos^3 \theta \sin \theta = -\frac{4}{3} \sin 2\theta \cos^2 \theta \right\}$ their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ Correct $\frac{dy}{dx}$	M1 A1 oe
	At $P(3, 2)$ , $\theta = \frac{\pi}{4}$ , $\frac{dy}{dx} = -\frac{8}{3} \cos^3 \left( \frac{\pi}{4} \right) \sin \left( \frac{\pi}{4} \right) \left\{ = -\frac{2}{3} \right\}$ Some evidence of substituting $\theta = \frac{\pi}{4}$ into their $\frac{dy}{dx}$ So, $m(N) = \frac{3}{2}$ applies $m(N) = \frac{-1}{m(T)}$	M1 M1
	Either N: $y - 2 = \frac{3}{2} (x - 3)$ or $2 = \left( \frac{3}{2} \right) (3) + c$ see notes	M1
	{ At Q, $y = 0$ , so, $-2 = \frac{3}{2} (x - 3)$ } giving $x = \frac{5}{3}$ $x = \frac{5}{3}$ or $1 \frac{2}{3}$ or awrt 1.67	A1 cso
(b)	$\left\{ \int y^2 dx = \int y^2 \frac{dx}{d\theta} d\theta \right\} = \left\{ \int \right\} (4 \cos^2 \theta)^2 3 \sec^2 \theta \{d\theta\}$ see notes So, $\pi \int y^2 dx = \pi \int (4 \cos^2 \theta)^2 3 \sec^2 \theta \{d\theta\}$ see notes $\int y^2 dx = \int 48 \cos^2 \theta d\theta$ $\int 48 \cos^2 \theta \{d\theta\}$ $= \{48\} \int \left( \frac{1 + \cos 2\theta}{2} \right) d\theta \left\{ = \int (24 + 24 \cos 2\theta) d\theta \right\}$ Applies $\cos 2\theta = 2 \cos^2 \theta - 1$	M1 A1 A1 M1
	$= \{48\} \left( \frac{1}{2} \theta + \frac{1}{4} \sin 2\theta \right) \left\{ = 24\theta + 12 \sin 2\theta \right\}$ Dependent on the first method mark. For $\pm \alpha \theta \pm \beta \sin 2\theta$ $\cos^2 \theta \rightarrow \left( \frac{1}{2} \theta + \frac{1}{4} \sin 2\theta \right)$	dM1 A1
	$\int_0^{\frac{\pi}{4}} y^2 dx \left\{ = 48 \left[ \frac{1}{2} \theta + \frac{1}{4} \sin 2\theta \right]_0^{\frac{\pi}{4}} \right\} = \{48\} \left( \left( \frac{\pi}{8} + \frac{1}{4} \right) - (0 + 0) \right) \{ = 6\pi + 12 \}$ Dependent on the third method mark. { So $V = \pi \int_0^{\frac{\pi}{4}} y^2 dx = 6\pi^2 + 12\pi$ } $V_{\text{cone}} = \frac{1}{3} \pi (2)^2 \left( 3 - \frac{5}{3} \right) \left\{ = \frac{16\pi}{9} \right\}$ $V_{\text{cone}} = \frac{1}{3} \pi (2)^2 (3 - \text{their } (a))$ $\left\{ \text{Vol}(S) = 6\pi^2 + 12\pi - \frac{16\pi}{9} \right\} \Rightarrow \text{Vol}(S) = \frac{92}{9} \pi + 6\pi^2$ $\frac{92}{9} \pi + 6\pi^2$ $\left\{ p = \frac{92}{9}, q = 6 \right\}$	dM1 M1 A1
		[9]

[6]

[9]

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Question 7 Notes		
7. (a)	<b>1<sup>st</sup> M1</b>	Applies their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ <b>or</b> applies $\frac{dy}{d\theta}$ multiplied by their $\frac{d\theta}{dx}$
	<b>SC</b>	Award <b>Special Case 1<sup>st</sup> M1</b> if <b>both</b> $\frac{dx}{d\theta}$ <b>and</b> $\frac{dy}{d\theta}$ are both correct.
	<b>1<sup>st</sup> A1</b>	Correct $\frac{dy}{dx}$ i.e. $\frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ or $-\frac{8}{3}\cos^3\theta\sin\theta$ or $-\frac{4}{3}\sin 2\theta\cos^2\theta$ or any equivalent form.
	<b>2<sup>nd</sup> M1</b>	<i>Some evidence</i> of substituting $\theta = \frac{\pi}{4}$ or $\theta = 45^\circ$ into their $\frac{dy}{dx}$
	<b>Note</b>	For 3 <sup>rd</sup> M1 and 4 <sup>th</sup> M1, $m(\mathbf{T})$ must be found by using $\frac{dy}{dx}$ .
	<b>3<sup>rd</sup> M1</b>	applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$ . Numerical value for $m(\mathbf{N})$ is required here.
	<b>4<sup>th</sup> M1</b>	<ul style="list-style-type: none"> <li>Applies <math>y - 2 = (\text{their } m_N)(x - 3)</math>, where <math>m(\mathbf{N})</math> is a numerical value,</li> <li>or <i>finds c</i> by solving <math>2 = (\text{their } m_N)3 + c</math>, where <math>m(\mathbf{N})</math> is a numerical value,</li> </ul> and $m_N = -\frac{1}{\text{their } m(\mathbf{T})}$ or $m_N = \frac{1}{\text{their } m(\mathbf{T})}$ or $m_N = -\text{their } m(\mathbf{T})$ .
	<b>Note</b>	This mark can be implied by subsequent working.
	<b>2<sup>nd</sup> A1</b>	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67 <b>from a correct solution only</b> .
	<b>1<sup>st</sup> M1</b>	Applying $\int y^2 dx$ as $y^2 \frac{dx}{d\theta}$ with their $\frac{dx}{d\theta}$ . Ignore $\pi$ or $\frac{1}{3}\pi$ outside integral.
(b)	<b>Note</b>	You can ignore the omission of an integral sign and/or $d\theta$ for the 1 <sup>st</sup> M1.
	<b>Note</b>	Allow 1 <sup>st</sup> M1 for $\int (\cos^2\theta)^2 \times \text{"their } 3\sec^2\theta"$ $d\theta$ or $\int 4(\cos^2\theta)^2 \times \text{"their } 3\sec^2\theta"$ $d\theta$
	<b>1<sup>st</sup> A1</b>	Correct expression $\left\{ \pi \int y^2 dx \right\} = \pi \int (4\cos^2\theta)^2 3\sec^2\theta \{d\theta\}$ (Allow the omission of $d\theta$ )
	<b>Note</b>	<b>IMPORTANT:</b> The $\pi$ can be recovered later, <b>but as a correct statement only</b> .
	<b>2<sup>nd</sup> A1</b>	$\left\{ \int y^2 dx \right\} = \int 48\cos^2\theta \{d\theta\}$ . (Ignore $d\theta$ ). <b>Note:</b> 48 can be written as 24(2) for example.
	<b>2<sup>nd</sup> M1</b>	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral. (Seen or <b>implied</b> .)
	<b>3<sup>rd</sup> dM1*</b>	<b>which is dependent on the 1<sup>st</sup> M1 mark.</b> Integrating $\cos^2\theta$ to give $\pm\alpha\theta \pm \beta\sin 2\theta$ , $\alpha \neq 0$ , $\beta \neq 0$ , un-simplified or simplified.
	<b>3<sup>rd</sup> A1</b>	<b>which is dependent on the 3<sup>rd</sup> M1 mark and the 1<sup>st</sup> M1 mark.</b> Integrating $\cos^2\theta$ to give $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$ , un-simplified or simplified.  This can be implied by $k\cos^2\theta$ giving $\frac{k}{2}\theta + \frac{k}{4}\sin 2\theta$ , un-simplified or simplified.
	<b>4<sup>th</sup> dM1</b>	<b>which is dependent on the 3<sup>rd</sup> M1 mark and the 1<sup>st</sup> M1 mark.</b> Some evidence of applying limits of $\frac{\pi}{4}$ and 0 (0 can be implied) to an integrated function in $\theta$
	<b>5<sup>th</sup> M1</b>	Applies $V_{\text{cone}} = \frac{1}{3}\pi(2)^2(3 - \text{their part (a) answer})$ .
	<b>Note</b>	Also allow the 5 <sup>th</sup> M1 for $V_{\text{cone}} = \pi \int_{\text{their } \frac{5}{3}}^3 \left( \frac{3}{2}x - \frac{5}{2} \right)^2 \{dx\}$ , which includes the correct limits.
	<b>4<sup>th</sup> A1</b>	$\frac{92}{9}\pi + 6\pi^2$ or $10\frac{2}{9}\pi + 6\pi^2$
	<b>Note</b> <b>Note</b>	A decimal answer of 91.33168464... (without a correct <b>exact</b> answer) is A0. The $\pi$ in the volume formula is only needed for the 1 <sup>st</sup> A1 mark and the final accuracy mark.

7.		<p><b><u>Working with a Cartesian Equation</u></b></p> <p>A cartesian equation for <math>C</math> is <math>y = \frac{36}{x^2 + 9}</math></p>
(a)	<p><b>1<sup>st</sup> M1</b></p> <p><b>1<sup>st</sup> A1</b></p> <p><b>2<sup>nd</sup> dM1</b></p>	<p><math>\frac{dy}{dx} = \pm \lambda x (\pm \alpha x^2 \pm \beta)^{-2}</math> <b>or</b> <math>\frac{dy}{dx} = \frac{\pm \lambda x}{(\pm \alpha x^2 \pm \beta)^2}</math></p> <p><math>\frac{dy}{dx} = -36(x^2 + 9)^{-2}(2x)</math> <b>or</b> <math>\frac{dy}{dx} = \frac{-72x}{(x^2 + 9)^2}</math> un-simplified or simplified.</p> <p><b>Dependent on the 1<sup>st</sup> M1 mark if a candidate uses this method</b></p> <p>For substituting <math>x = 3</math> into their <math>\frac{dy}{dx}</math></p> <p>i.e. at <math>P(3, 2)</math>, <math>\frac{dy}{dx} = \frac{-72(3)}{(3^2 + 9)^2} \left\{ = -\frac{2}{3} \right\}</math></p> <p>From this point onwards the original scheme can be applied.</p>
(b)	<p><b>1<sup>st</sup> M1</b></p> <p><b>A1</b></p>	<p>For <math>\int \left( \frac{\pm \lambda}{\pm \alpha x^2 \pm \beta} \right)^2 \{dx\}</math> (<math>\pi</math> not required for this mark)</p> <p>For <math>\pi \int \left( \frac{36}{x^2 + 9} \right)^2 \{dx\}</math> (<math>\pi</math> required for this mark)</p> <p>To integrate, a substitution of <math>x = 3 \tan \theta</math> is required which will lead to <math>\int 48 \cos^2 \theta d\theta</math> and so from this point onwards the original scheme can be applied.</p>
(a)	<p><b>1<sup>st</sup> M1</b></p> <p><b>1<sup>st</sup> A1</b></p> <p><b>2<sup>nd</sup> dM1</b></p>	<p>Another cartesian equation for <math>C</math> is <math>x^2 = \frac{36}{y} - 9</math></p> <p><math>\pm \alpha x = \pm \frac{\beta}{y^2} \frac{dy}{dx}</math> <b>or</b> <math>\pm \alpha x \frac{dx}{dy} = \pm \frac{\beta}{y^2}</math></p> <p><math>2x = -\frac{36}{y^2} \frac{dy}{dx}</math> <b>or</b> <math>2x \frac{dx}{dy} = -\frac{36}{y^2}</math></p> <p><b>Dependent on the 1<sup>st</sup> M1 mark if a candidate uses this method</b></p> <p>For substituting <math>x = 3</math> to find <math>\frac{dy}{dx}</math></p> <p>i.e. at <math>P(3, 2)</math>, <math>2(3) = -\frac{36}{4} \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \dots</math></p> <p>From this point onwards the original scheme can be applied.</p>

Question Number	Scheme	Marks
8.	$\overrightarrow{OA} = -2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$ , $\overrightarrow{OB} = -\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ & $\overrightarrow{OP} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$	
(a)	$\overrightarrow{AB} = \pm((- \mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) - (-2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k})); = \mathbf{i} - \mathbf{j} + \mathbf{k}$	M1; A1 [2]
(b)	$\{l_1 : \mathbf{r}\} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ or $\{\mathbf{r}\} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	B1ft [1]
(c)	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$	M1
	$\{\cos \theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{PB}}{ \overrightarrow{AB}   \overrightarrow{PB} } = \frac{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}}{\sqrt{(1)^2 + (-1)^2 + (1)^2} \cdot \sqrt{(-1)^2 + (1)^2 + (5)^2}}$ Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{PB}$ or $\overrightarrow{BP})$ .	M1
	$\{\cos \theta\} = \frac{-1-1+5}{\sqrt{3} \cdot \sqrt{27}} = \frac{3}{9} = \frac{1}{3}$ Correct proof	A1 cso [3]
(d)	$\{l_2 : \mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $\mathbf{p} + \lambda \mathbf{d}$ or $\mathbf{p} + \mu \mathbf{d}$ , $\mathbf{p} \neq 0$ , $\mathbf{d} \neq 0$ with either $\mathbf{p} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{d} =$ their $\overrightarrow{AB}$ , or a multiple of their $\overrightarrow{AB}$ . Correct vector equation.	M1 A1 ft [2]
(e)	$\overrightarrow{OC} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ or $\overrightarrow{OD} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ $\{C(1, 1, 4), D(-1, 3, 2)\}$ Either $\overrightarrow{OP} +$ their $\overrightarrow{AB}$ or $\overrightarrow{OP} -$ their $\overrightarrow{AB}$ At least one set of coordinates are correct. Both sets of coordinates are correct.	M1 A1 ft A1 ft [3]
(f) Way 1	$\frac{h}{\sqrt{(-1)^2 + (1)^2 + (5)^2}} = \sin \theta$ $h = \sqrt{27} \sin(70.5\dots) \left\{ = \sqrt{27} \frac{\sqrt{8}}{3} = 2\sqrt{6} = \text{awrt } 4.9 \right\}$ Area $ABCD = \frac{1}{2} 2\sqrt{6}(\sqrt{3} + 2\sqrt{3})$ $\left\{ = \frac{1}{2} 2\sqrt{6}(3\sqrt{3}) = 3\sqrt{18} \right\} = 9\sqrt{2}$	$\frac{h}{\text{their }  \overrightarrow{PB} } = \sin \theta$ $\sqrt{27} \sin(70.5\dots)$ or $\sqrt{27} \cdot \frac{\sqrt{8}}{3}$ or $2\sqrt{6}$ or awrt 4.9 or equivalent $\frac{1}{2}(\text{their } h)(\text{their } AB + \text{their } CD)$ $9\sqrt{2}$ M1 A1 oe dM1 A1 cao [4] 15

<p>8. (f)</p>	<p><b>Helpful Diagram!</b></p> <p>Area <math>\triangle APB = 4.2426\dots</math></p> <p><math>\vec{DA} = \vec{PB} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}</math></p> <p><math>\vec{PA} = \vec{CB} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}</math></p> <p><math>h = 2\sqrt{6} = 3\sqrt{3} \cdot \left(\frac{\sqrt{8}}{3}\right) = 4.8989\dots</math></p>
	<p><math>\vec{PA} = \vec{CB} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}</math> and <math>\vec{AB} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math>, so <math>BC \perp AB</math></p> <p>Candidates do not need to prove this result for part (f)</p>
<p>8. (f) Way 2</p>	<p><math>h =  \vec{CB}  = \sqrt{(-2)^2 + (2)^2 + (4)^2} = \sqrt{24} = 2\sqrt{6} = 4.8989\dots</math></p> <p>Area <math>ABCD = \frac{1}{2} \sqrt{24} (\sqrt{3} + 2\sqrt{3})</math> or <math>\frac{1}{2} \sqrt{24} \sqrt{3} + \sqrt{24} \sqrt{3}</math></p> <p><math>= 9\sqrt{2}</math></p> <p>Attempts <math> \vec{PA} </math> or <math> \vec{CB} </math></p> <p><math> \vec{PA}  =  \vec{CB}  = \sqrt{24}</math></p> <p><math>\frac{1}{2} h(\text{their } AB + \text{their } CD)</math></p> <p><math>9\sqrt{2}</math></p> <p>M1</p> <p>A1 oe</p> <p>dM1 oe</p> <p>A1 cso</p> <p>[4]</p>
<p>Way3 8. (f)</p>	<p><b>Finds the area of either triangle <math>APB</math> or <math>APD</math> or <math>BCP</math> and triples the result.</b></p> <p>Area <math>\triangle APB = \frac{1}{2} \sqrt{3} (3\sqrt{3}) \sin \theta</math></p> <p><math>= \frac{1}{2} \sqrt{3} (3\sqrt{3}) \sin(70.5\dots)</math></p> <p>Area <math>ABCD = 3 (3\sqrt{2})</math></p> <p><math>= 9\sqrt{2}</math></p> <p>Attempts <math>\frac{1}{2} (\text{their } AB)(\text{their } PB) \sin \theta</math></p> <p><math>\frac{1}{2} \sqrt{3} (3\sqrt{3}) \sin(70.5\dots)</math> or <math>3\sqrt{2}</math></p> <p>or awrt 4.24 or equivalent</p> <p><math>3 \times \text{Area of } \triangle APB</math></p> <p><math>9\sqrt{2}</math></p> <p>M1</p> <p>A1</p> <p>dM1</p> <p>A1 cso</p> <p>[4]</p>

Question 8 Notes	
8. (a)	<p><b>M1</b> Finding the difference (either way) between <math>\overrightarrow{OB}</math> and <math>\overrightarrow{OA}</math>. If no “subtraction” seen, you can award M1 for 2 out of 3 correct components of the difference.</p> <p><b>A1</b> <math>\mathbf{i} - \mathbf{j} + \mathbf{k}</math> or <math>\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math> or <math>(1, -1, 1)</math> or benefit of the doubt <math>\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math></p>
(b)	<p><b>B1ft</b> <math>\{\mathbf{r}\} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math> or <math>\{\mathbf{r}\} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math>, with <math>\overrightarrow{AB}</math> or <math>\overrightarrow{BA}</math> correctly followed through from (a).</p> <p><b>Note</b> <math>\mathbf{r} =</math> is not needed.</p>
(c)	<p><b>M1</b> An attempt to find either the vector <math>\overrightarrow{PB}</math> or <math>\overrightarrow{BP}</math>. If no “subtraction” seen, you can award M1 for 2 out of 3 correct components of the difference.</p> <p><b>M1</b> Applies dot product formula between their <math>(\overrightarrow{AB}</math> or <math>\overrightarrow{BA})</math> and their <math>(\overrightarrow{PB}</math> or <math>\overrightarrow{BP})</math>.</p> <p><b>A1</b> Obtains <math>\{\cos \theta\} = \frac{1}{3}</math> <b>by correct solution only.</b></p> <p><b>Note</b> If candidate starts by applying <math>\frac{\overrightarrow{AB} \cdot \overrightarrow{PB}}{ \overrightarrow{AB}   \overrightarrow{PB} }</math> correctly (without reference to <math>\cos \theta = \dots</math>) they can gain both 2<sup>nd</sup> M1 and A1 mark.</p> <p><b>Note</b> Award the final A1 mark if candidate achieves <math>\{\cos \theta\} = \frac{1}{3}</math> by either taking the dot product between</p> <p>(i) <math>\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math> and <math>\begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}</math> or (ii) <math>\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}</math> and <math>\begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}</math>. Ignore if any of these vectors are labelled incorrectly.</p> <p><b>Note</b> Award final A0, cso for those candidates who take the dot product between</p> <p>(iii) <math>\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math> and <math>\begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}</math> or (iv) <math>\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}</math> and <math>\begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}</math></p> <p>They will usually find <math>\{\cos \theta\} = -\frac{1}{3}</math> or may fudge <math>\{\cos \theta\} = \frac{1}{3}</math>.</p> <p>If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso</p>
(c)	<p><b>Alternative Method 1: The Cosine Rule</b></p> <p><math>\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}</math> or <math>\overrightarrow{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}</math></p> <p>Note <math> \overrightarrow{PB}  = \sqrt{27}</math>, <math> \overrightarrow{AB}  = \sqrt{3}</math> and <math> \overrightarrow{PA}  = \sqrt{24}</math></p> <p><math>(\sqrt{24})^2 = (\sqrt{27})^2 + (\sqrt{3})^2 - 2(\sqrt{27})(\sqrt{3})\cos \theta</math></p> <p><math>\cos \theta = \frac{27 + 3 - 24}{18} = \frac{1}{3}</math></p> <p>Mark in the same way as the main scheme. M1</p> <p>Applies the cosine rule the correct way round M1 oe</p> <p>Correct proof A1 cso</p> <p>[3]</p>

8. (c)	<b>Alternative Method 2: Right-Angled Trigonometry</b>		
	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix} \text{ or } \overrightarrow{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Either $(\sqrt{24})^2 + (\sqrt{3})^2 = (\sqrt{27})^2$		
	or $\overrightarrow{AB} \bullet \overrightarrow{PA} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix} = -2 - 2 + 4 = 0$	Confirms $\Delta PAB$ is right-angled	M1
	So, $\left\{ \cos \theta = \frac{AB}{PB} \Rightarrow \right\} \cos \theta = \frac{\sqrt{3}}{\sqrt{27}} = \frac{1}{3}$	Correct proof	A1 cso
<b>[3]</b>			
(d)	<b>M1</b>	Writing down a line in the form $\mathbf{p} + \lambda \mathbf{d}$ or $\mathbf{p} + \mu \mathbf{d}$ with either $\mathbf{a} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$ or $\mathbf{d} =$ their $\overrightarrow{AB}$ $\mathbf{d} =$ their $\overrightarrow{AB}$ , or a multiple of their $\overrightarrow{AB}$ found in part (a).	
	<b>A1ft</b>	Writing $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu \mathbf{d}$ , where $\mathbf{d} =$ their $\overrightarrow{AB}$ or a multiple of their $\overrightarrow{AB}$ found in part (a).	
	<b>Note</b>	$\mathbf{r} =$ is not needed.	
	<b>Note</b>	Using the same scalar parameter as in part (b) is fine for A1.	
(e)	<b>M1</b>	Either $\overrightarrow{OP} +$ their $\overrightarrow{AB}$ or $\overrightarrow{OP} -$ their $\overrightarrow{AB}$ .	
	<b>Note</b>	This can be implied at least two out of three correct components for either their $C$ or their $D$ .	
	<b>A1ft</b>	At least one set of coordinates are correct. Ignore labelling of $C, D$	
	<b>A1ft</b>	Both sets of coordinates are correct. Ignore labelling of $C, D$	
	<b>Note</b>	You can follow through either or both accuracy marks in this part using their $\overrightarrow{AB}$ from part (a).	
(f)	<b>M1</b>	Way 1: $\frac{h}{\text{their }  \overrightarrow{PB} } = \sin \theta$ Way 2: Attempts $ \overrightarrow{PA} $ or $ \overrightarrow{CB} $ Way 3: Attempts $\frac{1}{2} (\text{their } PB)(\text{their } AB) \sin \theta$	
	<b>Note</b>	Finding $AD$ by itself is M0.	
	<b>A1</b>	Either <ul style="list-style-type: none"><li><math>h = \sqrt{27} \sin(70.5\dots)</math> or <math> \overrightarrow{PA}  =  \overrightarrow{CB}  = \sqrt{24}</math> or equivalent. (See Way 1 and Way 2)</li></ul> or <ul style="list-style-type: none"><li>the area of either triangle <math>APB</math> or <math>APD</math> or <math>BDP = \frac{1}{2} \sqrt{3} (3\sqrt{3}) \sin(70.5\dots)</math> o.e. (See Way 3).</li></ul>	
	<b>dM1</b>	<b>which is dependent on the 1<sup>st</sup> M1 mark.</b>	
		A full method to find the area of trapezium $ABCD$ . (See Way 1, Way 2 and Way 3).	
	<b>A1</b>	$9\sqrt{2}$ from a correct solution only.	
	<b>Note</b>	A decimal answer of 12.7279... (without a correct <b>exact</b> answer) is A0.	





