

June 2005


6663 Core Mathematics C1

Mark Scheme

Question Number	Scheme	Marks
1. (a)	<u>2</u> Penalise \pm	B1 (1)
(b)	$8^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{64}}$ or $\frac{1}{(a)^2}$ or $\frac{1}{\sqrt[3]{8^2}}$ or $\frac{1}{8^{\frac{2}{3}}}$ $= \frac{1}{4}$ or 0.25 Allow \pm	M1 A1 (2) (3)
(b)	M1 for understanding that “-“ power means reciprocal $8^{\frac{2}{3}} = 4$ is M0A0 and $-\frac{1}{4}$ is M1A0	
2. (a)	$\frac{dy}{dx} = 6 + 8x^{-3}$ $x^n \rightarrow x^{n-1}$ both ($6x^0$ is OK)	M1 A1 (2)
(b)	$\int (6x - 4x^{-2})dx = \frac{6x^2}{2} + 4x^{-1} + c$	M1 A1 A1 (3) (5)
(b)	In (a) and (b) M1 is for a correct power of x in at least one term. This could be 6 in (a) or $+c$ in (b) 1 st A1 for one correct term in x : $\frac{6x^2}{2}$ or $+4x^{-1}$ (or better simplified versions) 2 nd A1 for all 3 terms as printed or better in one line. N.B. M1A0A1 is not possible. SC. For integrating their answer to part (a) just allow the M1 if $+c$ is present	

Question Number	Scheme	Marks
3. (a)	$x^2 - 8x - 29 \equiv (x - 4)^2 - 45$ $(x \pm 4)^2$ $(x - 4)^2 - 16 + (-29)$ $(x \pm 4)^2 - 45$	M1 A1 A1 (3)
ALT	Compare coefficients $-8 = 2a$ $a = -4$ <u>AND</u> $a^2 + b = -29$ $b = -45$ equation for a	M1 A1 A1 (3)
(b)	$(x - 4)^2 = 45$ $\Rightarrow x - 4 = \pm\sqrt{45}$ $x = 4 \pm 3\sqrt{5}$ (follow through their a and b from (a)) $c = 4$ $d = 3$ (\pm OK)	M1 A1 A1 (3) (6)
(a)	M1 for $(x \pm 4)^2$ or an equation for a (allow sign error ± 4 or ± 8 on ALT) 1stA1 for $(x - 4)^2 - 16(-29)$ can ignore -29 <u>or</u> for stating $a = -4$ and an equation for b 2 nd A1 for $b = -45$ Note M1A0 A1 is possible for $(x + 4)^2 - 45$ N.B. On EPEN these marks are called B1M1A1 but apply them as M1A1A1	
(b)	M1 for a full method leading to $x - 4 = \dots$ or $x = \dots$ (condone $x - 4 = \sqrt{-n}$) N.B. $(x - 4)^2 - 45 = 0$ leading to $(x - 4) \pm \sqrt{45} = 0$ is M0A0A0 A1 for c and A1 for d N.B. M1 and A1 for c do not need \pm (so this is a special case for the formula method) but \pm must be present for the d mark) <u>Note</u> Use of formula that ends with $\frac{8 \pm 6\sqrt{5}}{2}$ scores M1 A1 A0 (but must be $\sqrt{5}$) i.e. only penalise non-integers by one mark.	

Question Number	Scheme	Marks
4. (a)		Shape Points B1 B1 (2)
(b)		M1 -2 and 4 max A1 A1 (3) (5)
(a)	Marks for shape: graphs must have curved sides and round top. Don't penalise twice. (If both graphs are really straight lines then penalise B0 in part (a) only) 1 st B1 for \cap shape through (0, 0) and $((k, 0)$ where $k > 0$) 2 nd B1 for max at (3, 15) and 6 labelled or (6, 0) seen Condone (15, 3) if 3 and 15 are correct on axes. Similarly (5, 1) in (b)	
(b)	M1 for \cap shape <u>NOT</u> through (0, 0) but must cut x -axis twice. 1 st A1 for -2 and 4 labelled or (-2, 0) and (4, 0) seen 2 nd A1 for max at (1, 5). Must be clearly in 1 st quadrant	
5.	$x = 1 + 2y$ and sub $\rightarrow (1 + 2y)^2 + y^2 = 29$ $\Rightarrow 5y^2 + 4y - 28 = 0$ i.e. $(5y + 14)(y - 2) = 0$ $(y = 2 \text{ or } -\frac{14}{5})$ (o.e.) $y = 2 \Rightarrow x = 1 + 4 = 5$; $y = -\frac{14}{5} \Rightarrow x = -\frac{23}{5}$ (o.e.)	(both) M1 A1 M1 A1 M1A1 f.t. (6)
	1 st M1 Attempt to sub leading to equation in 1 variable Condone sign error such as $1 - 2y$, $x = -(1 + 2y)$ penalise 1 st A1 only 1 st A1 Correct 3TQ (condone = 0 missing) 2 nd M1 Attempt to solve 3TQ leading to 2 values for y . 2 nd A1 Condone mislabelling $x =$ for $y = \dots$ but then M0A0 in part (c). 3 rd M1 Attempt to find at least one x value (must use a correct equation) 3 rd A1 f.t. f.t. only in $x = 1 + 2y$ (3sf if not exact) Both values. N.B False squaring. (e.g. $x^2 + 4y^2 = 1$) can only score the last 2 marks.	

Question Number	Scheme	Marks
6. (a)	$6x + 3 > 5 - 2x \Rightarrow 8x > 2$ $x > \frac{1}{4}$ or 0.25 or $\frac{2}{8}$	M1 A1 (2)
(b)	$(2x - 1)(x - 3) (> 0)$ Critical values $x = \frac{1}{2}, 3$ 	M1 A1 (both)
	Choosing “outside” region $x > 3$ or $x < \frac{1}{2}$	M1 A1 f.t. (4)
(c)	$x > 3$ or $\frac{1}{4} < x < \frac{1}{2}$ [(3, ∞) or (1/4, 1/2) is OK]	B1f.t. B1f.t. (2) (8)
(a)	M1 Multiply out and collect terms (allow one slip and allow use of = here)	
(b)	1 st M1 Attempting to factorise 3TQ → x = ... 2 nd M1 Choosing the outside region	
	2 nd A1 f.t. f.t. their critical values N.B.(x>3, x > 1/2 is M0A0)	
(c)	f.t. their answers to (a) and (b)	
	1 st B1 a correct f.t. leading to an <u>infinite</u> region 2 nd B1 a correct f.t. leading to a <u>finite</u> region	
	Penalise ≤ or ≥ once only at first offence. For p < x < q where p > q penalise the final A1 in (b) .	
e.g.	(a) (b) (c) Mark	
	$x > \frac{1}{4}$ $\frac{1}{2} < x < 3$ $\frac{1}{2} < x < 3$ B0 B1	
	$x > \frac{1}{4}$ $x > 3, x > \frac{1}{2}$ $x > 3$ B1 B0	

Question Number	Scheme	Marks
7. (a)	$(3 - \sqrt{x})^2 = 9 - 6\sqrt{x} + x$ $\div by \sqrt{x} \rightarrow 9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}$	M1 A1 c.s.o. (2)
(b)	$\int (9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}) dx = \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} - 6x + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} (+c)$ <p>use $y = \frac{2}{3}$ and $x = 1$: $\frac{2}{3} = 18 - 6 + \frac{2}{3} + c$</p> <p style="text-align: right;">$c = -12$</p> <p>So $y = 18x^{\frac{1}{2}} - 6x + \frac{2}{3}x^{\frac{3}{2}} - 12$</p>	M1 A2/1/0 M1 A1 c.s.o. A1f.t. (6) (8)
(a)	M1 Attempt to multiply out $(3 - \sqrt{x})^2$. Must have 3 or 4 terms, allow one sign error A1 cso Fully correct solution to printed answer. Penalise invisible brackets or wrong working	
(b)	1 st M1 Some correct integration: $x^n \rightarrow x^{n+1}$ A1 At least 2 correct unsimplified terms Ignore + c A2 All 3 terms correct (unsimplified)	
	2 nd M1 Use of $y = \frac{2}{3}$ and $x = 1$ to find c . No + c is M0. A1c.s.o. for -12. (o.e.) Award this mark if “ $c = -12$ ” stated i.e. not as part of an expression for y A1f.t. for 3 simplified x terms with $y = \dots$ and a numerical value for c. Follow through their value of c but it must be a number.	
Question	Scheme	Marks

Number		
8. (a)	$y - (-4) = \frac{1}{3}(x - 9)$ or $\frac{y - (-4)}{x - 9} = \frac{1}{3}$ $3y - x + 21 = 0$ (o.e.) (condone 3 terms with integer coefficients e.g. $3y + 21 = x$)	M1 A1 A1 (3)
(b)	Equation of l_2 is: $y = -2x$ (o.e.) Solving l_1 and l_2 : $-6x - x + 21 = 0$ p is point where $x_p = 3$, $y_p = -6$	B1 M1 A1 A1f.t. ($-2x$) (4)
(c)	$(l_1 \text{ is } y = \frac{1}{3}x - 7)$ C is (0, -7) or $OC = 7$ Area of $\triangle OCP = \frac{1}{2}OC \times x_p = \frac{1}{2} \times 7 \times 3 = 10.5$ or $\frac{21}{2}$	B1f.t. M1 A1c.a.o. (3)
ALT	By Integration: M1 for $\pm \int_0^{x_p} (l_1 - l_2) dx$, B1 ft for correct integration (follow through their l_1), then A1cao.	(10)
(a)	M1 for full method to find equation of l_1 1stA1 any unsimplified form	
(b)	M1 Attempt to solve two linear equations leading to linear equation in one variable 2 nd A1 f.t. only f.t. their x_p or y_p in $y = -2x$ N.B. A fully correct solution by drawing, or correct answer with no working can score all the marks in part (b), but a partially correct solution by drawing only scores the first B1.	
(c)	B1f.t. Either a correct OC or f.t. from their l_1 M1 for correct attempt in letters or symbols for $\triangle OCP$ A1 c.a.o. $-\frac{1}{2} \times 7 \times 3$ scores M1 A0	
MR	(x-axis for y-axis) Get $C = (21, 0)$ Area of $\triangle OCP = \frac{1}{2}OC \times y_p = \frac{1}{2} \times 21 \times 6 = 63$ (B0M1A0)	

Question Number	Scheme	Marks
9 (a)	$(S \Rightarrow) a + (a + d) + \dots \dots + [a + (n - 1)d]$ $(S \Rightarrow) [a + (n - 1)d] + \dots \dots + a$ $2S = [2a + (n - 1)d] + \dots \dots + [2a + (n - 1)d] \quad \}$ either $2S = n[2a + (n - 1)d]$ $S = \frac{n}{2}[2a + (n - 1)d]$	B1 M1 dM1 A1 c.s.o (4)
(b)	$(a = 149, d = -2)$ $u_{21} = 149 + 20(-2) = \pounds 109$	M1 A1 (2)
(c)	$S_n = \frac{n}{2}[2 \times 149 + (n - 1)(-2)] \quad (= n(150 - n))$ $S_n = 5000 \Rightarrow n^2 - 150n + 5000 = 0 \quad (*)$	M1 A1 A1 c.s.o (3)
(d)	$(n - 100)(n - 50) = 0$ $n = 50 \text{ or } 100$	M1 A2/1/0 (3)
(e)	$u_{100} < 0 \quad \therefore n = 100 \text{ not sensible}$	B1 f.t. (1) (13)
(a)	B1 requires at least 3 terms, must include first and last terms, an adjacent term and dots! There must be + signs for the B1 (or at least implied see snippet 9D) 1 st M1 for reversing series. Must be arithmetic with a , n and d or l . (+ signs not essential here) 2 nd dM1 for adding, must have $2S$ and be a genuine attempt. Either line is sufficient. Dependent on 1 st M1 (NB Allow first 3 marks for use of l for last term but as given for final mark)	
(b)	M1 for using $a = 149$ and $d = \pm 2$ in $a + (n - 1)d$ formula.	
(c)	M1 for using their a, d in S_n A1 any correct expression A1cso for putting $S_n = 5000$ and simplifying to given expression. No wrong work NB EPEN has B1M1A1 here but apply marks as M1A1A1 as in scheme	
(d)	M1 Attempt to solve leading to $n = \dots$ A2/1/0 Give A1A0 for 1 correct value and A1A1 for both correct	
(e)	B1 f.t. Must mention 100 and state $u_{100} < 0$ (or loan paid or equivalent) If giving f.t. then must have $n \geq 76$.	

Question Number	Scheme	Marks
10 (a)	$x = 3, \quad y = 9 - 36 + 24 + 3 = 0$ (9 - 36 + 27 = 0 is OK)	B1 (1)
(b)	$\frac{dy}{dx} = \frac{3}{3}x^2 - 2 \times 4 \times x + 8 \quad (x^2 - 8x + 8)$ When $x = 3, \quad \frac{dy}{dx} = 9 - 24 + 8 \Rightarrow m = -7$ Equation of tangent: $y - 0 = -7(x - 3)$ $y = -7x + 21$	M1 A1 M1 M1 A1 c.a.o (5)
(c)	$\frac{dy}{dx} = m$ gives $x^2 - 8x + 8 = -7$ $(x^2 - 8x + 15 = 0)$ $(x - 5)(x - 3) = 0$ $x = (3) \text{ or } 5$ $\therefore y = \frac{1}{3}5^3 - 4 \times 5^2 + 8 \times 5 + 3$ $y = -15\frac{1}{3} \text{ or } -\frac{46}{3}$	M1 5 M1 A1 M1 A1 (5)
(b)	1 st M1 some correct differentiation ($x^n \rightarrow x^{n-1}$ for one term) 1 st A1 correct unsimplified (all 3 terms) 2 nd M1 substituting $x_p (= 3)$ in their $\frac{dy}{dx}$ clear evidence 3 rd M1 using their m to find tangent at p . The m must be from their $\frac{dy}{dx}$ at $x_p (= 3)$ Use of $\frac{1}{7}$ here scores M0A0 but Could get all 3 Ms in Part (c).	
(c)	1 st M1 forming a correct equation “their $\frac{dy}{dx} = \text{gradient of their tangent}$ ” 2 nd M1 for solving a quadratic based on their $\frac{dy}{dx}$ leading to $x = \dots$ The quadratic could be simply $\frac{dy}{dx} = 0$. 3 rd M1 for using their x value (obtained from their quadratic) in y to obtain y coordinate. Must have one of the other two M marks to score this.	
MR	For misreading (0, 3) for (3, 0) award B0 and then M1A1 as in scheme. Then allow all M marks but no A ft. (Max 7)	(11)