

# Mark Scheme (Results) Summer 2009

GCE

GCE Mathematics (6664/01)





## June 2009 6664 Core Mathematics C2 Mark Scheme

Question Number	Scheme		Marks
Q1	$\int \left(2x+3x^{\frac{1}{2}}\right) dx = \frac{2x^2}{2} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$	M1 .	A1A1
	$\int_{1}^{4} \left( 2x + 3x^{\frac{1}{2}} \right) dx = \left[ x^{2} + 2x^{\frac{3}{2}} \right]_{1}^{4} = 16 + 2 \times 8 - 1 + 2$	M1	
	= 29 (29 + <i>C</i> scores A0)	A1	(5) <b>[5]</b>
	1 <sup>st</sup> M1 for attempt to integrate $x$ $x^{\frac{1}{2}}$ or $x^{\frac{1}{2}}$		
	1 <sup>st</sup> A1 for $\frac{2x^2}{2}$ or a simplified version.		
	$2^{nd} A1$ for $\frac{2x^{\frac{3}{2}}}{\sqrt{2}}$ or $\frac{3}{\sqrt{2}}$ or a simplified version.		
	Ignore + $C$ , if seen, but two correct terms and an <u>extra non-constant</u> term scores M1A1A	A0.	
	2 <sup>nd</sup> M1 for correct use of correct limits ('top' – 'bottom'). Must be used in a 'changed function', not just the original. (The changed function may have been found by differentiation).	ý	
	Ignore 'poor notation' (e.g. missing integral signs) if the intention is clear.		
	No working: The answer 29 with no working scores M0A0A0M1A0 (1 mark).		



Que: Nun	stion nber	Scheme	Marks
Q2	(a)	The 7 or 21 can be in 'unsimplified' form.	M1
		$2 + kx^{7} = 2^{7} + 2^{6} \times 7 \times kx + 2^{5} \times \binom{7}{2} k^{2} x^{2}$	
		= 128; +448 $kx$ , +672 $k^2x^2$ [or 672 $(kx)^2$ ] (If 672 $kx^2$ follows 672 $(kx)^2$ , isw and allow A1)	B1; A1, A1 (4)
	(b)	$6 \times 448k = 672k^2$	M1
		k = 4 (Ignore $k = 0$ , if seen)	A1 (2) [6]
	(a)	The terms can be 'listed' rather than added. Ignore any extra terms.	
		<ul> <li>M1 for either the x term or the x<sup>2</sup> term. Requires correct binomial coefficient in any f with the correct power of x, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing.</li> <li><u>Allow</u> binomial coefficients such as <u>Allow</u> binomial coefficients such as <u>M1 for the simplified</u> versions seen above.</li> <li>B1, A1, A1 for the simplified versions seen above.</li> <li><u>Alternative</u>:</li> <li>Note that a factor 2<sup>7</sup> can be taken out first: 2<sup>7</sup> <u>x</u>, but the mark scheme still appling the state of the simplified versions.</li> </ul>	orm
		e.g. 128 48 $kx$ 672 $k^2x^2$ M1 B1 A1 A1 4 $kx$ 621 $k^2x^2$ isw (Full marks are still available in part (b)).	
	(b)	M1 for equating their coefficient of $x^2$ to 6 times that of x to get an equation in k, <u>or</u> equating their coefficient of x to 6 times that of $x^2$ , to get an equation in k. Allow this M mark even if the equation is trivial, providing their coefficients from pa have been used, e.g. 6 48k 572k, but beware $k = 4$ following from this, which is <u>An equation in k alone</u> is required for this M mark, so e.g. 6 48kx 572k <sup>2</sup> x <sup>2</sup> 100 11 or similar is M0 A0 (equation in coefficients only never seen), but	rt (a) s A0. is
		e.g. 6 48kx $572k^2x^2$ 48k $572k^2$ will get M1 A1 (as coefficients rather than terms have now been considered) The mistake 2 2 would give a maximum of 3 marks: M1B0A0A0, M1A1	ι.



Question Number	Scheme	Mar	ks
Q3 (a)	f(k) = -8	B1	(1)
(b)	$f(2) = 4 \Longrightarrow  4 = (6-2)(2-k) - 8$	M1	
	So $k = -1$	A1	(2)
(C)	$f(x) = 3x^2 - 2 + 3k x + 2k - 8 \qquad \qquad$	M1	
	=(3x-5)(x+2)	M1A1	(3) [6]
(b) (c)	MIAT M1 for substituting $x = 2$ (not x ) and equating to 4 to form an equation in k. If the expression is expanded in this part, condone 'slips' for this M mark. Treat the omission of the here as a 'slip' and allow the M mark. Beware: Substituting x and equating to 0 (M0 A0) also gives k . Alternative: M1 for dividing by (x ), to get $3x$ (function of k), with remainder as a function of k, and equating the remainder to 4. [Should be $3x$ 4 k), remainder k]. No working: k with no working scores M0 A0. 1 <sup>st</sup> M1 for multiplying out and substituting their (constant) value of k (in either order). The multiplying-out may occur earlier. Condone, for example, sign slips, but if the 4 (from part (b)) is included in the f(x) expression, this is M0. The 2 <sup>nd</sup> M1 is still available. 2 <sup>nd</sup> M1 for an attempt to factorise their three term quadratic (3TQ). A1 The correct answer, as a product of factors, is required. Allow 3 $= \frac{1}{3}$ x $=$ ) Ignore following work (such as a solution to a quadratic equation). If the 'acution' is colured but factors are never score the 2 <sup>nd</sup> M is not scored		



Question Number		Scheme	Marks	
Q4	(a)	$x = 2$ gives 2.236 (allow AWRT) Accept $\sqrt{5}$ x = 2.5 gives 2.580 (allow AWRT) Accept 2.58	B1 B1	(2)
	(b)	$\frac{1}{2}$ $\frac{1}$	B1,[M <sup>2</sup>	A1ft]
		= 6.133 (AWRT 6.13, even following minor slips)	A1	(4)
	(c)	Overestimate	B1	
		'Since the trapezia lie <u>above the curve</u> ', or an equivalent explanation, or sketch of (one or more) trapezia above the curve on a diagram (or on the given diagram, in which case there should be reference to this). (Note that there must be some reference to a trapezium or trapezia in the explanation or diagram).	dB1	(2) [8]
	(b)	B1 for $\frac{1}{2}$ $\frac{1}{2}$ or equivalent. For the M mark, the first bracket must contain the 'first and last' values, and the second bracket (which must be multiplied by 2) must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed. Bracketing mistake: i.e. $\frac{1}{2}$ $\frac{1}{2}$ 1.414 $\frac{1}{2}$ ) $\frac{1}{2}$ (1.554 $\frac{1}{2}$ .732 $\frac{1}{2}$ .957 $\frac{1}{2}$ .236 $\frac{1}{2}$ .580) scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). Alternative: Separate trapezia may be used, and this can be marked equivalently. $\frac{1}{4}$ (1.414 $\frac{1}{4}$ .554) $\frac{1}{4}$ (1.554 $\frac{1}{4}$ .732) $\frac{1}{4}$ $\frac{1}{4}$ (2.580 $\frac{1}{4}$ )		
	(C)	$1^{st}$ B1 for 'overestimate', ignoring earlier mistakes and ignoring any reasons given. $2^{nd}$ B1 is dependent upon the $1^{st}$ B1 (overestimate).		







Question Number	Scheme	Ma	rks
Q6 (a)	$x-3^{2}-9+y+2^{2}-4=12$ Centre is (3, -2)	M1 A1	, A1
(b) (c)	$x-3^{2} + y+2^{2} = 12 + "9" + "4" \qquad r = \sqrt{12 + "9" + "4"} = 5 \text{ (or } \sqrt{25} \text{ )}$ $PQ = \sqrt{(71)^{2} + (-5-1)^{2}} \text{ or } \sqrt{8^{2} + 6^{2}}$ $= 10 = 2 \times \text{radius, } \therefore \text{ diam. (N.B. For A1, need a comment or conclusion)}$ $[ALT: \text{ midpt. of } PQ \qquad p^{1} + p^{2} + p^{2} + p^{2} \text{ is } M1, \qquad = (3, -2) = \text{centre: A1}]$ $[ALT: \text{ eqn. of } PQ  3x  y^{2} + p^{2} \text{ is } M1, \qquad \text{verify } (3, -2) \text{ lies on this: A1}]$ $[ALT: \text{ find two grads, e.g. } PQ \text{ and } P \text{ to centre: M1, equal } \therefore \text{ diameter: A1}]$ $[ALT: \text{ show that point } S(\bigcirc PSQ = 90^{\circ}, \text{ semicircle } \therefore \text{ diameter: A1}]$ $R \text{ must lie on the circle (angle in a semicircle theorem) often implied by a diagram with R \text{ on the circle or by subsequent working)}$ $x = 0 \Rightarrow  y^{2} + 4y - 12 = 0$ $(y - 2)(y + 6) = 0  y  \dots  (M \text{ is dependent on previous } M)$ $y = -6 \text{ or } 2 \text{ (Ignore } y = -6 \text{ if seen, and 'coordinates' are not required)}$	M1 A1 M1 A1 B1 M1 dM1 A1	(5) (2) (4)
(a) (c)	1 <sup>st</sup> M1 for attempt to complete square. Allow $(x  binom{b})^2  binom{tr}$ , or $(y  binom{b})^2  binom{tr}$ , k  binom{tr}. 1 <sup>st</sup> A1 x-coordinate 3, 2 <sup>nd</sup> A1 y-coordinate -2 2 <sup>nd</sup> M1 for a full method leading to $r = \dots$ , with their 9 and their 4, 3 <sup>rd</sup> A1 5 or $\sqrt{2}$ : The 1 <sup>st</sup> M can be <u>implied</u> by ( <b>b</b> , <b>b</b> ) but a full method must be seen for the 2 <sup>nd</sup> M. Where the 'diameter' in part (b) has <u>clearly</u> been used to answer part (a), no marks in (a) but in this case the M1 ( <u>not</u> the A1) for part (b) can be given for work seen in (a). Alternative 1 <sup>st</sup> M1 for comparing with $x^2  binom{b}^2  binom{tr} gar b fy  binom{b}$ to write down centre ( <b>b</b> , <b>b</b> ) directly. Condone sign errors for this M mark. 2 <sup>nd</sup> M1 for using $r  binom{d} g^2  binom{tr}^2  binom{tr} g^2  binom{tr} gar b fy  binom{tr} gar b to write down centre (b, b) directly. Condone sign errors for this M mark. 1st M1 for setting x = 0 and getting a 3TQ in y by using eqn. of circle.2nd M1 (dep.) for attempt to solve a 3TQ leading to at least one solution for y.Alternative 1: (Requires the B mark as in the main scheme)1st M for using (3, 4, 5) triangle with vertices (3, b), (0, b), (0, y) to get a linear orquadratic equation in y (e.g. 3^2  binom{tr} y  binom{tr}^2  binom{tr} b b scored by simply solving a linear equation Alternative 2: (Not requiring realisation that R is on the circle) B1 for attempt at m_{PR}  binom{tr}_{QR}  binom{tr} (NOT m_{PQ}) or for attempt at Pythag. in triangle R1st M1 for setting x = 0, i.e. (0, y), and proceeding to get a 3TQ in y. Then main schemeAlternative 2 by 'verification':B1 for attempt at m_{PR}  binom{tr}_{QR}  binom{tr} m_{PQ} or for attempt at Pythag. in triangle R1st M1 for trying (0, 2).2nd M1 (dep.) for performing all required calculations.A1 for fully correct working and conclusion$	5 ), P <i>QR</i> . e. P <i>QR</i> .	



Question Number	Scheme	Marks
Q7 (i)	$\tan \theta = -1 \Longrightarrow  \theta = -45,  135$ $\sin \theta = \frac{2}{5} \Longrightarrow  \theta = 23.6,  156.4  (AWRT: 24, 156)$ $4 \sin x = \frac{3 \sin x}{4}$	B1, B1ft B1, B1ft (4) M1
	$\cos x$ $4 \sin x \cos x = 3 \sin x \implies \sin x (4 \cos x - 3) = 0$ Other possibilities (after squaring): $\sin^2 x (16 \sin^2 x \square) \square$ , $(16 \cos^2 x \square) (\cos^2 x \square) \square$ $x = 0, \ 180 \underline{\text{seen}}$ $x = 41.4, \ 318.6 \qquad (AWRT: 41, \ 319)$	M1 B1, B1 B1, B1ft (6)
(i) (ii)	1 <sup>st</sup> B1 for -45 seen (α, where $(a)$ (a) a snegative, or (α - 180) if α is positive. If tan $(a)$ is obtained from wrong working, 2 <sup>nd</sup> B1ft is still available. 3 <sup>rd</sup> B1 for awrt 24 (β, where $(a)$ ) if β is positive, or - (180 + β) if β is negative. If sin $(a)$ is obtained from wrong working, 4 <sup>th</sup> B1ft is still available. 1 <sup>st</sup> M1 for use of tan $x = \frac{\sin x}{\cos x}$ . Condone $\frac{3\sin x}{3\cos x}$ . 2 <sup>nd</sup> M1 for correct work leading to 2 factors (may be implied). 1 <sup>st</sup> B1 for awrt 41 (γ, where $(a)$ 80) 4 <sup>th</sup> B1 for awrt 41 (γ, where $(a)$ 80) 4 <sup>th</sup> B1 for awrt 41 (γ, where $(a)$ 80) 4 <sup>th</sup> B1 for awrt 41 (γ, where $(a)$ 80) 4 <sup>th</sup> B1 for squaring both sides 1 <sup>st</sup> M1 for squaring both sides and using a 'quadratic' identity. e.g. 16sin <sup>2</sup> (soc <sup>2</sup> ) 2 <sup>nd</sup> M1 for reaching a factorised form. e.g. (16cos <sup>2</sup> ) (cos <sup>2</sup> ) 2 <sup>nd</sup> M1 for reaching a factorised form. e.g. (16cos <sup>2</sup> ) 5 <sup>nd</sup> B1 for awrts are equivalent to the main scheme. Extra solutions, if not rejected, are perature the main scheme. For both parts of the question: Extra solutions outside required range: For each pair of B marks, the 2 <sup>nd</sup> B mark is lost if there are two correct values and one of more extra solution(s), e.g. tan $(a)$ B mark is lost if there are two correct values and one of more extra solution(s), e.g. tan $(a)$ B mark is lost if there are two correct values and one of more extra solution(s), e.g. tan $(a)$ B mark is lost if there are two correct values and one of more extra solution(s), e.g. tan $(a)$ B mark is lost if there are two correct values and one of more extra solution(s), e.g. tan $(a)$ B mark is lost if there are two correct values and one of more extra solution(s), e.g. tan $(a)$ B mark is lost if there are two correct values and one of more extra solution(s), e.g. tan $(a)$ B mark is lost if there are two correct values and one of more extra solution(s), e.g. tan $(a)$ B mark is lost if there are two correct values and one of more extra solution(s), e.g. tan $(a)$ B mark is lost if there are two correct values and	llised as in



Quest Numb	ion Der	Scheme	Mar	<sup>.</sup> ks
Q8	(a)	$\log_2 y = -3 \implies y = 2^{-3}$ $y = \frac{1}{2}$ or 0.125	M1 A1	(2)
	(b)	$32 2^5$ or $16 2^4$ or $512 2^9$	M1	
		$[or \log_2 32 = 5\log_2 2  or  \log_2 16 = 4\log_2 2  or  \log_2 512 \text{ Plog}_2 2]$		
		$[or \log_2 32 ] \frac{\log_{10} 32}{\log_{10} 2} or \log_2 16 ] \frac{\log_{10} 16}{\log_{10} 2} or \log_2 512 ] \frac{\log_{10} 512}{\log_{10} 2}]$		
		$\log_2 32 + \log_2 16 = 9$	A1	
		$(\log x)^2$ . or $(\log x)(\log x)$ . (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2)	M1	
		$\log_2 x = 3 \implies x = 2^3 = 8$	A1	
		$\log_2 x = -3  \Rightarrow  x = 2^{-3} = \frac{1}{8}$	A1ft	(5) <b>[7]</b>
	(b)	scores M1. A1 for the <u>exact</u> answer, e.g. $\log_{10} y$		
		$3^{rd}$ A1ft for an answer of $\frac{1}{their 8}$ . An ft answer may be non-exact. Possible mistakes: $\log_2 \int_{-2}^{9} \log_2 \int_{-2}^{2} \log_2 x \int_{-2}^{9} \log_2 x \int_{-$	) A1ft	



Questi	ion er	Scheme	Mark	s
Q9 (	(a)	(Arc length =) $r$ (Arc length =) $r$ (Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the <i>S</i> formula. (Requires use of $\theta = 1$ ).	B1	
		(Sector area =) $\frac{1}{2}r^2$ $r^2$ $r^2$ . Can be awarded by implication from later	B1	
		work, e.g. the correct volume formula. (Requires use of $\theta = 1$ ). Surface area = 2 sectors + 2 rectangles + curved face		
		$(= r^2 + 3rh)$ (See notes below for what is allowed here) Volume $= 300 - \frac{1}{2}r^2h$	M1	
		Sub for h: $S = r^2 + 3x^{600} - r^2 + 1800$ (*)	A1cso	(5)
(	(b)	$\frac{dS}{dr} = 2r - \frac{1800}{r^2} \text{ or } 2r  800r^{-1} \text{ or } 2r  800r^{-1}$	M1A1	
		$\frac{dS}{dr}$ $r = \sqrt[3]{900}$ , or AWRT 9.7 (NOT 9.7 or 9.7)	M1, A1	(4)
	(c)	$\frac{d^2S}{dr^2}$ and consider sign, $\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} > 0$ so point is a minimum	M1, A1ft	: (2)
(	(d)	$S_{\min} = 9.65^{2} + \frac{1800}{0.65}$		
		(Using their value of r, however found, in the given S formula) = 279.65 (AWRT: 280) (Dependent on full marks in part (b))	M1 A1	(2) [13]
(	(a)	M1 for attempting a formula (with terms added) for surface area. May be incomplete of may have extra term(s), but must have an $r^2$ (or $r^2$ ) term and an $rh$ (or $rh$ ) term.	or wrong a	and
(	(b)	In parts (b), (c) and (d), ignore labelling of parts $1^{\text{st}}$ M1 for attempt at differentiation (one term is sufficient) $r^n \blacksquare n^n$ $2^{\text{nd}}$ M1 for setting their derivative (a 'changed function') = 0 and solving as far as $r^3$ (depending upon their 'changed function', this could be $r \blacksquare$ . or $r^2 \blacksquare, \text{etc.}, T$ the algebra <u>must deal with a negative power</u> of $r$ and should be sound apart from possible <u>sign</u> errors, so that $r^n \blacksquare$ . is consistent with their derivative).	 but om	
	(c)	<ul> <li>M1 for attempting second derivative (one term is sufficient) r<sup>n</sup> m<sup>n</sup>, and considering its sign. Substitution of a value of r is not required. (Equating it to zero is M0).</li> <li>A1ft for a correct second derivative (or correct ft from their first derivative) and a valid reason (e.g. &gt; 0), and conclusion. The actual value of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum.</li> <li>Alternative:</li> <li>M1: Find value of dS/dr on each side of their value of r and consider sign.</li> </ul>		
		A1ft: Indicate sign change of negative to positive for $\frac{dS}{dr}$ , and conclude minimum.		
		<u>Alternative</u> : M1: Find <u>value</u> of <i>S</i> on each side of their value of <i>r</i> and compare with their 279.65. A1ft: Indicate that both values are more than 279.65, and conclude minimum.		