

June 2005

6664 Core Mathematics C2

Mark Scheme

Question number	Scheme	Marks
1.	$\frac{dy}{dx} = 4x - 12$ $4x - 12 = 0 \quad x = 3$ $y = -18$	B1 M1 A1ft A1 (4) <b>4</b>
	<p>M1: Equate <math>\frac{dy}{dx}</math> (not just <math>y</math>) to zero and proceed to <math>x = \dots</math></p> <p>A1ft: Follow through only from a linear equation in <math>x</math>.</p> <p><u>Alternative:</u></p> <p><math>y = 2x(x - 6) \Rightarrow</math> Curve crosses <math>x</math>-axis at 0 and 6 B1                      (By symmetry) <math>x = 3</math> M1 A1ft  <math>y = -18</math> A1</p> <p><u>Alternative:</u></p> <p><math>(x - 3)^2</math> B1 for <math>(x - 3)^2</math> seen somewhere  <math>y = 2(x^2 - 6x) = 2\{(x - 3)^2 - 9\} \quad x = 3</math>                      M1 for attempt to complete square and deduce <math>x = \dots</math>                      A1ft <math>[(x - a)^2 \Rightarrow x = a]</math>  <math>y = -18</math> A1</p>	

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2.	<p>(a) <math>x \log 5 = \log 8</math>, <math>x = \frac{\log 8}{\log 5}</math>, <math>= 1.29</math></p> <p>(b) <math>\log_2 \frac{x+1}{x}</math> (or <math>\log_2 7x</math>)</p> <p><math>\frac{x+1}{x} = 7</math> <math>x = \dots</math>, <math>\frac{1}{6}</math> (Allow 0.167 or better)</p>	<p>M1, A1, A1 (3)</p> <p>B1</p> <p>M1, A1 (3)</p> <p><b>6</b></p>
	<p>(a) Answer only 1.29 : Full marks.</p> <p>Answer only, which rounds to 1.29 (e.g. 1.292): M1 A1 A0</p> <p>Answer only, which rounds to 1.3 : M1 A0 A0</p> <p>Trial and improvement: Award marks as for “answer only”.</p> <p>(b) M1: Form (by legitimate log work) and solve an equation in <math>x</math>.</p> <p>Answer only: No marks unless verified (then full marks are available).</p>	

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3.	<p>(a) Attempt to evaluate <math>f(-4)</math> or <math>f(4)</math></p> $f(-4) = 2(-4)^3 + (-4)^2 - 25(-4) + 12 \quad (= 128 + 16 + 100 + 12) = 0,$ <p style="text-align: center;">so ..... is a factor.</p> <p>(b) <math>(x+4)(2x^2 - 7x + 3)</math></p> <p style="text-align: center;">.....<math>(2x-1)(x-3)</math></p>	<p>M1</p> <p>A1 (2)</p> <p>M1 A1</p> <p>M1 A1 (4)</p> <p><b>6</b></p>
	<p>(b) First M requires <math>(2x^2 + ax + b)</math>, <math>a \neq 0, b \neq 0</math>.</p> <p>Second M for the attempt to factorise the quadratic.</p> <p><u>Alternative:</u></p> <p><math>(x+4)(2x^2 + ax + b) = 2x^3 + (8+a)x^2 + (4a+b)x + 4b = 0</math>, then compare coefficients to find <u>values</u> of <math>a</math> and <math>b</math>. [M1]</p> <p style="text-align: center;"><math>a = -7, b = 3</math> [A1]</p> <p><u>Alternative:</u></p> <p>Factor theorem: Finding that <math>f\left(\frac{1}{2}\right) = 0, \therefore (2x-1)</math> is a factor [M1, A1]</p> <p>n.b. Finding that <math>f\left(\frac{1}{2}\right) = 0, \therefore (x-\frac{1}{2})</math> is a factor scores M1, A0, unless the factor 2 subsequently appears.</p> <p style="text-align: center;">Finding that <math>f(3) = 0, \therefore (x-3)</math> is a factor [M1, A1]</p>	

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4.	<p>(a) <math>1 + 12px + \frac{12 \times 11}{2}(px)^2</math></p> <p>(b) <math>12p(x) = -q(x) \quad 66p^2(x^2) = 11q(x^2) \quad (\text{Equate terms, or coefficients})</math></p> <p><math>\Rightarrow 66p^2 = -132p \quad (\text{Eqn. in } p \text{ or } q \text{ only})</math></p> <p><math>p = -2, \quad q = 24</math></p>	<p>B1, B1 (2)</p> <p>M1</p> <p>M1</p> <p>A1, A1 (4)</p> <p><b>6</b></p>
	<p>(a) Terms can be listed rather than added. First B1: Simplified form must be seen, but may be in (b).</p> <p>(b) First M: May still have <math>\binom{12}{2}</math> or <math>{}^{12}C_2</math></p> <p>Second M: <u>Not</u> with <math>\binom{12}{2}</math> or <math>{}^{12}C_2</math>. Dependent upon having <math>p</math>'s in each term.</p> <p>Zero solutions must be rejected for the final A mark.</p>	

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5.	<p>(a) <math>(x + 10 =) \quad 60 \quad \alpha</math>  <math>120</math> (M: <math>180 - \alpha</math> or <math>\pi - \alpha</math>)  <math>x = 50 \quad x = 110</math> ( or 50.0 and 110.0) (M: Subtract 10)</p> <p>(b) <math>(2x =) \quad 154.2 \quad \beta</math> Allow a.w.r.t. 154 or a.w.r.t. 2.69 (radians)  <math>205.8</math> (M: <math>360 - \beta</math> or <math>2\pi - \beta</math>)  <math>x = 77.1 \quad x = 102.9</math> (M: Divide by 2)</p>	<p>B1  M1  M1 A1 (4)  B1  M1  M1 A1 (4)  <b>8</b></p>
	<p>(a) First M: Must be subtracting from 180 <u>before</u> subtracting 10.  (b) First M: Must be subtracting from 360 <u>before</u> dividing by 2,  <u>or</u> dividing by 2 then subtracting from 180.</p> <p>In each part:  Extra solutions outside 0 to 180 : Ignore.  Extra solutions between 0 and 180 : A0.</p> <p><u>Alternative for (b): (double angle formula)</u></p> <p><math>1 - 2\sin^2 x = -0.9</math>      <math>2\sin^2 x = 1.9</math>      B1  <math>\sin x = \sqrt{0.95}</math>      M1  <math>x = 77.1</math>  <math>x = 180 - 77.1 = 102.9</math>      M1 A1</p>	

Question number	Scheme	Marks
6.	<p>(a) Missing y values: 1.6(00) 3.2(00) 3.394</p> <p>(b) <math>(A =) \frac{1}{2} \times 4, \{(0 + 0) + 2(1.6 + 2.771 + 3.394 + 3.2)\}</math> = 43.86 (or a more accurate value) (or 43.9, or 44)</p> <p>(c) Volume = <math>A \times 2 \times 60</math> = 5260 (m<sup>3</sup>) (or 5270, or 5280)</p>	<p>B1 B1 (2)</p> <p>B1, M1 A1ft A1 (4)</p> <p>M1 A1 (2)</p> <p><b>8</b></p>
	<p>(b) Answer only: No marks.</p> <p>(c) Answer only: Allow. (The M mark in this part can be “implied”).</p>	

Question number	Scheme	Marks
7.	<p>(a) <math>\frac{\sin x}{8} = \frac{\sin 0.5}{7}</math> or <math>\frac{8}{\sin x} = \frac{7}{\sin 0.5}</math>, <math>\sin x = \frac{8 \sin 0.5}{7}</math></p> <p><math>\sin x = 0.548</math></p> <p>(b) <math>x = 0.58</math> (<math>\alpha</math>) (This mark may be earned in (a)).</p> <p><math>\pi - \alpha = 2.56</math></p>	<p>M1 A1ft</p> <p>A1 (3)</p> <p>B1</p> <p>M1 A1ft (3)</p> <p><b>6</b></p>
	<p>(a) M: Sine rule attempt (sides/angles possibly the “wrong way round”).</p> <p>A1ft: follow through from sides/angles are the “wrong way round”.</p> <p><u>Too many d.p. given:</u></p> <p>Maximum 1 mark penalty in the complete question. (Deduct on first occurrence).</p>	

Question number	Scheme	Marks
8.	<p>(a) Centre (5, 0) (or <math>x = 5, y = 0</math>)</p> <p>(b) <math>(x \pm a)^2 \pm b \pm 9 + (y \pm c)^2 = 0 \Rightarrow r^2 = \dots</math> or <math>r = \dots</math> , Radius = 4</p> <p>(c) (1, 0), (9, 0) Allow just <math>x = 1, x = 9</math></p> <p>(d) Gradient of <math>AT = -\frac{2}{7}</math></p> <p><math>y = -\frac{2}{7}(x - 5)</math></p>	<p>B1 B1 (2)</p> <p>M1, A1 (2)</p> <p>B1ft, B1ft (2)</p> <p>B1</p> <p>M1 A1ft (3)</p> <p><b>9</b></p>
	<p>(a) (0, 5) scores B1 B0.</p> <p>(d) M1: Equation of straight line through centre, <u>any</u> gradient (except 0 or <math>\infty</math>) (The equation can be in any form).</p> <p>A1ft: Follow through from centre, but gradient must be <math>-\frac{2}{7}</math>.</p>	



Question number	Scheme	Marks
9.	<p>(a) (<math>S =</math>) <math>a + ar + \dots + ar^{n-1}</math> “<math>S =</math>” not required. Addition required.</p> <p><math>(rS =)</math> <math>ar + ar^2 + \dots + ar^n</math> “<math>rS =</math>” not required (M: Multiply by <math>r</math>)</p> <p><math>S(1 - r) = a(1 - r^n)</math> <math>S = \frac{a(1 - r^n)}{1 - r}</math> (M: Subtract and factorise) (*)</p> <p>(b) <math>ar^{n-1} = 35000 \times 1.04^3 = 39400</math> (M: Correct <math>a</math> and <math>r</math>, with <math>n = 3, 4</math> or <math>5</math>).</p> <p>(c) <math>n = 20</math> (Seen or implied)</p> <p><math>S_{20} = \frac{35000(1 - 1.04^{20})}{(1 - 1.04)}</math></p> <p>(M1: Needs <u>any</u> <math>r</math> value, <math>a = 35000</math>, <math>n = 19, 20</math> or <math>21</math>).</p> <p>(A1ft: ft from <math>n = 19</math> or <math>n = 21</math>, but <math>r</math> must be <math>1.04</math>).</p> <p><math>= 1\,042\,000</math></p>	<p>B1</p> <p>M1</p> <p>M1 A1cso (4)</p> <p>M1 A1 (2)</p> <p>B1</p> <p>M1 A1ft</p> <p>A1 (4)</p> <p><b>10</b></p>
	<p>(a) B1: At least the 3 terms shown above, and no extra terms. A1: Requires a completely correct solution. <u>Alternative for the 2 M marks:</u> M1: Multiply numerator and denominator by <math>1 - r</math>. M1: Multiply out numerator convincingly, and factorise.</p> <p>(b) M1 can also be scored by a “year by year” method. <u>Answer only:</u> 39 400 scores full marks, 39 370 scores M1 A0.</p> <p>(c) M1 can also be scored by a “year by year” method, <u>with terms added</u>. In this case the B1 will be scored if the correct number of years is considered. <u>Answer only:</u> Special case: 1 042 000 scores 2 B marks, scored as 1, 0, 0, 1 (Other answers score no marks).</p> <p><u>Failure to round correctly in (b) and (c):</u> Penalise once only (first occurrence).</p>	

Question number	Scheme	Marks
10.	<p>(a) <math>\int (2x + 8x^{-2} - 5)dx = x^2 + \frac{8x^{-1}}{-1} - 5x</math></p> $\left[ x^2 + \frac{8x^{-1}}{-1} - 5x \right]_1^4 = (16 - 2 - 20) - (1 - 8 - 5) \quad (= 6)$ <p><math>x = 1: y = 5</math> and <math>x = 4: y = 3.5</math></p> <p>Area of trapezium = <math>\frac{1}{2}(5 + 3.5)(4 - 1) \quad (= 12.75)</math></p> <p>Shaded area = <math>12.75 - 6 = 6.75</math> (M: Subtract either way round)</p> <p>(b) <math>\frac{dy}{dx} = 2 - 16x^{-3}</math></p> <p>(Increasing where) <math>\frac{dy}{dx} &gt; 0</math>; For <math>x &gt; 2, \frac{16}{x^3} &lt; 2, \therefore \frac{dy}{dx} &gt; 0</math> (Allow <math>\geq</math>)</p>	<p>M1 A1 A1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>M1 A1 (8)</p> <p>M1 A1</p> <p>dM1; A1 (4)</p> <p><b>12</b></p>
	<p>(a) Integration: One term wrong M1 A1 A0; two terms wrong M1 A0 A0. Limits: M1 for substituting limits 4 and 1 into a changed function, and subtracting the right way round.</p> <p><u>Alternative:</u> <math>x = 1: y = 5</math> and <math>x = 4: y = 3.5</math></p> <p>Equation of line: <math>y - 5 = -\frac{1}{2}(x - 1) \quad y = \frac{11}{2} - \frac{1}{2}x</math>, subsequently used in integration with limits.</p> $\left( \frac{11}{2} - \frac{1}{2}x \right) - \left( 2x + \frac{8}{x^2} - 5 \right) \quad (\text{M: Subtract either way round})$ $\int \left( \frac{21}{2} - \frac{5x}{2} - 8x^{-2} \right) dx = \frac{21x}{2} - \frac{5x^2}{4} - \frac{8x^{-1}}{-1}$ <p>(Penalise integration mistakes, not algebra for the ft marks)</p> $\left[ \frac{21x}{2} - \frac{5x^2}{4} - \frac{8x^{-1}}{-1} \right]_1^4 = (42 - 20 + 2) - \left( \frac{21}{2} - \frac{5}{4} + 8 \right) \quad (\text{M: Right way round})$ <p>Shaded area = 6.75</p> <p>(The follow through marks are for the subtracted version, and again deduct an accuracy mark for a wrong term: One wrong M1 A1 A0; two wrong M1 A0 A0.)</p> <p><u>Alternative for the last 2 marks in (b):</u> M1: Show that <math>x = 2</math> is a minimum, using, e.g., 2<sup>nd</sup> derivative. A1: Conclusion showing understanding of “increasing”, with accurate working.</p>	<p>B1</p> <p>3<sup>rd</sup> M1</p> <p>4<sup>th</sup> M1</p> <p>1<sup>st</sup> M1 A1ft A1ft</p> <p>2<sup>nd</sup> M1</p> <p>A1</p>