

June 2005

6664 Core Mathematics C2

Mark Scheme

Question number	Scheme	Marks
1.	$\frac{dy}{dx} = 4x - 12$ $4x - 12 = 0 \quad x = 3$ $y = -18$	B1 M1 A1ft A1 (4) 4
	<p>M1: Equate $\frac{dy}{dx}$ (not just y) to zero and proceed to $x = \dots$ A1ft: Follow through only from a linear equation in x.</p> <p><u>Alternative:</u> $y = 2x(x - 6) \Rightarrow$ Curve crosses x-axis at 0 and 6 B1 (By symmetry) $x = 3$ M1 A1ft $y = -18$ A1</p> <p><u>Alternative:</u> $(x - 3)^2$ B1 for $(x - 3)^2$ seen somewhere $y = 2(x^2 - 6x) = 2\{(x - 3)^2 - 9\} \quad x = 3$ M1 for attempt to complete square and deduce $x = \dots$ A1ft $[(x - a)^2 \Rightarrow x = a]$ $y = -18$ A1</p>	

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2.	<p>(a) $x \log 5 = \log 8$, $x = \frac{\log 8}{\log 5}$, $= 1.29$</p> <p>(b) $\log_2 \frac{x+1}{x}$ (or $\log_2 7x$)</p> <p>$\frac{x+1}{x} = 7$ $x = \dots$, $\frac{1}{6}$ (Allow 0.167 or better)</p>	<p>M1, A1, A1 (3)</p> <p>B1</p> <p>M1, A1 (3)</p> <p>6</p>
	<p>(a) Answer only 1.29 : Full marks.</p> <p>Answer only, which rounds to 1.29 (e.g. 1.292): M1 A1 A0</p> <p>Answer only, which rounds to 1.3 : M1 A0 A0</p> <p>Trial and improvement: Award marks as for “answer only”.</p> <p>(b) M1: Form (by legitimate log work) and solve an equation in x.</p> <p>Answer only: No marks unless verified (then full marks are available).</p>	

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3.	<p>(a) Attempt to evaluate $f(-4)$ or $f(4)$</p> $f(-4) = 2(-4)^3 + (-4)^2 - 25(-4) + 12 \quad (= 128 + 16 + 100 + 12) = 0,$ <p>so is a factor.</p> <p>(b) $(x+4)(2x^2 - 7x + 3)$</p> <p>.....$(2x-1)(x-3)$</p>	<p>M1</p> <p>A1 (2)</p> <p>M1 A1</p> <p>M1 A1 (4)</p> <p>6</p>
	<p>(b) First M requires $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$.</p> <p>Second M for the attempt to factorise the quadratic.</p> <p><u>Alternative:</u></p> <p>$(x+4)(2x^2 + ax + b) = 2x^3 + (8+a)x^2 + (4a+b)x + 4b = 0$, then compare coefficients to find <u>values</u> of a and b. [M1]</p> <p>$a = -7, b = 3$ [A1]</p> <p><u>Alternative:</u></p> <p>Factor theorem: Finding that $f\left(\frac{1}{2}\right) = 0, \therefore (2x-1)$ is a factor [M1, A1]</p> <p>n.b. Finding that $f\left(\frac{1}{2}\right) = 0, \therefore (x-\frac{1}{2})$ is a factor scores M1, A0 ,unless the factor 2 subsequently appears.</p> <p>Finding that $f(3) = 0, \therefore (x-3)$ is a factor [M1, A1]</p>	

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4.	<p>(a) $1 + 12px + \frac{12 \times 11}{2}(px)^2$</p> <p>(b) $12p(x) = -q(x) \quad 66p^2(x^2) = 11q(x^2) \quad (\text{Equate terms, or coefficients})$</p> <p>$\Rightarrow \quad 66p^2 = -132p \quad (\text{Eqn. in } p \text{ or } q \text{ only})$</p> <p>$p = -2, \quad q = 24$</p>	<p>B1, B1 (2)</p> <p>M1</p> <p>M1</p> <p>A1, A1 (4)</p> <p>6</p>
	<p>(a) Terms can be listed rather than added. First B1: Simplified form must be seen, but may be in (b).</p> <p>(b) First M: May still have $\binom{12}{2}$ or ${}^{12}C_2$</p> <p>Second M: <u>Not</u> with $\binom{12}{2}$ or ${}^{12}C_2$. Dependent upon having p's in each term.</p> <p>Zero solutions must be rejected for the final A mark.</p>	

Question number	Scheme	Marks
5.	<p>(a) $(x + 10 =) \quad 60 \quad \alpha$ 120 (M: $180 - \alpha$ or $\pi - \alpha$) $x = 50 \quad x = 110$ (or 50.0 and 110.0) (M: Subtract 10)</p> <p>(b) $(2x =) \quad 154.2 \quad \beta$ Allow a.w.r.t. 154 or a.w.r.t. 2.69 (radians) 205.8 (M: $360 - \beta$ or $2\pi - \beta$) $x = 77.1 \quad x = 102.9$ (M: Divide by 2)</p>	<p>B1 M1 M1 A1 (4)</p> <p>B1 M1 M1 A1 (4)</p> <p>8</p>
	<p>(a) First M: Must be subtracting from 180 <u>before</u> subtracting 10. (b) First M: Must be subtracting from 360 <u>before</u> dividing by 2, <u>or</u> dividing by 2 then subtracting from 180.</p> <p>In each part: Extra solutions outside 0 to 180 : Ignore. Extra solutions between 0 and 180 : A0.</p> <p><u>Alternative for (b): (double angle formula)</u></p> <p>$1 - 2\sin^2 x = -0.9$ $2\sin^2 x = 1.9$ B1 $\sin x = \sqrt{0.95}$ M1 $x = 77.1$ $x = 180 - 77.1 = 102.9$ M1 A1</p>	

Question number	Scheme	Marks
6.	<p>(a) Missing y values: 1.6(00) 3.2(00) 3.394</p> <p>(b) $(A =) \frac{1}{2} \times 4, \{(0 + 0) + 2(1.6 + 2.771 + 3.394 + 3.2)\}$ = 43.86 (or a more accurate value) (or 43.9, or 44)</p> <p>(c) Volume = $A \times 2 \times 60$ = 5260 (m³) (or 5270, or 5280)</p>	<p>B1 B1 (2)</p> <p>B1, M1 A1ft A1 (4)</p> <p>M1 A1 (2)</p> <p>8</p>
	<p>(b) Answer only: No marks.</p> <p>(c) Answer only: Allow. (The M mark in this part can be “implied”).</p>	

Question number	Scheme	Marks
7.	<p>(a) $\frac{\sin x}{8} = \frac{\sin 0.5}{7}$ or $\frac{8}{\sin x} = \frac{7}{\sin 0.5}$, $\sin x = \frac{8 \sin 0.5}{7}$</p> <p>$\sin x = 0.548$</p> <p>(b) $x = 0.58$ (α) (This mark may be earned in (a)).</p> <p>$\pi - \alpha = 2.56$</p>	<p>M1 A1ft</p> <p>A1 (3)</p> <p>B1</p> <p>M1 A1ft (3)</p> <p>6</p>
	<p>(a) M: Sine rule attempt (sides/angles possibly the “wrong way round”).</p> <p>A1ft: follow through from sides/angles are the “wrong way round”.</p> <p><u>Too many d.p. given:</u></p> <p>Maximum 1 mark penalty in the complete question. (Deduct on first occurrence).</p>	

Question number	Scheme	Marks
8.	<p>(a) Centre (5, 0) (or $x = 5, y = 0$)</p> <p>(b) $(x \pm a)^2 \pm b \pm 9 + (y \pm c)^2 = 0 \Rightarrow r^2 = \dots$ or $r = \dots$, Radius = 4</p> <p>(c) (1, 0), (9, 0) Allow just $x = 1, x = 9$</p> <p>(d) Gradient of $AT = -\frac{2}{7}$</p> <p>$y = -\frac{2}{7}(x - 5)$</p>	<p>B1 B1 (2)</p> <p>M1, A1 (2)</p> <p>B1ft, B1ft (2)</p> <p>B1</p> <p>M1 A1ft (3)</p> <p>9</p>
	<p>(a) (0, 5) scores B1 B0.</p> <p>(d) M1: Equation of straight line through centre, <u>any</u> gradient (except 0 or ∞) (The equation can be in any form).</p> <p>A1ft: Follow through from centre, but gradient must be $-\frac{2}{7}$.</p>	

Question number	Scheme	Marks
9.	<p>(a) ($S =$) $a + ar + \dots + ar^{n-1}$ “$S =$” not required. Addition required.</p> <p>($rS =$) $ar + ar^2 + \dots + ar^n$ “$rS =$” not required (M: Multiply by r)</p> <p>$S(1 - r) = a(1 - r^n)$ $S = \frac{a(1 - r^n)}{1 - r}$ (M: Subtract and factorise) (*)</p> <p>(b) $ar^{n-1} = 35000 \times 1.04^3 = 39400$ (M: Correct a and r, with $n = 3, 4$ or 5).</p> <p>(c) $n = 20$ (Seen or implied)</p> <p>$S_{20} = \frac{35000(1 - 1.04^{20})}{(1 - 1.04)}$</p> <p>(M1: Needs <u>any</u> r value, $a = 35000$, $n = 19, 20$ or 21).</p> <p>(A1ft: ft from $n = 19$ or $n = 21$, but r must be 1.04).</p> <p>$= 1\,042\,000$</p>	<p>B1</p> <p>M1</p> <p>M1 A1cso (4)</p> <p>M1 A1 (2)</p> <p>B1</p> <p>M1 A1ft</p> <p>A1 (4)</p> <p>10</p>
	<p>(a) B1: At least the 3 terms shown above, and no extra terms. A1: Requires a completely correct solution. <u>Alternative for the 2 M marks:</u> M1: Multiply numerator and denominator by $1 - r$. M1: Multiply out numerator convincingly, and factorise.</p> <p>(b) M1 can also be scored by a “year by year” method. <u>Answer only:</u> 39 400 scores full marks, 39 370 scores M1 A0.</p> <p>(c) M1 can also be scored by a “year by year” method, <u>with terms added</u>. In this case the B1 will be scored if the correct number of years is considered. <u>Answer only:</u> Special case: 1 042 000 scores 2 B marks, scored as 1, 0, 0, 1 (Other answers score no marks).</p> <p><u>Failure to round correctly in (b) and (c):</u> Penalise once only (first occurrence).</p>	

Question number	Scheme	Marks
10.	<p>(a) $\int (2x + 8x^{-2} - 5)dx = x^2 + \frac{8x^{-1}}{-1} - 5x$</p> $\left[x^2 + \frac{8x^{-1}}{-1} - 5x \right]_1^4 = (16 - 2 - 20) - (1 - 8 - 5) \quad (= 6)$ <p>$x = 1: y = 5$ and $x = 4: y = 3.5$</p> <p>Area of trapezium = $\frac{1}{2}(5 + 3.5)(4 - 1) \quad (= 12.75)$</p> <p>Shaded area = $12.75 - 6 = 6.75$ (M: Subtract either way round)</p> <p>(b) $\frac{dy}{dx} = 2 - 16x^{-3}$</p> <p>(Increasing where) $\frac{dy}{dx} > 0$; For $x > 2, \frac{16}{x^3} < 2, \therefore \frac{dy}{dx} > 0$ (Allow \geq)</p>	<p>M1 A1 A1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>M1 A1 (8)</p> <p>M1 A1</p> <p>dM1; A1 (4)</p> <p>12</p>
	<p>(a) Integration: One term wrong M1 A1 A0; two terms wrong M1 A0 A0. Limits: M1 for substituting limits 4 and 1 into a changed function, and subtracting the right way round.</p> <p><u>Alternative:</u> $x = 1: y = 5$ and $x = 4: y = 3.5$</p> <p>Equation of line: $y - 5 = -\frac{1}{2}(x - 1) \quad y = \frac{11}{2} - \frac{1}{2}x$, subsequently used in integration with limits.</p> $\left(\frac{11}{2} - \frac{1}{2}x \right) - \left(2x + \frac{8}{x^2} - 5 \right) \quad (\text{M: Subtract either way round})$ $\int \left(\frac{21}{2} - \frac{5x}{2} - 8x^{-2} \right) dx = \frac{21x}{2} - \frac{5x^2}{4} - \frac{8x^{-1}}{-1}$ <p>(Penalise integration mistakes, not algebra for the ft marks)</p> $\left[\frac{21x}{2} - \frac{5x^2}{4} - \frac{8x^{-1}}{-1} \right]_1^4 = (42 - 20 + 2) - \left(\frac{21}{2} - \frac{5}{4} + 8 \right) \quad (\text{M: Right way round})$ <p>Shaded area = 6.75</p> <p>(The follow through marks are for the subtracted version, and again deduct an accuracy mark for a wrong term: One wrong M1 A1 A0; two wrong M1 A0 A0.)</p> <p><u>Alternative for the last 2 marks in (b):</u> M1: Show that $x = 2$ is a minimum, using, e.g., 2nd derivative. A1: Conclusion showing understanding of “increasing”, with accurate working.</p>	<p>B1</p> <p>3rd M1</p> <p>4th M1</p> <p>1st M1 A1ft A1ft</p> <p>2nd M1</p> <p>A1</p>