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Mark Scheme (Results)
January 2009

GCE

GCE Mathematics (6664/01)

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January 2009
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1	$(3-2x)^5 = 243$, $\quad \ldots\ldots + 5\times(3)^4(-2x) = -810x \quad \ldots\ldots$	B1, B1
	$+\frac{5\times 4}{2}(3)^3(-2x)^2 = \qquad +1080x^2$	M1 A1 (4)
		[4]
Notes	First term must be 243 for **B1**, writing just 3^5 is B0 (Mark their final answers except in second line of special cases below). Term must be simplified to –810*x* for **B1** The *x* is required for this mark**.** The **method** mark **(M1) is generous** and is awarded for an attempt at Binomial to get the third term. There must be an x^2 (or no *x*- i.e. not wrong power) and attempt at Binomial Coefficient and at dealing with powers of 3 and 2. The power of 3 should not be one, but the power of 2 may be one (regarded as bracketing slip). So allow $\binom{5}{2}$ or $\binom{5}{3}$ or ${}^5C_2$ or ${}^5C_3$ or even $\left(\frac{5}{2}\right)$ or $\left(\frac{5}{3}\right)$ or use of '10' (maybe from Pascal's triangle) May see ${}^5C_2(3)^3(-2x)^2$ or ${}^5C_2(3)^3(-2x^2)$ or ${}^5C_2(3)^5(-\frac{2}{3}x^2)$ or $10(3)^3(2x)^2$ which would each score the **M1** **A1**is c.a.o and needs $1080x^2$ (if $1080x^2$ is written with no working this is awarded both marks i.e. **M1 A1.)**	
Special cases	$243+810x+1080x^2$ is **B1B0M1A1** (condone no negative signs) Follows correct answer with $27-90x+120x^2$ can isw here (sp case)– full marks for correct answer Misreads *ascending* and gives $-32x^5+240x^4-720x^3$ is marked as **B1B0M1A0** special case and must be completely correct. *(If any slips could get B0B0M1A0)* Ignores 3 and expands $(1\pm 2x)^5$ is **0/4** 243, -810*x*, 1080x^2 is full marks but 243, -810, 1080 is **B1,B0,M1,A0** NB Alternative method $3^5(1-\frac{2}{3}x)^5 = 3^5 - 5\times 3^5\times\left(\frac{2}{3}x\right)+\binom{5}{3}3^5\left(-\frac{2}{3}x\right)^2+..$ is **B0B0M1A0** – answers must be simplified to 243 –810*x* +1080x^2 for full marks (awarded as before) Special case $3(1-\frac{2}{3}x)^5 = 3-5\times 3\times\left(\frac{2}{3}x\right)+\binom{5}{3}3\left(-\frac{2}{3}x\right)^2+..$ is **B0, B0, M1, A0** Or $3(1-2x)^5$ is **B0B0M0A0**	

Question Number	Scheme	Marks
2	$y=(1+x)(4-x)=4+3x-x^2$ M: Expand, giving 3 (or 4) terms	M1
	$\int(4+3x-x^2)\mathrm{d}x=4x+\frac{3x^2}{2}-\frac{x^3}{3}$ M: Attempt to integrate	M1 A1
	$=[..............]_{-1}^{4}=\left(16+24-\frac{64}{3}\right)-\left(-4+\frac{3}{2}+\frac{1}{3}\right)=\frac{125}{6}$ $\left(=20\frac{5}{6}\right)$	M1 A1 (5) **[5]**
Notes	**M1** needs expansion, there may be a slip involving a sign or simple arithmetical error e.g. $1\times4=5$, but there needs to be a 'constant' an 'x term' and an 'x^2 term'. The x terms do not need to be collected. (Need not be seen if next line correct) Attempt to integrate means that $x^n\to x^{n+1}$ for at least one of the terms, then **M1** is awarded (even 4 becoming 4x is sufficient) – one correct power sufficient. **A1** is for correct answer only, not follow through. But allow $2x^2-\frac{1}{2}x^2$ or any correct equivalent. Allow **+ *c***, and even allow an evaluated extra constant term. **M1**: Substitute limit 4 and limit –1 into a changed function (must be –1) and indicate subtraction (either way round). **A1** must be exact, not 20.83 or similar. If recurring indicated can have the mark. Negative area, even if subsequently positive loses the A mark.	
Special cases	(i) Uses calculator method: **M1** for expansion (if seen) **M1** for limits if answer correct, so 0 , 1 or 2 marks out of 5 is possible (Most likely **M0 M0 A0 M1 A0**) (ii) Uses trapezium rule : not exact, no calculus – 0/5 unless expansion mark **M1** gained. (iii) Using original method, but then change all signs after expansion is likely to lead to: **M1 M1 A0, M1 A0 i.e. 3/5**	

Question Number	Scheme	Marks
3 (a)	3.84, 4.14, 4.58 (Any one correct B1 B0. All correct B1 B1)	B1 B1 (2)
(b)	$\frac{1}{2}\times 0.4,\ \{(3+4.58)+2(3.47+3.84+4.14+4.39)\}$	B1, M1 A1ft
	= 7.852 (awrt 7.9)	A1 (4)
		[6]

Notes

(a) **B1** for one answer correct Second **B1** for all three correct

Accept awrt ones given or exact answers so $\sqrt{21}$, $\sqrt{\left(\frac{369}{25}\right)}$ or $\frac{3\sqrt{41}}{5}$, and $\sqrt{\left(\frac{429}{25}\right)}$ or $\frac{\sqrt{429}}{5}$, score the marks.

(b) **B1** is for using 0.2 or $\frac{0.4}{2}$ as $\frac{1}{2}h$.

M1 requires first bracket to contain first plus last values
and second bracket to include no additional values from those in the table.
If the only mistake is to omit one value from 2nd bracket this may be regarded as a slip ar
can be allowed (An extra repeated term forfeits the **M** mark however)
x values: **M0** if values used in brackets are x values instead of y values.
Separate trapezia may be used : **B1** for 0.2, **M1** for $\frac{1}{2}h(a+b)$ used 4 or 5 times (and **A1**ft all
e.g.. $0.2(3+3.47)+0.2(3.47+3.84)+0.2(3.84+4.14)+0.2(4.14+4.58)$ is **M1 A0**
equivalent to missing one term in { } in main scheme
A1ft follows their answers to part (a) and is for {correct expression}

Final **A1** must be correct. (No follow through)

Special cases

Bracketing mistake: i.e. $\frac{1}{2}\times 0.4(3+4.58)+2(3.47+3.84+4.14+4.39)$
scores **B1 M1 A0 A0** unless the final answer implies that the calculation
has been done correctly (then full marks can be given).

Need to see trapezium rule – answer only (with no working) is 0/4.

Question Number	Scheme	Marks
4	$2\log_5 x = \log_5(x^2)$, $\quad \log_5(4-x) - \log_5(x^2) = \log_5 \dfrac{4-x}{x^2}$	B1, M1
	$\log\left(\dfrac{4-x}{x^2}\right) = \log 5 \quad 5x^2 + x - 4 = 0$ **or** $5x^2 + x = 4$ **o.e.**	M1 A1
	$(5x-4)(x+1) = 0 \quad x = \dfrac{4}{5} \quad (x = -1)$	dM1 A1 (6) **[6]**
Notes	**B1** is awarded for $2\log x = \log x^2$ anywhere. **M1** for correct use of log A – log B = log $\frac{A}{B}$ **M1** for replacing 1 by $\log_k k$. **A1** for correct quadratic ($\log(4-x) - \log x^2 = \log 5 \Rightarrow 4 - x - x^2 = 5$ is **B1M0M1A0 M0A0**) **dM1** for attempt to solve quadratic with usual conventions. (Only award if previous two **M** marks have been awarded) **A1** for 4/5 or 0.8 or equivalent (Ignore extra answer).	
Alternative 1	$\log_5(4-x) - 1 = 2\log_5 x$ so $\log_5(4-x) - \log_5 5 = 2\log_5 x$	M1
	$\log_5 \dfrac{4-x}{5} = 2\log_5 x$	M1
	then could complete solution with $2\log_5 x = \log_5(x^2)$	B1
	$\left(\dfrac{4-x}{5}\right) = x^2 \quad 5x^2 + x - 4 = 0$	A1
	Then as in first method $(5x-4)(x+1) = 0 \quad x = \dfrac{4}{5} \quad (x = -1)$	dM1 A1 (6) **[6]**
Special cases	Complete trial and error yielding 0.8 is **M3** and **B1** for 0.8 **A1, A1** awarded for each of two tries evaluated. i.e. 6/6 Incomplete trial and error with wrong or no solution is 0/6 Just answer 0.8 with no working is **B1**	
	If log base 10 or base e used throughout - can score **B1M1M1A0M1A0**	

Question Number	Scheme	Marks
5 (a)	PQ: $m_1 = \frac{10-2}{9-(-3)}$ $(=\frac{2}{3})$ **and** QR: $m_2 = \frac{10-4}{9-a}$	M1
(b)	$m_1 m_2 = -1$: $\frac{8}{12}\times\frac{6}{9-a} = -1$ $a = 13$ (*)	M1 A1 (3)
Alt for (a)	(a) Alternative method (Pythagoras) Finds **all three** of the following	
	$(9-(-3))^2 + (10-2)^2$, (*i.e.*208) , $(9-a)^2 + (10-4)^2$, $(a-(-3))^2 + (4-2)^2$	M1
	Using Pythagoras (correct way around) e.g. $a^2 + 6a + 9 = 240 + a^2 - 18a + 81$ to form equation	M1
	Solve (or verify) for a, a = 13 (*)	A1 (3)
	(b) Centre is at (5, 3)	B1
	$(r^2 =)$ $(10-3)^2 + (9-5)^2$ or equiv., or $(d^2 =)$ $(13-(-3))^2 + (4-2)^2$	M1 A1
	$(x-5)^2 + (y-3)^2 = 65$ or $x^2 + y^2 - 10x - 6y - 31 = 0$	M1 A1 (5)
Alt for (b)	Uses $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes (-3, 2), (9, 10) and (13, 4) then eliminates one unknown	M1
	Eliminates second unknown	M1
	Obtains $g = -5, f = -3, c = -31$ or $a = 5, b = 3$, $r^2 = 65$	A1, A1, B1cao (5) **[8]**
Notes (a)	**M1**-considers gradients of PQ and QR -must be y difference / x difference (or considers three lengths as in alternative method) **M1** Substitutes gradients into product = -1 (or lengths into Pythagoras' Theorem the correct way round) **A1** Obtains a = 13 with no errors by solution or verification. Verification can score 3/3.	
(b)	Geometrical method: **B1** for coordinates of centre – can be implied by use in part (b) **M1** for attempt to find r^2, d^2, r or d (allow one slip in a bracket). **A1** cao. These two marks may be gained implicitly from circle equation **M1** for $(x\pm5)^2 + (y\pm3)^2 = k^2$ or $(x\pm3)^2 + (y\pm5)^2 = k^2$ ft their (5,3) Allow k^2 non numerical. **A1** cao for whole equation and rhs must be 65 or $(\sqrt{65})^2$, (similarly B1 must be 65 or $(\sqrt{65})^2$, in alternative method for (b))	

Question Number	Scheme	Marks
Further alternatives	(i) A number of methods find gradient of PQ = 2/3 then give perpendicular gradient is –3/2 This is **M1** They then proceed using equations of lines through point Q or by using gradient QR to obtain equation such as $\frac{4-10}{a-9}=-\frac{3}{2}$ **M1** (may still have x in this equation rather than a and there may be a small slip) They then complete to give (a)= 13 **A1** (ii) A long involved method has been seen finding the coordinates of the centre of the circle first. This can be done by a variety of methods Giving centre as (c, 3) and using an equation such as $(c-9)^2+7^2=(c+3)^2+1^2$ (equal radii) or $\frac{3-6}{c-3}=-\frac{3}{2}$ **M1** (perpendicular from centre to chord bisects chord) Then using c (= 5) to find a is **M1** Finally a = 13 **A1** (iii) Vector Method: States **PQ. QR** = 0, with vectors stated 12i +8j and (9 – a)**i + 6**j is **M1** Evaluates scalar product so 108 – 12 a + 48 = 0 (**M1**) solves to give a = 13 (**A1**)	M1 M1 A1 M1 M1 A1 M1 M1 A1

Question Number	Scheme	Marks
6 (a)	$\mathrm{f}(2) = 16 + 40 + 2a + b$ **or** $\mathrm{f}(-1) = 1 - 5 - a + b$	M1 A1
	Finds 2nd remainder and equates to 1st $\Rightarrow$ $16 + 40 + 2a + b = 1 - 5 - a + b$	M1 A1
	$a = -20$	A1cso (5)
(b)	$\mathrm{f}(-3) = (-3)^4 + 5(-3)^3 - 3a + b = 0$	M1 A1ft
	81 – 135 + 60 + b = 0 gives b = -6	A1 cso (3) **[8]**
Alternative for (a)	(a) Uses long division, to get remainders as b + 2a + 56 or b – a - 4 or correct equivalent	M1 A1
	Uses second long division as far as remainder term, to get b + 2a + 56 = b – a - 4 or correct equivalent	M1 A1
	$a = -20$	A1cso (5)
Alternative for (b)	(b) Uses long division of $x^4 + 5x^3 - 20x + b$ by (x + 3) to obtain $x^3 + 2x^2 - 6x + a + 18$ (with their value for a)	M1 A1ft
	Giving remainder b + 6 = 0 and so b = -6	A1 cso (3) **[8]**
Notes (a)	**M1** : Attempts f(±2) or f(±1) **A1** is for the answer shown (or simplified with terms collected) for one remainder **M1:** Attempts other remainder and puts one equal to the other **A1:** for correct equation in a (and b) then **A1 for** $a = -20$ **cso**	
(b)	**M1** : Puts $\mathrm{f}(\pm 3) = 0$ **A1** is for f(-3) = 0, (where f is original function), with no sign or substitution errors (follow through on 'a' and could still be in terms of a) **A1:** b = -6 is cso.	
Alternatives	(a) **M1**: Uses long division of $x^4 + 5x^3 + ax + b$ by (x ±2) **or** by (x ±1) as far as three term quotient **A1**: Obtains at least one correct remainder **M1:** Obtains second remainder and puts two remainders (no x terms) equal **A1:** correct equation **A1**: correct answer a = -20 following correct work. (b) **M1**: complete long division as far as constant (ignore remainder) **A1ft:** needs correct answer for their a **A1:** correct answer	

Beware: It is possible to get **correct answers with wrong working**. If remainders are equated to 0 in part (a) both correct answers are obtained fortuitously. This could score M1A1M0A0A0M1A1A0

Question Number	Scheme	Marks
7 (a)	$\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times 2.2 = 39.6 \quad \left(\text{cm}^2\right)$	M1 A1 (2)
(b)	$\left(\frac{2\pi - 2.2}{2} = \right) \pi - 1.1 = 2.04 \quad \text{(rad)}$	M1 A1 (2)
	(c) $\Delta DAC = \frac{1}{2}\times 6\times 4\sin 2.04 \qquad (\approx 10.7)$	M1 A1ft
	Total area = sector + 2 triangles = 61 $\left(\text{cm}^2\right)$	M1 A1 (4) **[8]**
(a)	**M1:** Needs θ in radians for this formula. Could convert to degrees and use degrees formula. **A1:** Does not need units. Answer should be 39.6 exactly. Answer with no working is **M1 A1**. This **M1A1** can only be awarded in part (a).	
(b)	**M1:** Needs full method to give angle in radians **A1:** Allow answers which round to 2.04 (Just writes 2.04 – no working is 2/2)	
(c)	**M1:** Use $\frac{1}{2}\times 6\times 4\sin A$ (if any other triangle formula e.g. $\frac{1}{2}b\times h$ is used the method must be complete for this mark) (No value needed for A, but should not be using 2.2) **A1:** ft the value obtained in part (b) – need not be evaluated- could be in degrees **M1:** Uses Total area = sector + 2 triangles or other complete method **A1:** Allow answers which round to 61. (Do not need units) Special case degrees: Could get M0A0, M0A0, M1A1M1A0 Special case: Use $\Delta\, BDC - \Delta\, BAC$ Both areas needed for first **M1** Total area = sector + area found is second **M1** **NB** Just finding lengths BD, DC, and angle BDC then assuming area BDC is a sector to find area BDC is 0/4	

Question Number	Scheme	Marks
8 (a)	$4(1-\cos^2 x)+9\cos x-6=0$ $\quad 4\cos^2 x-9\cos x+2=0$ (*)	M1 A1 (2)
(b)	$(4\cos x-1)(\cos x-2)=0$ $\quad \cos x=...,\ \frac{1}{4}$	M1 A1
	$x=75.5\quad(\alpha)$	B1
	$360-\alpha,\quad 360+\alpha$ or $720-\alpha$	M1, M1
	284.5, 435.5, 644.5	A1 (6) **[8]**
(a)	**M1:** Uses $\sin^2 x=1-\cos^2 x$ (may omit bracket) **not** $\sin^2 x=\cos^2 x-1$ **A1:** Obtains the printed answer without error – **must have = 0**	
(b)	**M1:** Solves the quadratic with usual conventions **A1:** Obtains ¼ accurately- ignore extra answer 2 but penalise e.g. -2. **B1:** allow answers which round to 75.5 **M1:** $360-\alpha$ ft their value, **M1:** $360+\alpha$ ft their value or 720 - α ft **A1:** Three **and only three** correct exact answers in the range achieves the mark	
Special cases	In part (b) Error in solving quadratic (4cosx-1)(cosx+2) Could yield, **M1A0B1M1M1A1** losing one mark for the error Works in radians: Complete work in radians :Obtains 1.3 **B0**. Then allow **M1 M1** for $2\pi-\alpha$, $2\pi+\alpha$ or $4\pi-\alpha$ Then gets 5.0, 7.6, 11.3 **A0 so 2/4** Mixed answer 1.3, 360 – 1.3, 360 + 1.3, 720 – 1.3 still gets **B0M1M1A0**	

Question Number	Scheme	Marks
9 (a)	Initial step: Two of: $a = k + 4,\ ar = k,\ ar^2 = 2k - 15$ Or one of: $r = \frac{k}{k+4},\quad r = \frac{2k-15}{k},\quad r^2 = \frac{2k-15}{k+4}$, Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$	M1
	$k^2 = (k+4)(2k-15)$, so $k^2 = 2k^2 + 8k - 15k - 60$	M1, A1
	Proceed to $k^2 - 7k - 60 = 0$ (*)	A1 (4)
(b)	$(k-12)(k+5) = 0 \qquad k = 12$ (*)	M1 A1 (2)
(c)	Common ratio: $\frac{k}{k+4}$ or $\frac{2k-15}{k} = \frac{12}{16}\ \left(= \frac{3}{4} \text{ or } 0.75\right)$	M1 A1 (2)
(d)	$\frac{a}{1-r} = \frac{16}{\left(\frac{1}{4}\right)} = 64$	M1 A1 (2) **[10]**
(a)	**M1:** The 'initial step', scoring the first M mark, may be implied by next line of proof **M1**: Eliminates a and r to give valid equation in k only. Can be awarded for equation involving fractions. **A1 :** need some correct expansion and working and answer equivalent to required quadratic but with uncollected terms. Equations involving fractions do not get this mark. (No fractions, no brackets – could be a cubic equation) **A1:** as answer is printed this mark is for cso (Needs = 0) **All four marks must be scored in part (a)**	
(b)	**M1:** Attempt to solve quadratic **A1:** This is for correct factorisation or solution and $k = 12$. Ignore the extra solution ($k = -5$ or even $k = 5$), if seen. Substitute and verify is **M1 A0** **Marks must be scored in part (b)**	
(c)	**M1:** Complete method to find r Could have answer in terms of k **A1:** 0.75 or any correct equivalent **Both Marks must be scored in (c)**	
(d)	**M1:** Tries to use $\frac{a}{1-r}$, (even with r>1). Could have an answer still in terms of k. **A1:** This answer is 64 cao.	

Question Number	Scheme	Marks
10 (a)	$2\pi rh + 2\pi r^2 = 800$	B1
	$h = \dfrac{400 - \pi r^2}{\pi r}$, $V = \pi r^2\left(\dfrac{400 - \pi r^2}{\pi r}\right) = 400r - \pi r^3$ (*)	M1, M1 A1 (4)
(b)	$\dfrac{dV}{dr} = 400 - 3\pi r^2$	M1 A1
	$400 - 3\pi\ \ r^2 = 0$ $r^2 = ...,$ $r = \sqrt{\dfrac{400}{3\pi}}$ (= 6.5 (2 s.f.))	M1 A1
	$V = 400r - \pi r^3 = 1737 = \dfrac{800}{3}\sqrt{\dfrac{400}{3\pi}}$ (cm^3) (accept awrt 1737 or exact answer)	M1 A1 (6)
(c)	$\dfrac{d^2V}{dr^2} = -6\pi r$, Negative, $\therefore$ maximum (Parts (b) and (c) should be considered together when marking)	M1 A1 (2) **[12]**
Other methods for part (c):	Either: M: Find value of $\dfrac{dV}{dr}$ on each side of "$r = \sqrt{\dfrac{400}{3\pi}}$" and consider sign. A: Indicate sign change of positive to negative for $\dfrac{dV}{dr}$, and conclude max. Or: M: Find value of V on each side of "$r = \sqrt{\dfrac{400}{3\pi}}$" and compare with "1737". A: Indicate that both values are less than 1737 or 1737.25, and conclude max.	
Notes (a) (b)	**B1:** For any correct form of this equation (may be unsimplified, may be implied by 1st M1) **M1 :** Making h the subject of their three or four term formula **M1:** Substituting expression for h into $\pi r^2 h$ (independent mark) Must now be expression in r only. **A1:** cso **M1:** At least one power of r decreased by 1 **A1:** cao **M1:** Setting $\dfrac{dV}{dr}$ =0 and finding a value for correct power of r for candidate **A1 :** This mark may be credited if the value of V is correct. Otherwise answers should round to 6.5 (allow ± 6.5)or be exact answer **M1:** Substitute a positive value of r to give V **A1:** 1737 or 1737.25….. or exact answer	

(c)	**M1:** needs complete method **e.g.**attempts differentiation (power reduced) of their first derivative and considers its sign **A1(first method)** should be $-6\pi r$ (do not need to substitute *r* and can condone wrong *r* if found in (b)) Need to conclude maximum or indicate by a tick that it is maximum. Throughout allow confused notation such as d*y*/d*x* for d*V*/d*r*
Alternative for (a)	$A = 2\pi r^2 + 2\pi rh$, $\frac{A}{2} \times r = \pi r^3 + \pi r^2 h$ is **M1** Equate to $400r$ **B1** Then $V = 400r - \pi r^3$ is **M1 A1**