Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in Core Mathematics 3 (6665/01)

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Summer 2015
Publications Code UA041199

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\ \ q\neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.(a)	$\tan 2\theta° = \dfrac{2\tan\theta°}{1-\tan^2\theta°} = \dfrac{2p}{1-p^2}$ Final answer	M1A1 **(2)**
(b)	$\cos\theta° = \dfrac{1}{\sec\theta°} = \dfrac{1}{\sqrt{1+\tan^2\theta°}} = \dfrac{1}{\sqrt{1+p^2}}$ Final answer	M1A1 **(2)**
(c)	$\cot(\theta-45)° = \dfrac{1}{\tan(\theta-45)°} = \dfrac{1+\tan\theta°\tan 45°}{\tan\theta° - \tan 45°} = \dfrac{1+p}{p-1}$ Final answer	M1A1 **(2)** **(6 marks)**

(a)

M1 Attempt to use the double angle formula for tangent followed by the substitution $\tan\theta = p$.

For example accept $\tan 2\theta° = \dfrac{2\tan\theta°}{1\pm\tan^2\theta°} = \dfrac{2p}{1\pm p^2}$

Condone unconventional notation such as $\tan 2\theta° = \dfrac{2\tan\theta°}{1\pm\tan\theta^{2}°}$ followed by an attempt to substitute $\tan\theta = p$ for the M mark. Recovery from this notation is allowed for the A1.

Alternatively use $\tan(A+B) = \dfrac{\tan A + \tan B}{1\pm\tan A\tan B}$ with an attempt at substituting $\tan A = \tan B = p$. The unsimplified answer $\dfrac{p+p}{1-p\times p}$ is evidence

It is possible to use $\tan 2\theta° = \dfrac{\sin 2\theta°}{\cos 2\theta°} = \dfrac{2\sin\theta°\cos\theta°}{2\cos^2\theta° - 1} = \dfrac{2\times\dfrac{p}{\sqrt{1\pm p^2}}\times\dfrac{1}{\sqrt{1\pm p^2}}}{2\times\dfrac{1}{1\pm p^2}-1}$ but it is unlikely to succeed.

A1 Correct **simplified** answer of $\tan 2\theta° = \dfrac{2p}{1-p^2}$ or $\dfrac{2p}{(1-p)(1+p)}$.

Do not allow if they "simplify" to $\dfrac{2}{1-p}$

Allow the correct answer for both marks as long as no incorrect working is seen.

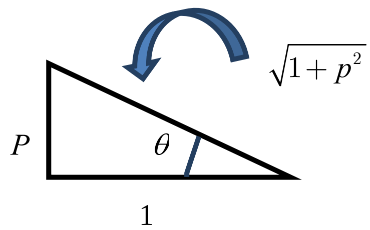
(b)

M1 Attempt to use **both** $\cos\theta = \frac{1}{\sec\theta}$ **and** $1+\tan^2\theta = \sec^2\theta$ with $\tan\theta = p$ in an attempt to obtain an expression for $\cos\theta$ in terms of p. Condone a slip in the sign of the second identity.

Evidence would be $\cos^2\theta = \frac{1}{\pm 1 \pm p^2}$

Alternatively use a triangle method, attempt Pythagoras' theorem and use $\cos\theta = \frac{adj}{hyp}$

The attempt to use Pythagoras must attempt to use the squares of the lengths.



A1 $\cos\theta° = \frac{1}{\sqrt{1+p^2}}$ Accept versions such as $\cos\theta° = \sqrt{\frac{1}{1+p^2}}$, $\cos\theta° = \pm\frac{1}{\sqrt{1+p^2}}$

Withhold this mark if the candidate goes on to write $\cos\theta° = \frac{1}{1+p}$

(c)

M1 Use the correct identity $\cot(\theta-45) = \frac{1}{\tan(\theta-45)}$ and an attempt to use the $\tan(A-B)$ formula with $A=\theta$, $B=45$ and $\tan\theta = p$.

For example accept an unsimplified answer such as $\frac{1}{\frac{\tan\theta \pm \tan 45}{1 \pm \tan\theta\tan 45}} = \frac{1}{\frac{p \pm \tan 45}{1 \pm p\tan 45}}$

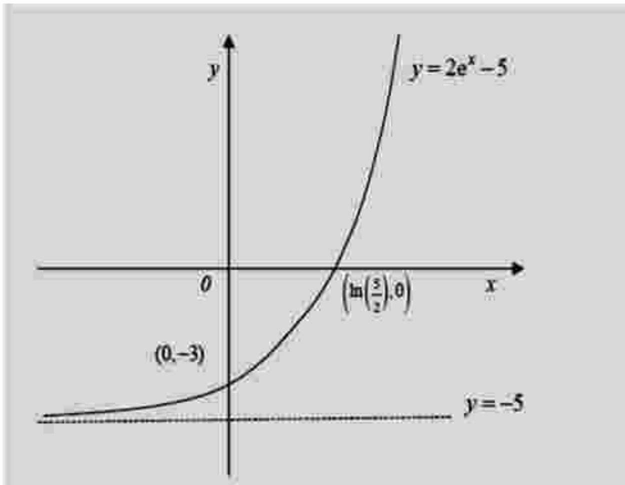
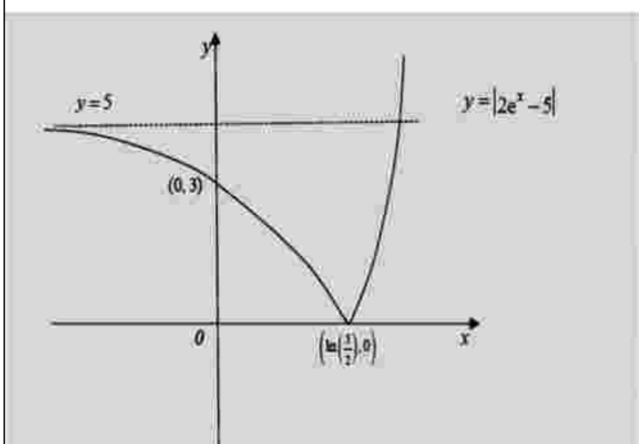
It is possible to use $\cot(\theta-45) = \frac{\cos(\theta-45)}{\sin(\theta-45)}$ and an attempt to use the formulae for $\sin(A-B)$ and $\cos(A-B)$ with $A=\theta$, $B=45$. $\sin\theta = \frac{p}{\sqrt{1\pm p^2}}$ and $\cos\theta = \frac{1}{\sqrt{1\pm p^2}}$

Sight of an expression $\frac{\frac{1}{\sqrt{1\pm p^2}}\cos 45 \pm \frac{p}{\sqrt{1\pm p^2}}\sin 45}{\frac{p}{\sqrt{1\pm p^2}}\cos 45 \pm \frac{1}{\sqrt{1\pm p^2}}\sin 45}$ is evidence.

A1 Uses $\tan 45 = 1$ or $\sin 45 = \cos 45 = \frac{\sqrt{2}}{2}$ *oe* and simplifies answer.

Accept $-\frac{1+p}{1-p}$ or $1+\frac{2}{p-1}$

Note that there is no isw in any parts of this question.

Question Number	Scheme		Marks
2.(ai)		Shape	B1
		$\left(\ln\left(\frac{5}{2}\right), 0\right)$ and $(0, -3)$	B1
		$y = -5$	B1
			(3)
(aii)		Shape inc cusp	B1ft
		$\left(\ln\left(\frac{5}{2}\right), 0\right)$ *and* $(0, 3)$	B1ft
		$y = 5$	B1ft
			(3)
(b)	$x \geqslant \ln\left(\frac{5}{2}\right)$		B1 ft
			(1)
(c)	$2e^x - 5 = -2 \Rightarrow (x) = \ln\left(\frac{3}{2}\right)$		M1A1
	$(x) = \ln\left(\frac{7}{2}\right)$		B1
			(3)
			(10 marks)

(a)(i)

B1 For an exponential (growth) shaped curve in any position. For this mark be tolerant on slips of the pen at either end. See Practice and Qualification for examples.

B1 Intersections with the axes at $\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$.

Allow $\ln\left(\frac{5}{2}\right)$ and -3 being marked on the correct axes.

Condone $\left(0,\ln\left(\frac{5}{2}\right)\right)$ and $(-3,0)$ being marked on the x and y axes respectively.

Do not allow $\left(\ln\left(\frac{5}{2}\right),0\right)$ appearing as awrt (0.92, 0) for this mark unless seen elsewhere. Allow if seen in body of script. If they are given in the body of the script and differently on the curve (save for the decimal equivalent) then **the ones on the curve take precedence.**

B1 Equation of the asymptote given as $y=-5$. Note that the curve must appear to have an asymptote at $y=-5$, not necessarily drawn. It is not enough to have -5 marked on the axis or indeed $x=-5$. An extra asymptote **with an equation** gets B0

(a)(ii)

B1ft For **either** the correct shape **or** a reflection of their curve from (a)(i) in the x- axis. For this to be scored it must have appeared both above and below the x - axis. The shape must be correct including the cusp. The curve to the lhs of the cusp must appear to have the correct curvature

B1ft Score for both intersections or follow through on both the intersections given in part (a)(i), including decimals, as long as the curve appeared both above and below the x - axis. See part (a) for acceptable forms

B1ft Score for an asymptote of $y=5$ or follow through on an asymptote of $\boldsymbol{y}=-C$ from part (a)(i). Note that the curve must appear to have an asymptote at $y=C$ but do not penalise if the first mark in (a)(ii) has been withheld for incorrect curvature on the lhs.

(b)

B1ft Score for $x \geqslant \ln\left(\frac{5}{2}\right)$, $x \geqslant \text{awrt } 0.92$ or follow through on the x intersection in part (a)

(c)

M1 Accept $2e^x-5=-2$ or $-2e^x+5=2 \Rightarrow x=..\ln(..)$

Allow squaring so $\left(2e^x-5\right)^2=4 \Rightarrow e^x=..\text{ and}.. \Rightarrow x=\ln(..), \ln(..)$

A1 $x=\ln\left(\frac{3}{2}\right)$ or exact equivalents such as $x=\ln 1.5$. You do not need to see the x.

Remember to isw a subsequent decimal answer 0.405

B1 $x=\ln\left(\frac{7}{2}\right)$ or exact equivalents such as $x=\ln 3.5$. You do not need to see the x.

Remember to isw a subsequent decimal answer 1.25

If both answers are given in decimals and there is no working $x=\text{awrt } 1.25, 0.405$ award SC 100

Question Number	Scheme	Marks
3(a)	$4\cos 2\theta + 2\sin 2\theta = R\cos(2\theta - \alpha)$	
	$R = \sqrt{4^2 + 2^2} = \sqrt{20} = \left(2\sqrt{5}\right)$	B1
	$\alpha = \arctan\left(\frac{1}{2}\right) = 26.565°... = \text{awrt } 26.57°$	M1A1
		(3)
(b)	$\sqrt{20}\cos(2\theta - 26.6) = 1 \Rightarrow \cos(2\theta - 26.57) = \frac{1}{\sqrt{20}}$	M1
	$\Rightarrow (2\theta - 26.57) = +77.1.. \Rightarrow \theta = ...$	dM1
	$\theta = \text{awrt } 51.8°$	A1
	$2\theta - 26.57 = \text{'}-77.1...\text{'} \Rightarrow \theta = -\text{awrt } 25.3°$	ddM1A1
		(5)
(c)	$k < -\sqrt{20}, k > \sqrt{20}$	B1ft either B1ft both
		(2)
		(10 marks)

You can marks parts (a) and (b) together as one.

(a)

B1 For $R = \sqrt{20} = 2\sqrt{5}$. Condone $R = \pm\sqrt{20}$

M1 For $\alpha = \arctan\left(\pm\frac{1}{2}\right)$ or $\alpha = \arctan(\pm 2)$ leading to a solution of α

Condone any solutions coming from $\cos\alpha = 4, \sin\alpha = 2$

Condone for this mark $2\alpha = \arctan\left(\pm\frac{1}{2}\right) \Rightarrow \alpha = ..$

If R has been used to find α award for only $\alpha = \arccos\left(\pm\frac{4}{\text{'}R\text{'}}\right)$ $\alpha = \arcsin\left(\pm\frac{2}{\text{'}R\text{'}}\right)$

A1 $\alpha = \text{awrt } 26.57°$

(b)

M1 Using part (a) and proceeding as far as $\cos\left(2\theta \pm \text{their } 26.57\right) = \dfrac{1}{\text{their } R}$.

This may be implied by $\left(2\theta \pm \text{their } 26.57\right) = \arccos\left(\dfrac{1}{\textit{theirR}}\right)$

Allow this mark for $\cos\left(\theta \pm \text{their } 26.57\right) = \dfrac{1}{\text{their } R}$

dM1 Dependent upon the first M1- it is for a correct method to find θ from their principal value
Look for the correct order of operations, that is dealing with the "26.57" before the "2".
Condone subtracting 26.57 instead of adding.
$\cos\left(2\theta \pm \text{their } 26.57\right) = ... \Rightarrow 2\theta \pm \text{their } 26.57 = \beta \Rightarrow \theta = \dfrac{\beta \pm \text{their } 26.57}{2}$

A1 awrt $\theta = 51.8°$

ddM1 For a correct method to find a secondary value of θ in the range
Either $2\theta \pm 26.57 = '-\beta' \Rightarrow \theta =$ OR $2\theta \pm 26.57 = 360 - '\beta' \Rightarrow \theta =$ THEN MINUS 180

A1 awrt $\theta = -25.3°$
Withhold this mark if there are extra solutions in the range.

Radian solution: Only lose the first time it occurs.

FYI. In radians desired accuracy is awrt 2 dp (a) $\alpha = 0.46$ and (b) $\theta_1 = 0.90, \theta_2 = -0.44$

Mixing degrees and radians only scores the first M

(c)

B1ft Follow through on their *R*. Accept decimals here including $\sqrt{20} \approx$ awrt 4.5 .

Score for one of the ends $k > \sqrt{20}, k < -\sqrt{20}$

Condone versions such as $g(\theta) > \sqrt{20}, y > \sqrt{20}$

or both ends including the boundaries $k \geqslant \sqrt{20}, k \leqslant -\sqrt{20}$

B1 ft For both intervals in terms of *k*.

Accept $k > \sqrt{20}$ or $k < -\sqrt{20}$. Accept $|k| > \sqrt{20}$ Accept $k \in \left(\sqrt{20}, \infty\right)\left(-\infty, -\sqrt{20}\right)$

Condone $k > \sqrt{20}, k < -\sqrt{20}$ $k > \sqrt{20}$ and $k < -\sqrt{20}$ for both marks

but $-\sqrt{20} > k > \sqrt{20}$ is B1 B0

Question Number	Scheme	Marks
4(a)	$(\theta =)20$	B1 **(1)**
(b)	Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$	
	$\Rightarrow e^{-40\lambda} = 0.5$	M1A1
	$\Rightarrow \lambda = \dfrac{\ln 2}{40}$	M1A1
		(4)
(c)	$\theta = 100 \Rightarrow T = \dfrac{\ln 0.2}{-\text{their}\,'\lambda'}$	M1
	$T = \text{awrt } 93$	A1
		(2)
		(7 marks)
Alt (b)	Sub $t = 40, \theta = 70 \Rightarrow 100e^{-40\lambda} = 50$	
	$\Rightarrow \ln 100 - 40\lambda = \ln 50$	M1A1
	$\Rightarrow \lambda = \dfrac{\ln 100 - \ln 50}{40} = \dfrac{\ln 2}{40}$	M1A1
		(4)

(a)

B1 Sight of $(\theta =)20$

(b)

M1 Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$ and proceed to $e^{\pm 40\lambda} = A$ where A is a constant. Allow sign slips and copying errors.

A1 $e^{-40\lambda} = 0.5$ or $e^{40\lambda} = 2$ or exact equivalent

M1 For undoing the e's by taking ln's and proceeding to $\lambda = ..$

May be implied by the correct decimal answer awrt 0.017 or $\lambda = \frac{\ln 0.5}{-40}$

A1 cso $\lambda = \frac{\ln 2}{40}$

Accept equivalents in the form $\frac{\ln a}{b}$, $a, b \in \mathbb{Z}$ such as $\lambda = \frac{\ln 4}{80}$

(c)

M1 Substitutes $\theta = 100$ and their numerical value of λ into $\theta = 120 - 100e^{-\lambda t}$ and proceed to $T = \pm \frac{\ln 0.2}{\text{their } '\lambda'}$ or $T = \pm \frac{\ln 5}{\text{their } '\lambda'}$ Allow inequalities here.

A1 awrt $T = 93$

Watch for candidates who lose the minus sign in (b) and use $\lambda = \frac{\ln \frac{1}{2}}{40}$ in (c). Many then reach $T = -93$ and ignore the minus. This is M1 A0

Question Number	Scheme	Marks
5.(a)	$p = 4\pi^2$ or $(2\pi)^2$	B1
		(1)
(b)	$x=(4y-\sin 2y)^2 \Rightarrow \frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	M1A1
	Sub $y=\frac{\pi}{2}$ into $\frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	
	$\Rightarrow \frac{dx}{dy}=24\pi$ (= 75.4) / $\frac{dy}{dx}=\frac{1}{24\pi}(=0.013)$	M1
	Equation of tangent $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$	M1
	Using $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$ with $x=0 \Rightarrow y=\frac{\pi}{3}$ cso	M1, A1
		(6)
		(7 marks)
Alt (b) I	$x=(4y-\sin 2y)^2 \Rightarrow x^{0.5}=4y-\sin 2y$	
	$\Rightarrow 0.5x^{-0.5}\frac{dx}{dy}=4-2\cos 2y$	M1A1
Alt (b) II	$x=\left(16y^2-8y\sin 2y+\sin^2 2y\right)$	
	$\Rightarrow 1=32y\frac{dy}{dx}-8\sin 2y\frac{dy}{dx}-16y\cos 2y\frac{dy}{dx}+4\sin 2y\cos 2y\frac{dy}{dx}$	M1A1
	Or $1dx=32y\,dy-8\sin 2y\,dy-16y\cos 2y\,dy+4\sin 2y\cos 2y\,dy$	

(a)

B1 $p = 4\pi^2$ or exact equivalent $(2\pi)^2$

Also allow $x = 4\pi^2$

(b)

M1 Uses the chain rule of differentiation to get a form $A(4y-\sin 2y)(B \pm C\cos 2y), \quad A,B,C \neq 0$ on the right hand side
Alternatively attempts to expand and then differentiate using product rule and chain rule to a form $x=\left(16y^2-8y\sin 2y+\sin^2 2y\right) \Rightarrow \frac{dx}{dy}=Py \pm Q\sin 2y \pm Ry\cos 2y \pm S\sin 2y\cos 2y \quad P,Q,R,S \neq 0$
A second method is to take the square root first. To score the method look for a differentiated expression of the form $Px^{-0.5}... = 4-Q\cos 2y$
A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.

A1 $\frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$ or $\frac{dy}{dx}=\frac{1}{2(4y-\sin 2y)(4-2\cos 2y)}$ with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs.
In the alternative $\frac{dx}{dy}=32y-8\sin 2y-16y\cos 2y+4\sin 2y\cos 2y$

M1 Sub $y=\frac{\pi}{2}$ into their $\frac{dx}{dy}$ or inverted $\frac{dx}{dy}$. Evidence could be minimal, eg $y=\frac{\pi}{2} \Rightarrow \frac{dx}{dy}=...$
It is not dependent upon the previous M1 but it must be a changed $x=(4y-\sin 2y)^2$

M1 Score for a correct method for finding the equation of the tangent at $\left('4\pi^2', \frac{\pi}{2}\right)$.

Allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical } \left(\frac{dx}{dy}\right)}\left(x-\text{their } 4\pi^2\right)$

Allow for $\left(y-\frac{\pi}{2}\right) \times \text{their numerical } \left(\frac{dx}{dy}\right)=\left(x-\text{their } 4\pi^2\right)$

Even allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical } \left(\frac{dx}{dy}\right)}(x-p)$

It is possible to score this by stating the equation $y=\frac{1}{24\pi}x+c$ as long as $\left('4\pi^2', \frac{\pi}{2}\right)$ is used in a subsequent line.

M1 Score for writing their equation in the form $y=mx+c$ and stating the value of 'c'
Or setting $x=0$ in their $y-\frac{\pi}{2}=\frac{1}{24\pi}\left(x-4\pi^2\right)$ and solving for y.
Alternatively using the gradient of the line segment AP = gradient of tangent.
Look for $\frac{\frac{\pi}{2}-y}{4\pi^2}=\frac{1}{24\pi} \Rightarrow y=..$ Such a method scores the previous M mark as well.
At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.

A1 cso $y=\frac{\pi}{3}$. You do not have to see $\left(0, \frac{\pi}{3}\right)$

Question Number	Scheme	Marks
6.(a)	$2^{x+1}-3=17-x\Rightarrow 2^{x+1}=20-x$	M1
	$(x+1)\ln 2=\ln(20-x)\Rightarrow x=\ldots$	dM1
	$x=\dfrac{\ln(20-x)}{\ln 2}-1$	A1*
		(3)
(b)	Sub $x_0=3$ into $x_{n+1}=\dfrac{\ln(20-x_n)}{\ln 2}-1,\Rightarrow$ $x_1=3.087$ (awrt)	M1A1
	$x_2=3.080,\ x_3=3.081$ (awrt)	A1
		(3)
(c)	$A=(3.1,13.9)$ cao	M1,A1
		(2) **(8 marks)**
6.(a)Alt	$2^{x+1}-3=17-x\Rightarrow 2^{x}=\dfrac{20-x}{2}$	M1
	$x\ln 2=\ln\dfrac{20-x}{2}\Rightarrow x=\ldots$	dM1
	$x=\dfrac{\ln(20-x)}{\ln 2}-1$	A1*
		(3)
6.(a) **backwards**	$x=\dfrac{\ln(20-x)}{\ln 2}-1\Rightarrow (x+1)\ln 2=\ln(20-x)$	M1
	$\Rightarrow 2^{x+1}=20-x$	dM1
	Hence $y=2^{x+1}-3$ meets $y=17-x$	A1*
		(3)

(a)

M1 Setting equations in x equal to each other and proceeding to make 2^{x+1} the subject

dM1 Take ln's or logs of both sides, use the power law and proceed to $x =$..

A1* This is a given answer and all aspects must be correct including ln or $\log_e$ rather than $\log_{10}$

Bracketing on both $(x+1)$ and $\ln(20-x)$ must be correct.

Eg $x+1\ln 2 = \ln(20-x) \Rightarrow x = \dfrac{\ln(20-x)}{\ln 2} - 1$ is A0*

Special case: Students who start from the point $2^{x+1} = 20 - x$ can score M1 dM1A0*

(b)

M1 Sub $x_0 = 3$ into $x_{n+1} = \dfrac{\ln(20-x_n)}{\ln 2} - 1$ to find $x_1 =$..

Accept as evidence $x_1 = \dfrac{\ln(20-3)}{\ln 2} - 1$, awrt $x_1 = 3.1$

Allow $x_0 = 3$ into the miscopied iterative equation $x_1 = \dfrac{\ln(20-3)}{\ln 2}$ to find $x_1 =$..

Note that the answer to this, 4.087, on its own without sight of $\dfrac{\ln(20-3)}{\ln 2}$ is M0

A1 awrt 3 dp $x_1 = 3.087$

A1 awrt $x_2 = 3.080$, $x_3 = 3.081$. Tolerate 3.08 for 3.080

Note that the subscripts are not important, just mark in the order seen

(c) Note that this appears as B1B1 on e pen. It is marked M1A1

M1 For sight of 3.1

Alternatively it can be scored for substituting their value of x or a rounded value of x from (b) into either $2^{x+1} - 3$ or $17 - x$ to find the y coordinate.

A1 $(3.1, 13.9)$

Question Number	Scheme	Marks
7.(a)	Applies $vu'+uv'$ to $\left(x^2-x^3\right)e^{-2x}$	
	$g'(x)=\left(x^2-x^3\right)\times -2e^{-2x}+\left(2x-3x^2\right)\times e^{-2x}$	M1 A1
	$g'(x)=\left(2x^3-5x^2+2x\right)e^{-2x}$	A1
		(3)
(b)	Sets $\left(2x^3-5x^2+2x\right)e^{-2x}=0 \Rightarrow 2x^3-5x^2+2x=0$	M1
	$x\left(2x^2-5x+2\right)=0 \Rightarrow x=(0),\frac{1}{2},2$	M1,A1
	Sub $x=\frac{1}{2}$, 2 into $g(x)=\left(x^2-x^3\right)e^{-2x} \Rightarrow g\left(\frac{1}{2}\right)=\frac{1}{8e},\quad g(2)=-\frac{4}{e^4}$	dM1,A1
	Range $-\frac{4}{e^4}\leqslant g(x)\leqslant\frac{1}{8e}$	A1
		(6)
(c)	Accept $g(x)$ is NOT a ONE to ONE function	
	Accept $g(x)$ is a MANY to ONE function	B1
	Accept $g^{-1}(x)$ would be ONE to MANY	**(1)**
		(10 marks)

Note that parts (a) and (b) can be scored together. Eg accept work in part (b) for part (a)

(a)

M1 **Uses the product rule** $vu'+uv'$ with $u=x^2-x^3$ and $v=e^{-2x}$ or vice versa. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $vu'+uv'$. If the rule is not quoted nor implied only accept expressions of the form $\left(x^2-x^3\right)\times \pm Ae^{-2x}+\left(Bx\pm Cx^2\right)\times e^{-2x}$ condoning bracketing issues

Method 2: multiplies out and **uses the product rule** on each term of $x^2e^{-2x}-x^3e^{-2x}$
Condone issues in the signs of the last two terms for the method mark
Uses the product rule for $uvw=u'vw+uv'w+uvw'$ applied as in method 1

Method 3:Uses **the quotient rule** with $u=x^2-x^3$ and $v=e^{2x}$. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $\frac{vu'-uv'}{v^2}$ If the rule is not quoted nor implied accept expressions of the form $\frac{e^{2x}\left(Ax-Bx^2\right)-\left(x^2-x^3\right)\times Ce^{2x}}{\left(e^{2x}\right)^2}$

condoning missing brackets on the numerator and e^{2x^2} on the denominator.

Method 4: Apply implicit differentiation to $ye^{2x}=x^2-x^3 \Rightarrow e^{2x}\times\frac{dy}{dx}+y\times 2e^{2x}=2x-3x^2$

Condone errors on coefficients and signs

A1 A correct (unsimplified form) of the answer

$g'(x) = (x^2 - x^3) \times -2e^{-2x} + (2x - 3x^2) \times e^{-2x}$ by one use of the product rule

or $g'(x) = x^2 \times -2e^{-2x} + 2xe^{-2x} - x^3 \times -2e^{-2x} - 3x^2 \times e^{-2x}$ using the first alternative

or $g'(x) = 2x(1-x)e^{-2x} + x^2 \times -1 \times e^{-2x} + x^2(1-x) \times -2e^{-2x}$ using the product rule on 3 terms

or $g'(x) = \dfrac{e^{2x}(2x - 3x^2) - (x^2 - x^3) \times 2e^{2x}}{(e^{2x})^2}$ using the quotient rule.

A1 Writes $g'(x) = (2x^3 - 5x^2 + 2x)e^{-2x}$. You do not need to see $f(x)$ stated and award even if a correct $g'(x)$ is followed by an incorrect $f(x)$. If the f(x) is not simplified at this stage you need to see it simplified later for this to be awarded.

(b) Note: The last mark in e-pen has been changed from a 'B' to an A mark

M1 For setting their f(x) =0. The = 0 may be implied by subsequent working.
Allow even if the candidate has failed to reach a 3TC for f(x).
Allow for $f(x) \geqslant 0$ or $f(x) \leqslant 0$ as they can use this to pick out the relevant sections of the curve

M1 For solving their 3TC = 0 by ANY correct method.
Allow for division of x or factorising out the x followed by factorisation of 3TQ. Check first and last terms of the 3TQ. Allow for solutions from either $f(x) \geqslant 0$ or $f(x) \leqslant 0$
Allow solutions **from the cubic equation** just appearing from a Graphical Calculator

A1 $x = \frac{1}{2}, 2$. Correct answers from a correct $g'(x)$ would imply all 3 marks so far in (b)

dM1 Dependent upon both previous M's being scored. For substituting their **two** (non zero) values of x into g(x) to find both y values. Minimal evidence is required $x = .. \Rightarrow y = ..$ is OK.

A1 Accept decimal answers for this mark. $g\left(\frac{1}{2}\right) = \frac{1}{8e}$ =awrt 0.046 **AND** $g(2) = -\frac{4}{e^4}$ =awrt -0.073

A1 CSO Allow $-\frac{4}{e^4} \leqslant \text{Range} \leqslant \frac{1}{8e}$, $-\frac{4}{e^4} \leqslant y \leqslant \frac{1}{8e}$, $\left[-\frac{4}{e^4}, \frac{1}{8e}\right]$. Condone $y \geqslant -\frac{4}{e^4}$ $y \leqslant \frac{1}{8e}$

Note that the question states hence and part (a) must have been used for all marks. Some students will just write down the answers for the range from a graphical calculator.
Seeing just $-\frac{4}{e^4} \leqslant g(x) \leqslant \frac{1}{8e}$ or $-0.073 \leqslant g(x) \leqslant 0.046$ special case 100000.
They know what a range is!

(c)

B1 If the candidate states 'NOT ONE TO ONE' then accept unless the explicitly link it to $g^{-1}(x)$.
So accept 'It is not a one to one function'. 'The function is not one to one' '$g(x)$ is not one to one'
If the candidate states 'IT IS MANY TO ONE' then accept unless the candidate explicitly links it to $g^{-1}(x)$. So accept 'It is a many to one function.' 'The function is many to one' '$g(x)$ is many to one'
If the candidate states 'IT IS ONE TO MANY' then accept unless the candidate explicitly links it to $g(x)$
Accept an explanation like " one value of x would map/ go to more than one value of y"
Incorrect statements scoring B0 would be $g^{-1}(x)$ is not one to one, $g^{-1}(x)$ is many to one and $g(x)$ is one to many.

Question Number	Scheme	Marks
8(a)	$\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$	B1
	$= \dfrac{1+\sin 2A}{\cos 2A}$	M1
	$= \dfrac{1+2\sin A\cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \dfrac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	
	$= \dfrac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}$	M1
	$= \dfrac{\cos A + \sin A}{\cos A - \sin A}$	A1*
		(5)
(b)	$\sec 2\theta + \tan 2\theta = \dfrac{1}{2} \Rightarrow \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \dfrac{1}{2}$	
	$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$	
	$\Rightarrow \tan\theta = -\dfrac{1}{3}$	M1 A1
	$\Rightarrow \theta = awrt\ 2.820, 5.961$	dM1A1
		(4)
		(9 marks)

(a)

B1 A correct identity for $\sec 2A = \dfrac{1}{\cos 2A}$ **OR** $\tan 2A = \dfrac{\sin 2A}{\cos 2A}$.

It need not be in the proof and it could be implied by the sight of $\sec 2A = \dfrac{1}{\cos^2 A - \sin^2 A}$

M1 For setting their expression as a single fraction. The denominator must be correct for their fractions and at least two terms on the numerator.

This is usually scored for $\dfrac{1+\cos 2A\tan 2A}{\cos 2A}$ or $\dfrac{1+\sin 2A}{\cos 2A}$

M1 For getting an expression in just sin A and cos A by using the double angle identities

$\sin 2A = 2\sin A\cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$, $2\cos^2 A - 1$ or $1 - 2\sin^2 A$.

Alternatively for getting an expression in just sin A and cos A by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ with $\tan A = \dfrac{\sin A}{\cos A}$.

For example $= \dfrac{1}{\cos^2 A - \sin^2 A} + \dfrac{2\sin A / \cos A}{1 - \sin^2 A / \cos^2 A}$ is B1M0M1 so far

M1 In the main scheme it is for replacing 1 by $\cos^2 A + \sin^2 A$ **and** factorising both numerator and denominator

A1* Cancelling to produce given answer with no errors.
Allow a consistent use of another variable such as θ, but mixing up variables will lose the A1*.

(b)
M1 For using part (a), cross multiplying, dividing by $\cos\theta$ to reach $\tan\theta = k$
Condone $\tan 2\theta = k$ for this mark only

A1 $\tan\theta = -\frac{1}{3}$

dM1 Scored for $\tan\theta = k$ leading to at least one value (with 1 dp accuracy) for θ between 0 and 2π. You may have to use a calculator to check. Allow answers in degrees for this mark.

A1 $\theta =$ awrt $2.820, 5.961$ with no extra solutions within the range. Condone 2.82 for 2.820.

You may condone different/ mixed variables in part (b)

....

There are some long winded methods. Eg. M1, dM1 applied as in main scheme

$$\Rightarrow (2\cos\theta + 2\sin\theta)^2 = (\cos\theta - \sin\theta)^2 \Rightarrow 4 + 4\sin 2\theta = 1 - \sin 2\theta$$

$$\Rightarrow \sin 2\theta = -\frac{3}{5} \text{ is M1 (for } \sin 2\theta = k\text{) A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ for dM1 (for } \theta = \frac{\arcsin k}{2}\text{) A1}$$

....

$$\cos\theta + 3\sin\theta = 0 \Rightarrow (\sqrt{10})\cos(\theta - 1.25) = 0 \text{ M1 for..} \cos(\theta - \alpha) = 0, \alpha = \arctan\left(\pm\frac{3}{1} \text{ or } \pm\frac{1}{3}\right)\text{) A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

$$\cos\theta + 3\sin\theta = 0 \Rightarrow (\sqrt{10})\sin(\theta + 0.32) = 0 \text{ M1 A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

....

$$\cos\theta = -3\sin\theta \Rightarrow \cos^2\theta = 9\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{10} \Rightarrow \sin\theta = (\pm)\sqrt{\frac{1}{10}} \text{ M1 A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

$$\cos\theta = -3\sin\theta \Rightarrow \cos^2\theta = 9\sin^2\theta \Rightarrow \cos^2\theta = \frac{9}{10} \Rightarrow \cos\theta = (\pm)\sqrt{\frac{9}{10}} \text{ M1 A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

....

Question Number	Scheme	Marks
Alt I **From RHS**	$\dfrac{\cos A+\sin A}{\cos A-\sin A} = \dfrac{\cos A+\sin A}{\cos A-\sin A}\times\dfrac{\cos A+\sin A}{\cos A+\sin A}$ $=\dfrac{\cos^2 A+\sin^2 A+2\sin A\cos A}{\cos^2 A-\sin^2 A}$ $=\dfrac{1+\sin 2A}{\cos 2A}$ $=\dfrac{1}{\cos 2A}+\dfrac{\sin 2A}{\cos 2A}$ $=\sec 2A+\tan 2A$	(Pythagoras) **M1** (Double Angle) **M1** (Single Fraction) **M1** **B1**(Identity), **A1***
Alt II **Both sides**	Assume true $\sec 2A+\tan 2A=\dfrac{\cos A+\sin A}{\cos A-\sin A}$ $\dfrac{1}{\cos 2A}+\dfrac{\sin 2A}{\cos 2A}=\dfrac{\cos A+\sin A}{\cos A-\sin A}$ $\dfrac{1+\sin 2A}{\cos 2A}=\dfrac{\cos A+\sin A}{\cos A-\sin A}$ $\dfrac{1+2\sin A\cos A}{\cos^2 A-\sin^2 A}=\dfrac{\cos A+\sin A}{\cos A-\sin A}$ $\times(\cos A-\sin A)\Rightarrow\dfrac{1+2\sin A\cos A}{\cos A+\sin A}=\cos A+\sin A$ $1+2\sin A\cos A=\cos^2 A+2\sin A\cos A+\sin^2 A=1+2\sin A\cos A$ **True**	B1 (identity) M1 (single fraction) M1(double angles) M1(Pythagoras)A1*
Alt 111 **Very difficult**	$\sec 2A+\tan 2A=\dfrac{1}{\cos 2A}+\tan 2A$ $=\dfrac{1}{\cos 2A}+\dfrac{2\tan A}{1-\tan^2 A}$ $=\dfrac{1-\tan^2 A+2\tan A\cos 2A}{\cos 2A(1-\tan^2 A)}$ $=\dfrac{1-\tan^2 A+2\tan A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(1-\tan^2 A)}$ $=\dfrac{1-\dfrac{\sin^2 A}{\cos^2 A}+2\dfrac{\sin A}{\cos A}(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)\left(1-\dfrac{\sin^2 A}{\cos^2 A}\right)}$ $\times\cos^2 A=\dfrac{\cos^2 A-\sin^2 A+2\sin A\cos A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(\cos^2 A-\sin^2 A)}$ $=\dfrac{\cancel{(\cos^2 A-\sin^2 A)}(1+2\sin A\cos A)}{\cancel{(\cos^2 A-\sin^2 A)}(\cos^2 A-\sin^2 A)}$ Final two marks as in main scheme	(Identity) **B1** (Single fraction) **M1** (Double Angle and in just sin and cos) **M1** **M1A1***

Question Number	Scheme	Marks
9.(a)	$x^2-3kx+2k^2=(x-2k)(x-k)$	B1
	$2-\dfrac{(x-5k)(x-k)}{(x-2k)(x-k)}=2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$	M1
	$=\dfrac{x+k}{(x-2k)}$	A1*
		(3)
(b)	Applies $\dfrac{vu'-uv'}{v^2}$ to $y=\dfrac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$	
	$\Rightarrow \mathrm{f}'(x)=\dfrac{(x-2k)\times1-(x+k)\times1}{(x-2k)^2}$	M1, A1
	$\Rightarrow \mathrm{f}'(x)=\dfrac{-3k}{(x-2k)^2}$	A1
		(3)
(c)	If $\mathrm{f}'(x)=\dfrac{-Ck}{(x-2k)^2}\Rightarrow \mathrm{f}(x)$ is an increasing function as $\mathrm{f}'(x)>0$,	M1
	$\mathrm{f}'(x)=\dfrac{-3k}{(x-2k)^2}>0$ for all values of x as $\dfrac{\text{negative}\times\text{negative}}{\text{positive}}=\text{positive}$	A1
		(2)
		(8 marks)

(a)

B1 For seeing $x^2-3kx+2k^2=(x-2k)(x-k)$ anywhere in the solution

M1 For writing as a single term or two terms with the same denominator

Score for $2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$ or

$$2-\frac{(x-5k)(x-k)}{(x-2k)(x-k)}=\frac{2(x-2k)(x-k)-(x-5k)(x-k)}{(x-2k)(x-k)} \qquad \left(=\frac{x^2-k^2}{x^2-3kx+2k^2}\right)$$

A1* Proceeds without any errors (including bracketing) to $=\dfrac{x+k}{(x-2k)}$

(b)

M1 Applies $\frac{vu'-uv'}{v^2}$ to $y=\frac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$.

If the rule it is stated it must be correct. It can be implied by $u=x+k$ and $v=x-2k$ with their u', v' and $\frac{vu'-uv'}{v^2}$

If it is neither stated nor implied only accept expressions of the form $\mathrm{f}'(x)=\frac{x-2k-x\pm k}{(x-2k)^2}$

The mark can be scored for applying the product rule to $y=(x+k)(x-2k)^{-1}$ If the rule it is stated it must be correct. It can be implied by $u=x+k$ and $v=(x-2k)^{-1}$ with their u', v' and $vu'+uv'$

If it is neither stated nor implied only accept expressions of the form
$\mathrm{f}'(x)=(x-2k)^{-1}\pm(x+k)(x-2k)^{-2}$

Alternatively writes $y=\frac{x+k}{x-2k}$ as $y=1+\frac{3k}{x-2k}$ and differentiates to $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{A}{(x-2k)^2}$

A1 Any correct form (unsimplified) form of $\mathrm{f}'(x)$.

$\mathrm{f}'(x)=\frac{(x-2k)\times 1-(x+k)\times 1}{(x-2k)^2}$ by quotient rule

$\mathrm{f}'(x)=(x-2k)^{-1}-(x+k)(x-2k)^{-2}$ by product rule

and $\mathrm{f}'(x)=\frac{-3k}{(x-2k)^2}$ by the third method

A1 cao $\mathrm{f}'(x)=\frac{-3k}{(x-2k)^2}$. Allow $\mathrm{f}'(x)=\frac{-3k}{x^2-4kx+4k^2}$

As this answer is not given candidates you may allow recovery from missing brackets

(c) Note that this is B1 B1 on e pen. We are scoring it M1 A1

M1 If in part (b) $\mathrm{f}'(x)=\frac{-Ck}{(x-2k)^2}$, look for f($x$) is an increasing function as $\mathrm{f}'(x)$ / gradient >0

Accept a version that states as $k<0\Rightarrow -Ck>0$ hence increasing

If in part (b) $\mathrm{f}'(x)=\frac{(+)Ck}{(x-2k)^2}$, look for f($x$) is an decreasing function as $\mathrm{f}'(x)$ / gradient <0

Similarly accept a version that states as $k<0\Rightarrow (+)Ck<0$ hence decreasing

A1 Must have $\mathrm{f}'(x)=\frac{-3k}{(x-2k)^2}$ and give a reason that links the gradient with its sign.

There must have been reference to the sign of both numerator and denominator to justify the overall positive sign.

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