

Mark Scheme (Results)

January 2009

GCE

GCE Mathematics (6667/01)

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6667 Further Pure Mathematics FP1 (new)
Mark Scheme

Question Number	Scheme	Marks
1	$x - 3$ is a factor $f(x) = (x - 3)(2x^2 - 2x + 1)$ Attempt to solve quadratic i.e. $x = \frac{2 \pm \sqrt{4 - 8}}{4}$ $x = \frac{1 \pm i}{2}$	B1 M1 A1 M1 A1 [5]

Notes:

First and last terms in second bracket required for first M1

Use of correct quadratic formula for their equation for second M1

Question Number	Scheme	Marks
2	<p>(a)</p> $6\sum r^2 + 4\sum r - \sum 1 = 6\frac{n}{6}(n+1)(2n+1) + 4\frac{n}{2}(n+1) - n$ $= \frac{n}{6}(12n^2 + 18n + 6 + 12n + 12 - 6) \text{ or } n(n+1)(2n+1) + (2n+1)n$ $= \frac{n}{6}(12n^2 + 30n + 12) = n(2n^2 + 5n + 2) = n(n+2)(2n+1) \quad *$ <p>(b)</p> $\sum_{r=1}^{20} (6r^2 + 4r - 1) - \sum_{r=1}^{10} (6r^2 + 4r - 1) = 20 \times 22 \times 41 - 10 \times 12 \times 21$ $= 15520$	<p>M1 A1, B1</p> <p>M1</p> <p>A1</p> <p>(5)</p> <p>M1</p> <p>A1</p> <p>(2)</p> <p>[7]</p>

Notes:

(a) First M1 for first 2 terms, B1 for $-n$
 Second M1 for attempt to expand and gather terms.
 Final A1 for correct solution only

(b) Require (r from 1 to 20) subtract (r from 1 to 10) and attempt to substitute for M1

Question Number	Scheme	Marks
3	(a) $xy = 25 = 5^2$ or $c = \pm 5$	B1 (1)
	(b) A has co-ords (5, 5) and B has co-ords (25, 1)	B1
	Mid point is at (15, 3)	M1A1 (3) [4]

Notes:

(a) $xy = 25$ only B1, $c^2 = 25$ only B1, $c = 5$ only B1

(b) Both coordinates required for B1
Add theirs and divide by 2 on both for M1

Question Number	Scheme	Marks
4	<p>When $n = 1$, $\text{LHS} = \frac{1}{1 \times 2} = \frac{1}{2}$, $\text{RHS} = \frac{1}{1+1} = \frac{1}{2}$. So $\text{LHS} = \text{RHS}$ and result true for $n = 1$</p> <p>Assume true for $n = k$; $\sum_{r=1}^k \frac{1}{r(r+1)} = \frac{k}{k+1}$ and so $\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$</p> $\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{k+2}$ <p>and so result is true for $n = k + 1$ (and by induction true for $n \in \mathbf{Z}^+$)</p>	<p>B1</p> <p>M1</p> <p>M1 A1</p> <p>B1 [5]</p>

Notes:

Evaluate both sides for first B1

Final two terms on second line for first M1

Attempt to find common denominator for second M1.

Second M1 dependent upon first.

$\frac{k+1}{k+2}$ for A1

‘Assume true for $n = k$ ’ and ‘so result true for $n = k + 1$ ’ and correct solution for final B1

Question Number	Scheme	Marks
5	<p>(a) attempt evaluation of $f(1.1)$ and $f(1.2)$ (– looking for sign change)</p> <p>$f(1.1) = 0.30875$, $f(1.2) = -0.28199$ Change of sign in $f(x) \Rightarrow$ root in the interval</p> <p>(b) $f'(x) = \frac{3}{2}x^{-\frac{1}{2}} - 9x^{-1\frac{1}{2}}$</p> <p>(c) $f(1.1) = 0.30875..$ $f'(1.1) = -6.37086...$</p> <p>$x_1 = 1.1 - \frac{0.30875...}{-6.37086..}$</p> <p>$= 1.15(\text{to 3 sig.figs.})$</p>	<p>M1</p> <p>A1 (2)</p> <p>M1 A1 A1 (3)</p> <p>B1 B1</p> <p>M1</p> <p>A1 (4)</p> <p>[9]</p>

Notes:

(a) awrt 0.3 and -0.3 and indication of sign change for first A1

(b) Multiply by power and subtract 1 from power for evidence of differentiation and award of first M1

(c) awrt 0.309 B1 and awrt -6.37 B1 if answer incorrect

Evidence of Newton-Raphson for M1

Evidence of Newton-Raphson and awrt 1.15 award 4/4

Question Number	Scheme	Marks
6	<p>At $n = 1$, $u_n = 5 \times 6^0 + 1 = 6$ and so result true for $n = 1$</p> <p>Assume true for $n = k$; $u_k = 5 \times 6^{k-1} + 1$, and so $u_{k+1} = 6(5 \times 6^{k-1} + 1) - 5$</p> <p>$\therefore u_{k+1} = 5 \times 6^k + 6 - 5 \quad \therefore u_{k+1} = 5 \times 6^k + 1$</p> <p>and so result is true for $n = k + 1$ and by induction true for $n \geq 1$</p>	<p>B1</p> <p>M1, A1</p> <p>A1</p> <p>B1</p> <p>[5]</p>

Notes:

6 and so result true for $n = 1$ award B1

Sub u_k into u_{k+1} or M1 and A1 for correct expression on right hand of line 2

Second A1 for $\therefore u_{k+1} = 5 \times 6^k + 1$

‘Assume true for $n = k$ ’ and ‘so result is true for $n = k + 1$ ’ and correct solution for final B1

Question Number	Scheme	Marks
7 (a)	The determinant is $a - 2$	M1
	$\mathbf{X}^{-1} = \frac{1}{a-2} \begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$	M1 A1 (3)
(b)	$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B1
	Attempt to solve $2 - \frac{1}{a-2} = 1$, or $a - \frac{a}{a-2} = 0$, or $-1 + \frac{1}{a-2} = 0$, or $-1 + \frac{2}{a-2} = 1$	M1
	To obtain $a = 3$ only	A1 cso (3) [6]
	Alternatives for (b) If they use $\mathbf{X}^2 + \mathbf{I} = \mathbf{X}$ they need to identify \mathbf{I} for B1, then attempt to solve suitable equation for M1 and obtain $a = 3$ for A1 If they use $\mathbf{X}^2 + \mathbf{X}^{-1} = \mathbf{O}$, they can score the B1 then marks for solving If they use $\mathbf{X}^3 + \mathbf{I} = \mathbf{O}$ they need to identify \mathbf{I} for B1, then attempt to solve suitable equation for M1 and obtain $a = 3$ for A1	

Notes:

(a) Attempt $ad-bc$ for first M1 $\frac{1}{\det} \begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$ for second M1

(b) Final A1 for correct solution only

Question Number	Scheme	Marks
8	<p>(a) $\frac{dy}{dx} = a^{\frac{1}{2}} x^{-\frac{1}{2}}$ or $2y \frac{dy}{dx} = 4a$</p> <p>The gradient of the tangent is $\frac{1}{q}$</p> <p>The equation of the tangent is $y - 2aq = \frac{1}{q}(x - aq^2)$</p> <p>So $yq = x + aq^2$ *</p> <p>(b) R has coordinates $(0, aq)$</p> <p>The line l has equation $y - aq = -qx$</p> <p>(c) When $y = 0$ $x = a$ (so line l passes through $(a, 0)$ the focus of the parabola.)</p> <p>(d) Line l meets the directrix when $x = -a$: Then $y = 2aq$. So coordinates are $(-a, 2aq)$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1A1</p> <p>B1</p> <p>M1:A1</p> <p>(4)</p> <p>(3)</p> <p>(1)</p> <p>(2)</p> <p>[10]</p>

Notes:

(a) $\frac{dy}{dx} = \frac{2a}{2aq}$ OK for M1

Use of $y = mx + c$ to find c OK for second M1

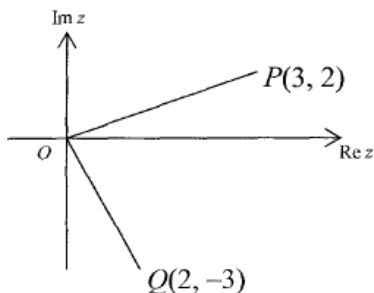
Correct solution only for final A1

(b) $-1/(\text{their gradient in part a})$ in equation OK for M1

(c) They must attempt $y = 0$ or $x = a$ to show correct coordinates of R for B1

(d) Substitute $x = -a$ for M1.

Both coordinates correct for A1.

Question Number	Scheme	Marks
9	<p>(a) $z_2 = \frac{12-5i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{36-24i-15i-10}{13}$ $= 2-3i$</p> <hr/> <p>(b)  P: B1, Q: B1ft</p> <hr/> <p>(c) $\text{grad. } OP \times \text{grad. } OQ = \frac{2}{3} \times -\frac{3}{2}$ $= -1 \Rightarrow \angle POQ = \frac{\pi}{2} \quad (*)$</p> <p>OR $\angle POX = \tan^{-1} \frac{2}{3}, \angle QOX = \tan^{-1} \frac{3}{2}$ $\tan(\angle POQ) = \frac{\frac{2}{3} + \frac{3}{2}}{1 - \frac{2}{3} \times \frac{3}{2}} \quad \text{M1}$ $\Rightarrow \angle POQ = \frac{\pi}{2} \quad (*) \quad \text{A1}$</p> <hr/> <p>(d) $z = \frac{3+2}{2} + \frac{2+(-3)}{2}i$ $= \frac{5}{2} - \frac{1}{2}i$</p> <hr/> <p>(e) $r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$ $= \frac{\sqrt{26}}{2} \text{ or exact equivalent}$</p>	<p>M1 A1 (2)</p> <p>B1, B1ft (2)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2) [10]</p>

Notes:

(a) $\times \frac{3-2i}{3-2i}$ for M1

(b) Position of points not clear award B1B0

(c) Use of calculator / decimals award M1A0

(d) Final answer must be in complex form for A1

(e) Radius or diameter for M1

Question Number	Scheme	Marks
10	<p>(a) A represents an enlargement scale factor $3\sqrt{2}$ (centre O)</p> <p>B represents reflection in the line $y = x$</p> <p>C represents a rotation of $\frac{\pi}{4}$, i.e. 45° (anticlockwise) (about O)</p> <p>(b) $\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}$</p> <p>(c) $\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$</p> <p>(d) $\begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 0 & -15 & 4 \\ 0 & 15 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 90 & 51 \\ 0 & 0 & 75 \end{pmatrix}$ so $(0, 0)$, $(90, 0)$ and $(51, 75)$</p> <p>(e) Area of $\triangle OR'S'$ is $\frac{1}{2} \times 90 \times 75 = 3375$</p> <p>Determinant of E is -18 or use area scale factor of enlargement So area of $\triangle ORS$ is $3375 \div 18 = 187.5$</p>	<p>M1 A1</p> <p>B1 B1 (4)</p> <p>M1 A1 (2)</p> <p>B1 (1)</p> <p>M1A1A1A1 (4)</p> <p>B1</p> <p>M1A1 (3) [14]</p>

Notes:

(a) Enlargement for M1

 $3\sqrt{2}$ for A1(b) Answer incorrect, require **CD** for M1(c) Answer given so require **DB** as shown for B1(d) Coordinates as shown or written as $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 90 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 51 \\ 75 \end{pmatrix}$ for each A1

(e) 3375 B1

Divide by theirs for M1