

Mark Scheme (Results)

January 2011

GCE

GCE Further Pure Mathematics FP1 (6667) Paper 1



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General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M marks:** method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A marks:** Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B marks** are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod - benefit of doubt
- ft - follow through
- the symbol \checkmark will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- \square The second mark is dependent on gaining the first mark

January 2011
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme	Marks
1.	$z = 5 - 3i, w = 2 + 2i$ $z^2 = (5 - 3i)(5 - 3i)$ $= 25 - 15i - 15i + 9i^2$ $= 25 - 15i - 15i - 9$ $= 16 - 30i$	<p>An attempt to multiply out the brackets to give four terms (or four terms implied). zw is M0</p> <p>M1</p> <p>16 - 30i A1</p> <p>Answer only 2/2 (2)</p>
(b)	$\frac{z}{w} = \frac{(5 - 3i)}{(2 + 2i)}$ $= \frac{(5 - 3i)}{(2 + 2i)} \times \frac{(2 - 2i)}{(2 - 2i)}$ $= \frac{10 - 10i - 6i - 6}{4 + 4}$ $= \frac{4 - 16i}{8}$ $= \frac{1}{2} - 2i$	<p>Multiplies $\frac{z}{w}$ by $\frac{(2 - 2i)}{(2 - 2i)}$</p> <p>M1</p> <p>Simplifies realising that a real number is needed on the denominator and applies $i^2 = -1$ on their numerator expression and denominator expression.</p> <p>M1</p> <p>$\frac{1}{2} - 2i$ or $a = \frac{1}{2}$ and $b = -2$ or equivalent A1</p> <p>Answer as a single fraction A0</p> <p>(3) [5]</p>

Question Number	Scheme	Marks
2.		
(a)	$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$ $\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$ $= \begin{pmatrix} 2(-3) + 0(5) & 2(-1) + 0(2) \\ 5(-3) + 3(5) & 5(-1) + 3(2) \end{pmatrix}$ $= \begin{pmatrix} -6 & -2 \\ 0 & 1 \end{pmatrix}$	<p>A correct method to multiply out two matrices. Can be implied by two out of four correct elements. M1</p> <p>Any three elements correct A1</p> <p>Correct answer A1</p> <p>Correct answer only 3/3 (3)</p>
(b)	<p>Reflection; about the y-axis.</p> <p style="text-align: right;"><u>Reflection</u> <u>y-axis</u> (or $x = 0$.)</p>	<p>M1</p> <p>A1</p> <p>(2)</p>
(c)	$\mathbf{C}^{100} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	<p>$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or \mathbf{I} B1</p> <p>(1)</p> <p>[6]</p>

Question Number	Scheme	Marks
3.		
(a)	$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geq 0$ $f(1.6) = -1.29543081...$ $f(1.8) = 0.5401863372...$ $\frac{\alpha - 1.6}{\text{"1.29543081..."}} = \frac{1.8 - \alpha}{\text{"0.5401863372..."}}$ $\alpha = 1.6 + \left(\frac{\text{"1.29543081..."}}{\text{"0.5401863372..." + "1.29543081..."}} \right) 0.2$ $= 1.741143899...$	awrt -1.30 B1 awrt 0.54 B1 Correct linear interpolation method with signs correct. Can be implied by working below. M1 awrt 1.741 A1 Correct answer seen 4/4 (4)
(b)	$f'(x) = 10x - 6x^{\frac{1}{2}}$	At least one of $\pm ax$ or $\pm bx^{\frac{1}{2}}$ correct. M1 Correct differentiation. A1 (2)
(c)	$f(1.7) = -0.4161152711...$ $f'(1.7) = 9.176957114...$ $\alpha_2 = 1.7 - \left(\frac{\text{"-0.4161152711..."}}{\text{"9.176957114..."}} \right)$ $= 1.745343491...$ $= 1.745 \text{ (3dp)}$	$f(1.7) =$ awrt -0.42 B1 $f'(1.7) =$ awrt 9.18 B1 Correct application of Newton-Raphson formula using their values. M1 1.745 A1 cao Correct answer seen 4/4 (4) [10]

Question Number	Scheme	Marks
4.	$z^2 + pz + q = 0, z_1 = 2 - 4i$	
(a)	$z_2 = 2 + 4i$	B1 (1)
(b)	$(z - 2 + 4i)(z - 2 - 4i) = 0$ $\Rightarrow z^2 - 2z - 4iz - 2z + 4 - 8i + 4iz - 8i + 16 = 0$ $\Rightarrow z^2 - 4z + 20 = 0$	<p>An attempt to multiply out brackets of two complex factors and no i^2. Any one of $p = -4, q = 20$. Both $p = -4, q = 20$. $\Rightarrow z^2 - 4z + 20 = 0$ only 3/3</p> M1 A1 A1 (3) [4]

Question Number	Scheme	Marks
5	$\sum_{r=1}^n r(r+1)(r+5)$ <p>(a) $= \sum_{r=1}^n r^3 + 6r^2 + 5r$</p> $= \frac{1}{4}n^2(n+1)^2 + 6 \cdot \frac{1}{6}n(n+1)(2n+1) + 5 \cdot \frac{1}{2}n(n+1)$ $= \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + \frac{5}{2}n(n+1)$ $= \frac{1}{4}n(n+1)(n(n+1) + 4(2n+1) + 10)$ $= \frac{1}{4}n(n+1)(n^2 + n + 8n + 4 + 10)$ $= \frac{1}{4}n(n+1)(n^2 + 9n + 14)$	<p>Multiplying out brackets and an attempt to use at least one of the standard formulae correctly. M1</p> <p><u>Correct expression.</u> A1</p> <p>Factorising out at least $n(n+1)$ dM1</p> <p>Correct 3 term quadratic factor A1</p>
	$= \frac{1}{4}n(n+1)(n+2)(n+7) *$	<p>Correct proof. No errors seen. A1</p> <p>(5)</p>
(b)	$S_n = \sum_{r=20}^{50} r(r+1)(r+5)$ $= S_{50} - S_{19}$ $= \frac{1}{4}(50)(51)(52)(57) - \frac{1}{4}(19)(20)(21)(26)$ $= 1889550 - 51870$ $= 1837680$	<p>Use of $S_{50} - S_{19}$ M1</p> <p>1837680 A1</p> <p>Correct answer only 2/2</p> <p>(2) [7]</p>

Question Number	Scheme	Marks
6.	$C: y^2 = 36x \Rightarrow a = \frac{36}{4} = 9$	
(a)	$S(9, 0)$ (9, 0)	B1 (1)
(b)	$x + 9 = 0$ or $x = -9$ $x + 9 = 0$ or $x = -9$ or ft using their a from part (a).	B1 $\sqrt{}$ (1)
(c)	$PS = 25 \Rightarrow \underline{QP = 25}$ Either 25 by itself or $PQ = 25$. Do not award if just $PS = 25$ is seen.	B1 (1)
(d)	x -coordinate of $P \Rightarrow x = 25 - 9 = 16$ $x = 16$ $y^2 = 36(16)$ Substitutes their x -coordinate into equation of C . $\underline{y} = \sqrt{576} = \underline{24}$ $\underline{y} = 24$ Therefore $P(16, 24)$	B1 $\sqrt{}$ M1 A1 (3)
(e)	Area $OSPQ = \frac{1}{2}(9 + 25)24$ $= \underline{408} \text{ (units)}^2$ $\frac{1}{2}(\text{their } a + 25)(\text{their } y)$ or rectangle and 2 distinct triangles, correct for their values. 408	M1 A1 (2) [8]

Question Number	Scheme	Marks
7. (a)	<p>Correct quadrant with $(-24, -7)$ indicated.</p>	B1 (1)
(b)	$\arg z = -\pi + \tan^{-1}\left(\frac{7}{24}\right)$ $= -2.857798544... = -2.86 \text{ (2 dp)}$	$\tan^{-1}\left(\frac{7}{24}\right)$ or $\tan^{-1}\left(\frac{24}{7}\right)$ awrt -2.86 or awrt 3.43 M1 A1 (2)
(c)	$ w = 4, \arg w = \frac{5\pi}{6} \Rightarrow r = 4, \theta = \frac{5\pi}{6}$ $w = r \cos \theta + i r \sin \theta$ $w = 4 \cos\left(\frac{5\pi}{6}\right) + 4i \sin\left(\frac{5\pi}{6}\right)$ $= 4\left(\frac{-\sqrt{3}}{2}\right) + 4i\left(\frac{1}{2}\right)$ $= -2\sqrt{3} + 2i$ $a = -2\sqrt{3}, b = 2$	Attempt to apply $r \cos \theta + i r \sin \theta$. Correct expression for w . either $-2\sqrt{3} + 2i$ or awrt $-3.5 + 2i$ M1 A1 A1 (3)
(d)	$ z = \sqrt{(-24)^2 + (-7)^2} = \underline{25}$ $ zw = z \times w = (25)(4)$ $= \underline{100}$	$ z = 25$ or $zw = (48\sqrt{3} + 14) + (14\sqrt{3} - 48)i$ or awrt 97.1-23.8i Applies $ z \times w $ or $ zw $ $\underline{100}$ B1 M1 A1 (3) [9]

Question Number	Scheme	Marks
8.		
(a)	$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$ $\det \mathbf{A} = 2(3) - (-1)(-2) = 6 - 2 = \underline{4}$	<p>4 B1 (1)</p>
(b)	$\mathbf{A}^{-1} = \frac{1}{4} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	<p> $\frac{1}{\det \mathbf{A}} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ M1 $\frac{1}{4} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ A1 (2) </p>
(c)	$\text{Area}(R) = \frac{72}{4} = \underline{18} \text{ (units)}^2$	<p> $\frac{72}{\text{their } \det \mathbf{A}}$ or $72(\text{their } \det \mathbf{A})$ M1 <u>18</u> or ft answer. A1 $\sqrt{\quad}$ (2) </p>
(d)	$\mathbf{AR} = \mathbf{S} \Rightarrow \mathbf{A}^{-1} \mathbf{AR} = \mathbf{A}^{-1} \mathbf{S} \Rightarrow \mathbf{R} = \mathbf{A}^{-1} \mathbf{S}$ $\mathbf{R} = \frac{1}{4} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 8 & 12 \\ 4 & 16 & 4 \end{pmatrix}$ $= \frac{1}{4} \begin{pmatrix} 8 & 56 & 44 \\ 8 & 40 & 20 \end{pmatrix}$ $= \begin{pmatrix} 2 & 14 & 11 \\ 2 & 10 & 5 \end{pmatrix}$ <p>Vertices are (2, 2), (14, 10) and (11, 5).</p>	<p>At least one attempt to apply \mathbf{A}^{-1} by any of the three vertices in \mathbf{S}. M1</p> <p>At least one correct column o.e. A1 $\sqrt{\quad}$</p> <p>At least two correct columns o.e. A1</p> <p>All three coordinates correct. A1 (4) [9] </p>

Question Number	Scheme	Marks
9.	<p>$u_{n+1} = 4u_n + 2$, $u_1 = 2$ and $u_n = \frac{2}{3}(4^n - 1)$</p> <p>$n = 1$; $u_1 = \frac{2}{3}(4^1 - 1) = \frac{2}{3}(3) = 2$</p> <p>So u_n is true when $n = 1$.</p> <p>Assume that for $n = k$ that, $u_k = \frac{2}{3}(4^k - 1)$ is true for $k \in \mathbb{Z}^+$.</p> <p>Then $u_{k+1} = 4u_k + 2$</p> <p>$= 4\left(\frac{2}{3}(4^k - 1)\right) + 2$</p> <p>$= \frac{8}{3}(4)^k - \frac{8}{3} + 2$</p> <p>$= \frac{2}{3}(4)(4)^k - \frac{2}{3}$</p> <p>$= \frac{2}{3}4^{k+1} - \frac{2}{3}$</p> <p>$= \frac{2}{3}(4^{k+1} - 1)$</p> <p>Therefore, the general statement, $u_n = \frac{2}{3}(4^n - 1)$ is true when $n = k + 1$. (As u_n is true for $n = 1$,) then u_n is true for all positive integers by mathematical induction</p>	<p>Check that $u_n = \frac{2}{3}(4^n - 1)$ yields 2 when $n = 1$.</p> <p>Substituting $u_k = \frac{2}{3}(4^k - 1)$ into $u_{n+1} = 4u_n + 2$.</p> <p>An attempt to multiply out the brackets by 4 or $\frac{8}{3}$</p> <p>$\frac{2}{3}(4^{k+1} - 1)$</p> <p>Require 'True when $n=1$', 'Assume true when $n=k$' and 'True when $n = k + 1$' then true for all n o.e.</p> <p>(5) [5]</p>

Question Number	Scheme	Marks
10.	$xy = 36$ at $(6t, \frac{6}{t})$.	
(a)	$y = \frac{36}{x} = 36x^{-1} \Rightarrow \frac{dy}{dx} = -36x^{-2} = -\frac{36}{x^2}$ An attempt at $\frac{dy}{dx}$. or $\frac{dy}{dt}$ and $\frac{dx}{dt}$	M1
	At $(6t, \frac{6}{t})$, $\frac{dy}{dx} = -\frac{36}{(6t)^2}$ An attempt at $\frac{dy}{dx}$ in terms of t	M1
	So, $m_T = \frac{dy}{dx} = -\frac{1}{t^2}$ $\frac{dy}{dx} = -\frac{1}{t^2} *$	A1
	T: $y - \frac{6}{t} = -\frac{1}{t^2}(x - 6t)$ T: $y - \frac{6}{t} = -\frac{1}{t^2}x + \frac{6}{t}$ T: $y = -\frac{1}{t^2}x + \frac{6}{t} + \frac{6}{t}$ T: $y = -\frac{1}{t^2}x + \frac{12}{t} *$ Must see working to award here Applies $y - \frac{6}{t} = \text{their } m_T(x - 6t)$	M1
	Correct solution .	A1 cso
(b)	Both T meet at $(-9, 12)$ gives $12 = -\frac{1}{t^2}(-9) + \frac{12}{t}$ Substituting $(-9, 12)$ into T . $12 = \frac{9}{t^2} + \frac{12}{t} \quad (\times t^2)$ $12t^2 = 9 + 12t$ $12t^2 - 12t - 9 = 0$ An attempt to form a "3 term quadratic" $4t^2 - 4t - 3 = 0$ An attempt to factorise. $(2t - 3)(2t + 1) = 0$ $t = \frac{3}{2}, -\frac{1}{2}$ $t = \frac{3}{2}, -\frac{1}{2}$ An attempt to substitute either their $t = \frac{3}{2}$ or their $t = -\frac{1}{2}$ into x and y . At least one of $(9, 4)$ or $(-3, -12)$. Both $(9, 4)$ and $(-3, -12)$. $t = \frac{3}{2} \Rightarrow x = 6(\frac{3}{2}) = 9, y = \frac{6}{(\frac{3}{2})} = 4 \Rightarrow (9, 4)$ $t = -\frac{1}{2} \Rightarrow x = 6(-\frac{1}{2}) = -3,$ $y = \frac{6}{(-\frac{1}{2})} = -12 \Rightarrow (-3, -12)$	M1
		M1
		M1
		A1
		M1
		A1
		A1
		(7)
		[12]

Other Possible Solutions

Question Number	Scheme	Marks
4. (a) (i) <i>Aliter</i> (ii) Way 2	$z^2 + pz + q = 0, z_1 = 2 - 4i$ $z_2 = 2 + 4i$ Product of roots $= (2 - 4i)(2 + 4i)$ $= 4 + 16 = 20$ or $b^2 - 4ac = (8i)^2$ Sum of roots $= (2 - 4i) + (2 + 4i) = 4$ $= z^2 - 4z + 20 = 0$	B1 M1 A1 A1 (4)
4. (a) (i) <i>Aliter</i> (ii) Way 3	$z^2 + pz + q = 0, z_1 = 2 - 4i$ $z_2 = 2 + 4i$ $(2 - 4i)^2 + p(2 - 4i) + q = 0$ $-12 - 16i + p(2 - 4i) + q = 0$ Imaginary part: $-16 - 4p = 0$ Real part: $-12 + 2p + q = 0$ $4p = -16 \Rightarrow p = -4$ $q = 12 - 2p \Rightarrow q = 12 - 2(-4) = 20$	B1 M1 A1 A1 (4)

[illegible]

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