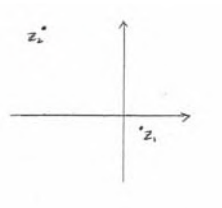
June 2009
6667 Further Pure Mathematics FP1 (new)
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)		B1 (1)
(b)	$\lvert z_1 \rvert = \sqrt{2^2 + (-1)^2} = \sqrt{5}$ (or awrt 2.24)	M1 A1 (2)
(c)	$\alpha = \arctan\left(\frac{1}{2}\right)$ or $\arctan\left(-\frac{1}{2}\right)$	M1
	$\arg z_1 = -0.46$ or 5.82 (awrt) (answer in degrees is A0 unless followed by correct conversion)	A1 (2)
(d)	$\frac{-8+9\text{i}}{2-\text{i}} \times \frac{2+\text{i}}{2+\text{i}}$	M1
	$= \frac{-16-8\text{i}+18\text{i}-9}{5} = -5+2\text{i}$ i.e. a = -5 and b =2 or $-\frac{2}{5}a$	A1 A1ft (3) **[8]**
	Alternative method to part (d) $-8+9\text{i} = (2-i)(a+b\text{i})$, and so $2a+b=-8$ and $2b-a=9$ and attempt to solve as far as equation in one variable So a = -5 and $b = 2$	M1 A1 A1cao
Notes	(a) B1 needs both complex numbers as either points or vectors, in correct quadrants and with 'reasonably correct' relative scale (b) M1 Attempt at Pythagoras to find modulus of either complex number A1 condone correct answer even if negative sign not seen in (-1) term A0 for $\pm\sqrt{5}$ (c) arctan 2 is M0 unless followed by $\frac{3\pi}{2}+\arctan 2$ or $\frac{\pi}{2}-\arctan 2$ Need to be clear that argz = - 0.46 or 5.82 for A1 (d) M1 Multiply numerator and denominator by conjugate of their denominator A1 for –5 and A1 for 2i (should be simplified) Alternative scheme for (d) Allow slips in working for first M1	

Question Number	Scheme	Marks
Q2 (a)	$r(r+1)(r+3) = r^3 + 4r^2 + 3r$, so **use** $\sum r^3 + 4\sum r^2 + 3\sum r$	M1
	$= \frac{1}{4}n^2(n+1)^2 + 4\left(\frac{1}{6}n(n+1)(2n+1)\right) + 3\left(\frac{1}{2}n(n+1)\right)$	A1 A1
	$= \frac{1}{12}n(n+1)\{3n(n+1) + 8(2n+1) + 18\}$ or $= \frac{1}{12}n\{3n^3 + 22n^2 + 45n + 26\}$	
	or $= = \frac{1}{12}(n+1)\{3n^3 + 19n^2 + 26n\}$	M1 A1
	$= \frac{1}{12}n(n+1)\{3n^2 + 19n + 26\} = \frac{1}{12}n(n+1)(n+2)(3n+13)$ $(k = 13)$	M1 A1cao (7)
(b)	$\sum_{21}^{40} = \sum_{1}^{40} - \sum_{1}^{20}$	M1
	$= \frac{1}{12}(40\times41\times42\times133) - \frac{1}{12}(20\times21\times22\times73) = 763420 - 56210 = 707210$	A1 cao (2) **[9]**
Notes	(a) M1 expand and must start to use at least one standard formula First 2 A marks: One wrong term A1 A0, two wrong terms A0 A0. M1: Take out factor $kn(n+1)$ or kn or $k(n+1)$ directly or from quartic A1: See scheme (cubics must be simplified) M1: Complete method including a quadratic factor and attempt to factorise it A1 Completely correct work. Just gives $k = 13$, no working is **0** marks for the question. **Alternative method** Expands $(n+1)(n+2)(3n+k)$ and confirms that it equals $\{3n^3 + 22n^2 + 45n + 26\}$ together with statement $k = 13$ can earn last **M1A1** The previous **M1A1** can be implied if they are using a quartic**.** **(b)** M 1 is for substituting 40 and 20 into their **answer** to (a) and subtracting. (NB not 40 and 21) Adding terms is M0A0 as the question said "Hence"	

Question Number	Scheme	Marks
Q3 (a)	$x^2+4=0 \quad \Rightarrow \quad x = k\text{i}, \quad x = \pm 2\text{i}$	M1, A1
	Solving 3-term quadratic	M1
	$x = \dfrac{-8 \pm \sqrt{64-100}}{2} \quad = -4+3\text{i}$ and $-4-3\text{i}$	A1 A1ft
		(5)
(b)	$2\text{i} + (-2\text{i}) + (-4+3\text{i}) + (-4-3\text{i}) = -8$	M1 A1cso
		(2)
		[7]
	Alternative method : Expands f(x) as quartic and chooses $\pm$ coefficient of x^3	M1
	-8	A1 cso
Notes	(a) Just $x = 2\text{i}$ is M1 A0 $x = \pm 2$ is M0A0 M1 for solving quadratic follows usual conventions, then A1 for a correct root (simplified as here) and A1ft for conjugate of first answer. Accept correct answers with no working here. Do not give accuracy marks for factors unless followed by roots. (b) M1 for adding four roots of which at least two are complex conjugates and getting a real answer. A1 for –8 following **correct** roots or the alternative method. If any incorrect working in part (a) this A mark will be A0	

Question Number	Scheme	Marks
Q4 (a)	$\text{f}(2.2) = 2.2^3 - 2.2^2 - 6 \quad (= -0.192)$ $\text{f}(2.3) = 2.3^3 - 2.3^2 - 6 \quad (= 0.877)$	M1
	Change of sign $\Rightarrow$ Root need numerical values correct (to 1 s.f.).	A1 (2)
(b)	$\text{f}'(x) = 3x^2 - 2x$	B1
	$\text{f}'(2.2) = 10.12$	B1
	$x_1 = x_0 - \dfrac{\text{f}(x_0)}{\text{f}'(x_0)} = 2.2 - \dfrac{-0.192}{10.12}$	M1 A1ft
	$= 2.219$	A1cao (5)
(c)	$\dfrac{\alpha - 2.2}{\pm '0.192'} = \dfrac{2.3 - \alpha}{\pm '0.877'}$ (or equivalent such as $\dfrac{k}{\pm '0.192'} = \dfrac{0.1 - k}{\pm '0.877'}$.)	M1
	$\alpha(0.877 + 0.192) = 2.3 \times 0.192 + 2.2 \times 0.877$ or $k(0.877 + 0.192) = 0.1 \times 0.192$, where $\alpha = 2.2 + k$	A1
	so $\alpha \approx 2.218$ (2.21796…) (Allow awrt)	A1 (3) **[10]**
Alternative	Uses equation of line joining (2.2, -0.192) to (2.3, 0.877) and substitutes $y = 0$	M1
	$y + 0.192 = \dfrac{0.192 + 0.877}{0.1}(x - 2.2)$ and $y = 0$, so $\alpha \approx 2.218$ or awrt as before (NB Gradient = 10.69)	A1, A1
Notes	(a) M1 for attempt at f(2.2) and f(2.3) A1 need indication that there is a change of sign – (could be –0.19<0, 0.88>0) and need conclusion. (These marks may be awarded in other parts of the question if not done in part (a)) (b) B1 for seeing correct derivative (but may be implied by later correct work) B1 for seeing 10.12 or this may be implied by later work M1 Attempt Newton-Raphson with their values A1ft may be implied by the following answer (but does not require an evaluation) Final A1 must 2.219 exactly as shown. So answer of 2.21897 would get 4/5 If done twice ignore second attempt (c) M1 Attempt at ratio with their values of ± f(2.2) and ± f(2.3). N.B. If you see $0.192 - \alpha$ or $0.877 - \alpha$ in the fraction then this is M0 A1 correct linear expression and definition of variable if not α (may be implied by final correct answer- does not need 3 dp accuracy) A1 for awrt 2.218 If done twice ignore second attempt	

Question Number	Scheme	Marks
Q5 (a)	$\mathbf{R}^2 = \begin{pmatrix} a^2+2a & 2a+2b \\ a^2+ab & 2a+b^2 \end{pmatrix}$	M1 A1 A1 (3)
(b)	Puts their $a^2+2a = 15$ or their $2a+b^2 = 15$ or their $(a^2+2a)(2a+b^2)-(a^2+ab)(2a+2b) = 225$ (or to 15) ,	M1,
	Puts their $a^2+ab=0$ or their $2a + 2b = 0$	M1
	Solve to find either a or b	M1
	$a=3,\quad b=-3$	A1, A1 (5) **[8]**
Alternative for (b)	Uses $\mathbf{R}^2\times$ column vector = $15\times$ column vector, and equates rows to give two equations in a and b only Solves to find either a or b as above method	M1, M1 M1 A1 A1
Notes	(a) 1 term correct: M1 A0 A0 2 or 3 terms correct: M1 A1 A0 (b) M1 M1 as described in scheme (In the alternative scheme column vector can be general or specific for first M1 but must be specific for 2nd M1) M1 requires solving equations to find a and/or b (though checking that correct answer satisfies the equations will earn this mark) This mark can be given independently of the first two method marks. So solving $\mathbf{M}^2 = 15\mathbf{M}$ for example gives M0M0M1A0A0 in part (b) Also putting leading diagonal = 0 and other diagonal = 15 is M0M0M1A0A0 (No possible solutions as a >0) A1 A1 for correct answers only **Any Extra answers given**, e.g. a = -5 and b = 5 or wrong answers – **deduct last A1 awarded** So the two sets of answers would be A1 A0 Just the answer . a = -5 and b = 5 is A0 A0 Stopping at two values for a or for b – no attempt at other is A0A0 Answer with no working at all is 0 marks	

Question Number	Scheme	Marks
Q6 (a)	$y^2 = (8t)^2 = 64t^2$ and $16x = 16 \times 4t^2 = 64t^2$ Or identifies that $a = 4$ and uses general coordinates $(at^2, 2at)$	B1 (1)
(b)	$(4, 0)$	B1 (1)
(c)	$y = 4x^{\frac{1}{2}}$ $\quad \frac{dy}{dx} = 2x^{-\frac{1}{2}}$	B1
	Replaces x by $4t^2$ to give **gradient** $\quad [2(4t^2)^{-\frac{1}{2}} = \frac{2}{2t} = \frac{1}{t}]$	M1,
	Uses Gradient of normal is $-\frac{1}{\text{gradient of curve}}$ $\quad [-t]$	M1
	$y - 8t = -t(x - 4t^2) \quad \Rightarrow \quad y + tx = 8t + 4t^3$ $\quad$ (*)	M1 A1cso (5)
(d)	At N, $y = 0$, so $x = 8 + 4t^2$ or $\frac{8t + 4t^3}{t}$	B1
	Base $SN = (8 + 4t^2) - 4 \quad (= 4 + 4t^2)$	B1ft
	Area of $\Delta\, PSN = \frac{1}{2}(4 + 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$ for $t > 0$	M1 A1 (4)
	{Also Area of $\Delta\, PSN = \frac{1}{2}(4 + 4t^2)(-8t) = -16t(1 + t^2)$ for $t < 0$ } *this is not required*	**[11]**
	Alternatives: (c) $\frac{dx}{dt} = 8t$ and $\frac{dy}{dt} = 8$ $\quad$ B1 $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{1}{t}$ $\quad$ M1 , then as in main scheme. (c) $2y\frac{dy}{dx} = 16$ $\quad$ B1 (or uses $x = \frac{y^2}{8}$ to give $\frac{dx}{dy} = \frac{2y}{8}$) $\frac{dy}{dx} = \frac{8}{y} = \frac{8}{8t} = \frac{1}{t}$ $\quad$ M1 , then as in main scheme.	
Notes	(c) Second M1 – need not be function of t Third M1 requires linear equation (not fraction) and should include the parameter t but could be given for equation of tangent (So tangent equation loses 2 marks only and could gain B1M1M0M1A0) (d) Second B1 does not require simplification and may be a constant rather than an expression in t. M1 needs correct area of triangle formula using ½ 'their SN' $\times 8t$ Or may use two triangles in which case need $(4t^2 - 4)$ and $(4t^2 + 8 - 4t^2)$ for B1ft Then Area of $\Delta\, PSN = \frac{1}{2}(4t^2 - 4)(8t) + \frac{1}{2}(4t^2 + 8 - 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$	

Question Number	Scheme	Marks
Q7 (a)	Use $4a-(-2\times -1)=0 \quad \Rightarrow \quad a,=\frac{1}{2}$	M1, A1 (2)
(b)	Determinant: $(3\times 4)-(-2\times -1)=10 \quad (\Delta)$	M1
	$\mathbf{B}^{-1}=\frac{1}{10}\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}$	M1 A1cso (3)
(c)	$\frac{1}{10}\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}\begin{pmatrix}k-6\\ 3k+12\end{pmatrix}, \quad =\frac{1}{10}\begin{pmatrix}4(k-6)+2(3k+12)\\ (k-6)+3(3k+12)\end{pmatrix}$	M1, A1ft
	$\begin{pmatrix}k\\ k+3\end{pmatrix}$ Lies on $y=x+3$	A1 (3) **[8]**
	Alternatives: (c) $\begin{pmatrix}3 & -2\\ -1 & 4\end{pmatrix}\begin{pmatrix}x\\ x+3\end{pmatrix}, \quad =\begin{pmatrix}3x-2(x+3)\\ -x+4(x+3)\end{pmatrix}$,	M1, A1,
	$=\begin{pmatrix}x-6\\ 3x+12\end{pmatrix}$**,** which was of the form $(k - 6, 3k +12)$	A1
	Or $\begin{pmatrix}3 & -2\\ -1 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}, \quad =\begin{pmatrix}3x-2y\\ -x+4y\end{pmatrix}=\begin{pmatrix}k-6\\ 3k+12\end{pmatrix}$, and solves simultaneous equations	M1
	Both equations correct and eliminate one letter to get $x = k$ or $y = k + 3$ or $10x-10y=-30$ or equivalent.	A1
	Completely correct work (to $x = k$ and $y = k + 3$), and conclusion lies on $y=x+3$	A1
Notes	(a) Allow sign slips for first M1 (b) Allow sign slip for determinant for first M1 (This mark may be awarded for 1/10 appearing in inverse matrix.) Second M1 is for correctly treating the 2 by 2 matrix, ie for $\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}$ Watch out for determinant (3 + 4) – (-1 + -2) = 10 – M0 then final answer is A0 (c) M1 for multiplying matrix by appropriate column vector A1 correct work (ft wrong determinant) A1 for conclusion	

Question Number	Scheme	Marks
Q8 (a)	$f(1) = 5 + 8 + 3 = 16$, (which is divisible by 4). ($\therefore$ True for $n = 1$).	B1
	Using the formula to write down f(k + 1), $f(k+1) = 5^{k+1} + 8(k+1) + 3$	M1 A1
	$f(k+1) - f(k) = 5^{k+1} + 8(k+1) + 3 - 5^k - 8k - 3$	M1
	$= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$	A1
	$f(k+1) = 4(5^k + 2) + f(k)$, **which is divisible by** 4	A1ft
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. True for $n = 1$, $\therefore$ true for all n.	A1cso (7)
(b)	For $n = 1$, $\begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1$ ($\therefore$ True for $n = 1$.)	B1
	$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$	M1 A1 A1
	$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$	M1 A1
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. **True for *n*** = 1, $\therefore$ **true for all *n***	A1 cso (7) **[14]**
(a) Alternative for 2[nd] M:	$f(k+1) = 5(5^k) + 8k + 8 + 3$ M1 $= 4(5^k) + 8 + (5^k + 8k + 3)$ A1 or $= 5(5^k + 8k + 3) - 32k - 4$ $= 4(5^k + 2) + f(k)$, or $= 5f(k) - 4(8k + 1)$ which is divisible by 4 A1 (or similar methods)	

Notes

(a) B1 Correct values of 16 or 4 for $n = 1$ or for $n = 0$ (Accept "is a multiple of")
M1 Using the formula to write down f(k + 1) A1 Correct expression of f(k+1) (or for f(n +1)
M1 Start method to connect f(k+1) with f(k) as shown
A1 correct working toward multiples of 4, A1 ft result including f(k +1) as subject, A1cso conclusion

(b) B1 correct statement for $n = 1$ or $n = 0$
First M1: Set up product of two appropriate matrices – product can be either way round
A1 A0 for one or two slips in simplified result
A1 A1 all correct simplified
A0 A0 more than two slips
M1: States in terms of (k + 1)
A1 Correct statement A1 for induction conclusion

Part (b) Alternative

May write $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$. Then may or may not complete the proof.
This can be awarded the second M (substituting k + 1)and following A (simplification) in part (b). The first three marks are awarded as before . Concluding that they have reached the same matrix and therefore a result will then be part of final A1 cso but also need other statements as in the first method.