



# Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in Further Pure  
Mathematics (6667/01)

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Publications Code UA041563

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**EDEXCEL GCE MATHEMATICS****General Instructions for Marking**

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Further Pure Mathematics Marking

*(But note that specific mark schemes may sometimes override these general principles).*

### **Method mark for solving 3 term quadratic:**

#### **1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$ , where  $|pq| = |c|$ , leading to  $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = \dots$

#### **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

#### **3. Completing the square**

Solving  $x^2 + bx + c = 0$ :  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$ ,  $q \neq 0$ , leading to  $x = \dots$

### **Method marks for differentiation and integration:**

#### **1. Differentiation**

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

#### **2. Integration**

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

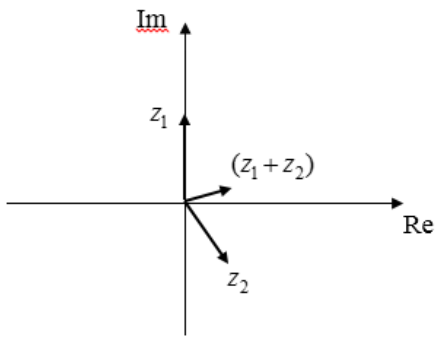
**June 2015**  
**Further Pure Mathematics FP1 6667**  
**Mark Scheme**

| Question Number | Scheme  | Marks   |
|-----------------|---|---|
| 1.              | <p><math>(x - 5)</math> is a factor of <math>f(x)</math> so <math>f(x) = (x - 5)(9x^2 \dots)</math></p> <p><math>f(x) = (x - 5)(9x^2 + 12x + 5)</math></p> <p>Solve <math>(9x^2 + 12x + 5) = 0</math> to give <math>x =</math></p> <p><math>(x =) -\frac{2}{3} - \frac{1}{3}i, -\frac{2}{3} + \frac{1}{3}i</math> or <math>-\frac{2}{3} \pm \frac{1}{3}i</math> or <math>\frac{-2 \pm i}{3}</math> oe ( and 5)</p>  | <p>M1</p> <p>A1</p> <p>M1</p> <p>A1cao A1ft (5)<br/> <b>(5 marks)</b></p> |
|                 | <p style="text-align: center;"><b>Notes</b></p> <p>M1: Uses <math>(x-5)</math> as factor and begins division or process to obtain quadratic with <math>9x^2</math>. Award if no working but quadratic factor completely correct.</p> <p>A1: <math>9x^2 + 12x + 5</math></p> <p>M1: Solves their quadratic by usual rules leading to <math>x =</math></p> <p>Award if one complex root correct with no working.</p> <p>Award for <math>(9x^2 + \dots)</math> incorrectly factorised to <math>(3x + p)(3x + q)</math>, where <math> pq  = 5</math></p> <p>A1: One correct complex root. Accept any exact equivalent form. Accept single fraction and <math>\pm</math></p> <p>A1ft: Conjugate of their first complex root.</p> |   |

| Question Number   | Scheme  | Marks     |
|---|---|-----------|
| 2. (a)  | Let $f(x) = 3 + x \sin\left(\frac{x}{4}\right)$ then $f(13) = 1.593$ [and $f(14) = -1.911$ need not be seen in (a)]               |           |
|   | $f(13.5) = -0.122$ , so root in $[13, 13.5]$  | M1 A1     |
|   | $f(13.25) = 0.746$ so root in $[13.25, 13.5]$   | A1        |
| (b)   | $\frac{\alpha - 13}{14 - \alpha} = \frac{1.593}{1.911} \quad \text{or} \quad \frac{\alpha - 13}{1} = \frac{1.593}{1.593 + 1.911}$ | M1 A1     |
|   | So $\alpha(1.911 + 1.593) = 1.593 \times 14 + 13 \times 1.911$ and $\alpha = \frac{47.145}{3.504} = 13.455$                       | dM1 A1    |
|   |   | (4)       |
|   |   | (7 marks) |
| <b>Notes</b>  |   |           |
| (a)   |   |           |
| M1: Evaluate $f(13)$ and $f(13.5)$ giving at least positive, negative <b>OR</b> evaluate $f(13.5)$ and $f(13.25)$ to give at least negative, positive. Do not award if using degrees. |   |           |
| A1: $f(13.5) = \text{awrt } -0.1$ , $f(13.25) = \text{awrt } 0.7(5)$ .  |   |           |
| A1: Correct interval $[13.25, 13.5]$ or equivalent form with or without boundaries.   |   |           |
| (b)   |   |           |
| M1: Attempt at linear interpolation on either side of equation with correct signs.  |   |           |
| A1: Correct equivalent statement  |   |           |
| dM1: Makes alpha subject of formula   |   |           |
| A1: cao. Award A0 for 13.456 and 13.454   |   |           |
| <b>ALT (b)</b>  |   |           |
| Using equation of line  |   |           |
| M1: Attempt to find gradient $\frac{y_1 - y_0}{x_1 - x_0} = \frac{-1.911 - 1.593}{14 - 13} (= -3.504...)$ , attempt to use $y - y_0 = m(x - x_0)$                                     |   |           |
| with either 13 or 14 (gives $y = -3.504x + 47.145$ ) and substitute $y = 0$   |   |           |
| A1: Correct statement after substituting $y = 0$ in their equation i.e. $0 = -3.504x + 47.145$  |   |           |
| dM1: Makes $x$ the subject of the formula   |   |           |
| A1: cao. Award A0 for 13.456 and 13.454   |   |           |



| Question Number   | Scheme   | Marks   |
|---|--|---|
| 3. (a)  | $\sum_{r=1}^n (r+1)(r+4)$ $= \sum_{r=1}^n r^2 + 5r + 4$ $= \frac{n}{6}(n+1)(2n+1) + 5\frac{n}{2}(n+1) + 4n$ $= \frac{n}{6}\{(n+1)(2n+1) + 15(n+1) + 24\}$ $= \frac{n}{6}\{2n^2 + 3n + 1 + 15n + 15 + 24\}$ $= \frac{n}{6}(2n^2 + 18n + 40) \text{ or } = \frac{n}{3}(n^2 + 9n + 20)$ $= \frac{n}{3}(n+4)(n+5) \text{ ** given answer**}$ | <p>B1</p> <p>M1 A1</p> <p>dM1</p> <p>A1*</p> <p>(5)</p>           |
| (b)   | $\sum_{r=n+1}^{2n} (r+1)(r+4) = \frac{2n}{3}(2n+4)(2n+5) - \frac{n}{3}(n+4)(n+5)$ $= \frac{n}{3}\{8n^2 + 36n + 40 - n^2 - 9n - 20\}$ $= \frac{n}{3}\{7n^2 + 27n + 20\} = \frac{n}{3}(n+1)(7n+20) \text{ or } a = 7, b = 20$  | <p>M1</p> <p>dM1</p> <p>A1</p> <p>(3)</p> <p><b>(8 marks)</b></p> |
| <p style="text-align: center;"><b>Notes</b></p> <p>(a)</p> <p>B1: Expands bracket correctly to <math>r^2 + 5r + 4</math></p> <p>M1: Uses <math>\frac{n}{6}(n+1)(2n+1)</math> or <math>\frac{n}{2}(n+1)</math> correctly.</p> <p>A1: Completely correct expression.</p> <p>dM1: Attempts to remove factor <math>\frac{n}{6}</math> or <math>\frac{n}{3}</math> to obtain a quadratic factor. Need not be 3 term.</p> <p>A1: Completely correct work including a step with a collected <b>3 term</b> quadratic prior in the bracket with correct printed answer.</p> <p>Accept approach which starts with LHS and then RHS which meet at <math>\frac{n^3}{3} + 3n^2 + \frac{20n}{3}</math>. Award marks as above.</p> <p>NB If induction attempted then typically this may only score the first B1.</p> <p>However, consider the solution carefully and award as above if seen in the body of the induction attempt.</p> <p>(b) M1: Uses <math>f(2n) - f(n)</math> or <math>f(2n) - f(n+1)</math> correctly. Require all 3 terms in <math>2n</math> (and <math>n+1</math> if used).</p> <p>dM1: Attempts to remove factor <math>\frac{n}{6}</math> or <math>\frac{n}{3}</math> to obtain a quadratic factor. Need not be 3 term.</p> <p>A1: Either in expression or as above.</p> |  |   |

| Question Number   | Scheme   | Marks  |
|---|--|--|
| 4. (a)  | $z_2 = \frac{6(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})} = \frac{6(1-i\sqrt{3})}{4}$ $z_2 = \frac{6(1-i\sqrt{3})}{4} \left( = \frac{3}{2} - i\frac{3}{2}\sqrt{3} \right)$  | <p>M1</p> <p>A1</p> <p>(2)</p>                     |
| (b)   | $ z_2  = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{27}{4}}$ <p>The modulus of <math>z_2</math> is 3</p> <p><math>\tan \theta = (\pm)\sqrt{3}</math> and attempts to find <math>\theta</math></p> <p>and the argument is <math>-\frac{\pi}{3}</math></p> | <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>(4)</p> |
| (c)   |    | <p>M1</p> <p>A1</p> <p>(2)</p>                     |
| <p style="text-align: center;"><b>Notes</b></p> <p>(a) M1: Multiplies numerator and denominator by <math>1-i\sqrt{3}</math><br/> A1: any correct equivalent with real denominator.</p> <p>(b) M1: Uses correct method for modulus for their <math>z_2</math> in part (a)<br/> A1: for 3 only<br/> M1: Uses tan or inverse tan<br/> A1: <math>-\frac{\pi}{3}</math> accept <math>\frac{5\pi}{3}</math><br/> NB Answers only then award 4/4 but arg must be in terms of <math>\pi</math></p> <p>(c) M1:<br/> <b>Either</b> <math>z_1</math> on imaginary axis and labelled with <math>z_1</math> or <math>3i</math> or <math>(0,3)</math> or axis labelled 3;<br/> <b>or</b> their <math>z_2</math> in the correct quadrant labelled <math>z_2</math> or <math>\frac{3}{2} - i\frac{3}{2}\sqrt{3}</math> or <math>\left(\frac{3}{2}, -\frac{3}{2}\sqrt{3}\right)</math> or axes labelled<br/> or their <math>a+bi</math> or their <math>(a,b)</math> or axes labelled.<br/> Axes need not be labelled Re and Im.<br/> A1: All 3 correct ie <math>z_1</math> on positive imaginary axis, <math>z_2</math> in 4<sup>th</sup> quadrant and <math>z_1+z_2</math> in the first quadrant.<br/> Accept points or lines. Arrows not required.</p> |  |  |
|   |  | <b>(8 marks)</b>                                   |

| Question Number   | Scheme   | Marks                                |
|---|--|--------------------------------------|
| 5. (a)  | $\frac{dy}{dx} = -\frac{9}{x^2}$ or $\frac{dy}{dx} = -\frac{y}{x}$ or $\frac{dy}{dx} = -\frac{1}{t^2}$<br>so gradient at $x = 6$ or $t = 2$ is $-\frac{9}{36}$ or $-\frac{\frac{3}{2}}{6}$ or $-\frac{1}{4}$ o.e.<br><br>Gradient of normal is $-\frac{1}{m}$ ( $= 4$ )<br>Equation of normal is $y - \frac{3}{2} = 4(x - 6)$<br>So $2y - 8x + 45 = 0$ <b>**given answer**</b> | M1<br>A1<br>M1<br>dM1<br>A1 *<br>(5) |
| (b)   | $\frac{18}{x} - 8x + 45 = 0$ or $2y - \frac{72}{y} + 45 = 0$ or $x(4x - 22.5) = 9$ or $y\left(\frac{y}{4} + \frac{45}{8}\right) = 9$ o.e.<br>$8x^2 - 45x - 18 = 0$ or $2y^2 + 45y - 72 = 0$<br><br>So $x = -\frac{3}{8}$ or $y = -24$<br>Finds other ordinate: $\left(-\frac{3}{8}, -24\right)$  | M1<br>A1<br>M1 A1                    |
| ALT   | Sub $\left(3t, \frac{3}{t}\right)$ in $2y - 8x + 45 = 0 \Rightarrow t = -\frac{1}{8}$<br>Sub $t = -\frac{1}{8}$ in $\left(3t, \frac{3}{t}\right) \Rightarrow \left(-\frac{3}{8}, -24\right)$   | M1A1<br>M1A1<br>(4)                  |
| <b>(9 marks)</b><br><br><b>Notes</b><br>(a) M1: Differentiates to obtain $\frac{k}{x^2}$ and substitutes $x = 6$<br>or uses implicit differentiation $\frac{dy}{dx} = -\frac{y}{x}$ and substitutes $x$ and $y$<br>or uses parametric differentiation $\frac{dy}{dx} = -\frac{1}{t^2}$ and substitutes $t = 2$<br>A1: For grad of tangent – accept any equivalent i.e. - 0.25 etc<br>M1: Uses negative reciprocal of their gradient.<br>dM1: $y - y_1 = m(x - x_1)$ with $\left(6, \frac{3}{2}\right)$ or $y = mx + c$ and sub $\left(6, \frac{3}{2}\right)$ to find $c$ .<br>A1: cso: Correct answer with no errors seen in the solution.<br><br>(b) M1: Obtains equation in one variable, $x$ or $y$<br>A1: Correct value of $x$ or correct value of $y$<br>M1: Finds second coordinate using $xy = 9$ or solving second quadratic or equation of the normal<br>A1: Correct coordinates that can be written as $x = \dots, y = \dots$ |  |                                      |

| Question Number   | Scheme   | Marks  |
|---|--|--|
| 6. (i)  | <p>If <math>n = 1</math>, <math>\begin{pmatrix} 1 &amp; 0 \\ -1 &amp; 5 \end{pmatrix}^1 = \begin{pmatrix} 1 &amp; 0 \\ -\frac{1}{4}(5^1 - 1) &amp; 5^1 \end{pmatrix}</math> so <b>true</b> for <math>n = 1</math></p> <p>Assume result true for <math>n = k</math></p> <p><math>\begin{pmatrix} 1 &amp; 0 \\ -1 &amp; 5 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 &amp; 0 \\ -\frac{1}{4}(5^k - 1) &amp; 5^k \end{pmatrix} \begin{pmatrix} 1 &amp; 0 \\ -1 &amp; 5 \end{pmatrix}</math> or <math>\begin{pmatrix} 1 &amp; 0 \\ -1 &amp; 5 \end{pmatrix} \begin{pmatrix} 1 &amp; 0 \\ -\frac{1}{4}(5^k - 1) &amp; 5^k \end{pmatrix}</math></p> <p><math>\begin{pmatrix} 1 &amp; 0 \\ -1 &amp; 5 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 &amp; 0 \\ -\frac{1}{4}(5^k - 1) - 5^k &amp; 5 \times 5^k \end{pmatrix}</math> or <math>\begin{pmatrix} 1 &amp; 0 \\ -1 - 5 \cdot \frac{1}{4}(5^k - 1) &amp; 5 \times 5^k \end{pmatrix}</math></p> <p><math>= \begin{pmatrix} 1 &amp; 0 \\ -\frac{1}{4}5^k + \frac{1}{4} - 5^k &amp; 5^{k+1} \end{pmatrix}</math> or <math>\begin{pmatrix} 1 &amp; 0 \\ -1 - \frac{1}{4}5^{k+1} + \frac{5}{4} &amp; 5^{k+1} \end{pmatrix} = \begin{pmatrix} 1 &amp; 0 \\ -\frac{1}{4}(5^{k+1} - 1) &amp; 5^{k+1} \end{pmatrix}</math></p> <p><b>True</b> for <math>n = k + 1</math> if <b>true</b> for <math>n = k</math>, (and <b>true</b> for <math>n = 1</math>) so <b>true</b> by induction for all <math>n \in \mathbf{Z}^+</math>.</p> | <p>B1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p>A1cso</p>  |
| (ii)  | <p>If <math>n = 1</math>, <math>\sum_{r=1}^n (2r-1)^2 = 1</math> and <math>\frac{1}{3}n(4n^2 - 1) = 1</math>, so <b>true</b> for <math>n = 1</math>.</p> <p>Assume result true for <math>n = k</math> so <math>\sum_{r=1}^{k+1} (2r-1)^2 = \frac{1}{3}k(4k^2 - 1) + (2(k+1) - 1)^2</math></p> <p><math>= \sum_{r=1}^{k+1} (2r-1)^2 = \frac{1}{3}(2k+1)\{(2k^2 - k) + (3(2k+1))\}</math></p> <p><math>= \frac{1}{3}(2k+1)\{(2k^2 + 5k + 3)\}</math> or <math>\frac{1}{3}(k+1)(4k^2 + 8k + 3)</math> or <math>\frac{1}{3}((2k+3)(2k^2 + 3k + 1))</math></p> <p><math>= \frac{1}{3}(k+1)(2k+1)(2k+3) = \frac{1}{3}(k+1)(4(k+1)^2 - 1)</math></p> <p><b>True</b> for <math>n = k + 1</math> if <b>true</b> for <math>n = k</math>, ( and <b>true</b> for <math>n = 1</math>) so <b>true</b> by induction for all <math>n \in \mathbf{Z}^+</math></p>   | <p>B1</p> <p>M1</p> <p>M1 A1</p> <p>dA1</p> <p>A1cso</p> |
| <p style="text-align: center;"><b>Notes</b></p> <p>(i) B1: Checks <math>n = 1</math> on both sides and states true for <math>n = 1</math> seen anywhere.<br/> M1: Assumes true for <math>n = k</math> and indicates intention to multiply power <math>k</math> by power 1 either way around.<br/> M1: Multiplies matrices. Condone one slip. A1: Correct unsimplified matrix<br/> A1: Intermediate step required cao<br/> A1: cso Makes correct induction statement including at least statements in bold.<br/> Statement <b>true</b> for <math>n = 1</math> here could contribute to B1 mark earlier.</p> <p>(ii) B1: Checks <math>n = 1</math> on both sides and states true for <math>n = 1</math> seen anywhere.<br/> M1: Assumes true for <math>n = k</math> and adds <math>(k+1)^{\text{th}}</math> term to sum of <math>k</math> terms. Accept <math>4(k+1)^2 - 4(k+1) + 1</math> or <math>(2k+1)^2</math> for <math>(k+1)^{\text{th}}</math> term. M1: Factorises out a linear factor of the three possible - usually <math>2k+1</math><br/> A1: Correct expression with one linear and one quadratic factor.<br/> dA1: Need to see <math>\frac{1}{3}(k+1)(4(k+1)^2 - 1)</math> somewhere dependent upon previous A1.</p> <p>Accept assumption plus <math>(k+1)^{\text{th}}</math> term and <math>\frac{1}{3}(k+1)(4(k+1)^2 - 1)</math> both leading to <math>\frac{1}{3}(4k^3 + 12k^2 + 11k + 3)</math> then award for expressions seen as above.<br/> A1: cso Makes correct complete induction statement including at least statements in bold. Statement true for <math>n = 1</math> here could contribute to B1 mark earlier.</p> |  |  |
|   |  | <b>12 marks</b>  |

| Question Number   | Scheme   | Marks   |
|---|--|---|
| 7. (i)  | $5k(k+1) - 3(3k-1)=0$<br>$5k^2 + 5k + 9k - 3 = 0$<br>$(5k-1)(k+3)=0$ so $k =$<br>$k = \frac{1}{5}$ or $-3$   | M1<br>A1<br>M1<br>A1<br>(4)   |
| (ii)(a)   | $\mathbf{B}^{-1} = \frac{1}{45} \begin{pmatrix} 3 & -5 \\ 3 & 10 \end{pmatrix}$  | M1 A1<br>(2)  |
| (b)   | $\frac{1}{45} \begin{pmatrix} 3 & -5 \\ 3 & 10 \end{pmatrix} \begin{pmatrix} 0 & -20 & 10c \\ 0 & 6 & 6c \end{pmatrix} =$<br>$\frac{1}{45} \begin{pmatrix} 0 & -90 & 0 \\ 0 & 0 & 90c \end{pmatrix}$<br>Vertices at $(0, 0)$ $(-2, 0)$ $(0, 2c)$   | M1<br>A1,A1<br>(3)  |
| ALT   | $\begin{pmatrix} 10 & 5 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} a & d & f \\ b & e & g \end{pmatrix} = \begin{pmatrix} 0 & -20 & 10c \\ 0 & 6 & 6c \end{pmatrix}$ and attempt to form simultaneous equations<br>$10a + 5b = 0, -3a + 3b = 0$<br>$10d + 5e = -20, -3d + 3e = 6$ all correct oe<br>$10f + 5g = 10c, -3f + 3g = 6c$<br>Vertices at $(0, 0)$ $(-2, 0)$ $(0, 2c)$ | M1<br>A1<br>A1<br>B1<br>M1<br>A1<br>(3)   |
| (c)   | Area of $T$ is $\frac{1}{2} \times 2 \times 2c = 2c$<br>Area of $T \times \text{determinant} = 135$<br>So $c = \frac{3}{2}$  | OR Area of $T' = \frac{1}{2} \begin{vmatrix} 0 & -20 & 10c & 0 \\ 0 & 6 & 6c & 0 \end{vmatrix} = 90c$<br>Their area = 135<br>So $c = \frac{3}{2}$<br>(12 marks) |
| Notes   |  |   |
| (i) M1: Puts determinant equal to zero<br>A1: cao as three or four term quadratic<br>M1: Solve their quadratic to find $k$<br>A1: cao – need both correct answers<br>(ii) (a) M1 Uses correct method for inverse with fraction $\frac{1}{45}$ or $\frac{1}{\text{their det}}$<br>A1: All correct oe<br>(b) M1: Post multiplies their inverse by <b>2 by 3 matrix or 2 by 2 matrix excluding the origin</b> or does not use inverse and attempts to form simultaneous equations. Can exclude origin.<br>A1: $(-2,0)$ and $(0,2c)$ . Can be written as column vectors. Accept seen in final two columns of single matrix<br>A1: $(0,0)$ . Can be written as column vectors. Award if seen as first column of single matrix.<br>(c) B1: Area of $T$ given as $2c$ or area of $T' = 90c$ Accept $\pm$<br>M1: Either method using their area of $T$ and their det or their area of $T'$<br>A1: $c = \frac{3}{2}$ cao |  |   |

| Question Number   | Scheme  | Marks   |
|---|---|---|
| 8(a)  | $SP = \sqrt{(3p^2 - a)^2 + 36p^2}$ , with $a = 3$<br>$SP = \sqrt{9p^4 + 18p^2 + 9} = 3(1 + p^2)$ <b>**given answer**</b>  | M1, B1<br>A1 *<br>(3)                         |
| ALT   | For parabola, perpendicular distance from $P$ to directrix = $SP$<br>Directrix $x = -3$<br>So $SP = 3 + 3p^2 = 3(1 + p^2)$  | M1<br>B1<br>A1                                |
| (b)   | $y^2 = 12x \Rightarrow 2y \frac{dy}{dx} = 12$ or $y = \sqrt{12x} \Rightarrow \frac{dy}{dx} = \sqrt{3x}^{-\frac{1}{2}}$ or $\frac{dy}{dx} = \frac{dp}{dx}$ or $\frac{dy}{dx} = \frac{dq}{dx}$<br>The tangent at $P$ has gradient $= \frac{1}{p}$ or the tangent at $Q$ has gradient $\frac{1}{q}$<br>and equation is $y - 6p = \frac{1}{p}(x - 3p^2)$ or $py = x + 3p^2$ o.e.<br>Tangent at $Q$ is $y - 6q = \frac{1}{q}(x - 3q^2)$ or $qy = x + 3q^2$ o.e.<br>Eliminate $x$ or $y$ : So $x = 3pq$ or $y = 3(p + q) = 3p + 3q$<br>Substitute for second variable so $x = 3pq$ and $y = 3(p + q) = 3p + 3q$ | M1<br>A1<br>A1<br>B1<br>M1 A1<br>M1 A1<br>(8) |
| (c)   | $SR^2 = (3 - 3pq)^2 + (3p + 3q)^2 (= 9 + 9p^2q^2 + 9p^2 + 9q^2)$<br>$SP.SQ = 3(1 + p^2) 3(1 + q^2) (= 9 + 9p^2q^2 + 9p^2 + 9q^2)$<br>So $SR^2 = SP.SQ$ as required  | M1<br>M1<br>A1<br>(3)                         |
| <b>Notes</b><br>(a) M1: Uses distance between two points or states perpendicular distance from $P$ to directrix required.<br>B1: States or uses focus at $(3,0)$ or focus at $a = 3$ or directrix as $x = -3$<br>A1: cso<br>(b) M1: Calculus method for finding gradient and substitutes $x$ value at either point<br>A1: Either correct. Accept unsimplified.<br>A1: One equation of tangent correct. B1: Both correct<br>M1: Eliminate $x$ or $y$ . A1: Obtain first variable<br>M1: Substitute or eliminate again. A1: Both variables correct in simplest form as above.<br>(c) M1: Find their $SR^2 = (3 - 3pq)^2 + (3p + 3q)^2 (= 9 + 9p^2q^2 + 9p^2 + 9q^2)$<br>M1: Find their $SP.SQ = 3(1 + p^2) 3(1 + q^2) (= 9 + 9p^2q^2 + 9p^2 + 9q^2)$<br>A1: Deduce equal after no errors seen. Concluding statement required cso. |   |   |

(14 marks)



