

# Mark Scheme (Results)

## Summer 2015

Pearson Edexcel GCE in Core Mathematics 4 (6666/01)



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#### **General Marking Guidance**

• All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

• Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

• Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

• There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

• All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

• Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

• Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

#### EDEXCEL GCE MATHEMATICS

## **General Instructions for Marking**

- 1. The total number of marks for the paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.
- 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol  $\sqrt{}$  will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

- 6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

## **General Principles for Core Mathematics Marking**

(But note that specific mark schemes may sometimes override these general principles).

#### Method mark for solving 3 term quadratic:

#### 1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q)$$
, where  $|pq| = |c|$ , leading to  $x = ...$ 

 $(ax^2 + bx + c) = (mx + p)(nx + q)$ , where |pq| = |c| and |mn| = |a|, leading to x = ...

#### 2. Formula

Attempt to use the correct formula (with values for a, b and c).

#### 3. Completing the square

Solving  $x^2 + bx + c = 0$ :  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$ ,  $q \neq 0$ , leading to  $x = \dots$ 

#### Method marks for differentiation and integration:

#### 1. Differentiation

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

#### 2. Integration

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

#### Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

#### Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

## June 2015 6666/01 Core Mathematics 4 Mark Scheme

Question Number		Scheme	Marks	
<b>1.</b> (a)	(4 + 5	$(x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}} \left(1 + \frac{5x}{4}\right)^{\frac{1}{2}} = \underline{2} \left(1 + \frac{5x}{4}\right)^{\frac{1}{2}} \qquad \underline{(4)^{\frac{1}{2}} \text{ or } \underline{2}}$	<u>B1</u>	
	= {2}	$\left[1 + \left(\frac{1}{2}\right)(kx) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2 + \dots\right]$ see notes	M1 A1ft	
	= {2}	$\left[1 + \left(\frac{1}{2}\right)\left(\frac{5x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5x}{4}\right)^2 + \dots\right]$		
	= 2	$1 + \frac{5}{8}x - \frac{25}{128}x^2 + \dots$ See notes below!		
	= 2 +	$\frac{5}{4}x; -\frac{25}{64}x^2 + \dots$ isw	A1; A1	
(b)	$\begin{cases} x = -\frac{1}{2} \end{cases}$	$\frac{1}{10} \Rightarrow (4+5(0.1))^{\frac{1}{2}} = \sqrt{4.5} = \sqrt{\frac{9}{2}} = \frac{3}{\frac{\sqrt{2}}{\sqrt{2}}} = \frac{3}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}}$	[5]	
		$=\frac{3}{2}\sqrt{2}$ $\frac{3}{2}\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e.	B1	
			[1]	
(c)	$\frac{3}{2}\sqrt{2}$	or $1.5\sqrt{2}$ or $\frac{3}{\sqrt{2}} = 2 + \frac{5}{4} \left(\frac{1}{10}\right) - \frac{25}{64} \left(\frac{1}{10}\right)^2 + \dots = 2.121\dots$ See notes	M1	
	So, $\frac{3}{2}$	$\sqrt{2} = \frac{543}{256}$ or $\frac{3}{\sqrt{2}} = \frac{543}{256}$		
	yields,	$\sqrt{2} = \frac{181}{128}$ or $\sqrt{2} = \frac{256}{181}$ $\frac{181}{128}$ or $\frac{362}{256}$ or $\frac{543}{384}$ or $\frac{256}{181}$ etc.	A1 oe	
			[2] 8	
		Question 1 Notes	, , , , , , , , , , , , , , , , , , ,	
<b>1.</b> (a)	B1	$(4)^{\frac{1}{2}}$ or <u>2</u> outside brackets or <u>2</u> as candidate's constant term in their binomial expansion	1.	
	M1	Expands $(+kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified,		
		Eg: $1 + \left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(kx)^2$ or $1 + \dots + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(kx)^2$		
	where k is a numerical value and where $k \neq 1$ .			
	A1	A correct simplified or un-simplified $1 + \left(\frac{1}{2}\right)(kx) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(kx)^2$ expansion with <b>consis</b>	stent (kx).	
	Note	( <i>kx</i> ), $k \neq 1$ , must be consistent (on the RHS, not necessarily on the LHS) in a candidate	's expansion.	

<b>1.</b> (a) ctd.	Note	Award B1M1A0 for $2\left[1+\left(\frac{1}{2}\right)(5x)+\frac{\binom{1}{2}(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+\right]$ because $(kx)$ is not consistent.			
	Note	<b>Incorrect bracketing:</b> $2\left[1 + \left(\frac{1}{2}\right)\left(\frac{5x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5x^2}{4}\right) + \dots\right]$ is B1M1A0 unless recovered.			
	A1	$2 + \frac{5}{4}x$ (simplified fractions) or allow $2 + 1.25x$ or $2 + 1\frac{1}{4}x$			
	A1	Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$			
	SC	If a candidate <i>would otherwise score</i> 2 <sup>nd</sup> A0, 3 <sup>rd</sup> A0 then allow Special Case 2 <sup>nd</sup> A1 for either			
		SC: $2\left[1+\frac{5}{8}x;\right]$ or SC: $2\left[1+\frac{25}{128}x^2+\right]$ or SC: $\lambda\left[1+\frac{5}{8}x-\frac{25}{128}x^2+\right]$			
		or SC: $\left[\lambda + \frac{5\lambda}{8}x - \frac{25\lambda}{128}x^2 +\right]$ (where $\lambda$ can be 1 or omitted), where each term in the $\left[\right]$			
		is a simplified fraction or a decimal, $10 - 50$			
		<b>OR SC:</b> for $2 + \frac{10}{8}x - \frac{30}{128}x^2 +$ (i.e. for not simplifying their correct coefficients.)			
	Note	Candidates who write $2\left[1 + \left(\frac{1}{2}\right)\left(-\frac{5x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{5x}{4}\right)^2 + \dots\right]$ , where $k = -\frac{5}{4}$ and not $\frac{5}{4}$			
		and achieve $2 - \frac{5}{4}x - \frac{25}{64}x^2 +$ will get B1M1A1A0A1			
	Note	Ignore extra terms beyond the term in $x^2$ .			
	Note	You can ignore subsequent working following a correct answer. 2 2			
(b)	B1	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $k = \frac{3}{2}$ or 1.5 o.e. (Ignore how $k = \frac{3}{2}$ is found.)			
(c)	M1	Substitutes $x = \frac{1}{10}$ or 0.1 into their binomial expansion found in part (a) which must contain both			
		an x term and an $x^2$ term (or even an $x^3$ term) and equates this to either $\frac{3}{\sqrt{2}}$ or their $k\sqrt{2}$ from (b),			
		where $k$ is a numerical value.			
	Note	M1 can be implied by $\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\sqrt{2}}$ = awrt 2.121			
	Note	M1 <i>can be implied</i> by $\frac{1}{k}$ (their $\frac{543}{256}$ ), with their <i>k</i> found in part (b).			
	Note	M1 <i>cannot be implied</i> by $(k)$ (their $\frac{543}{256}$ ), with their k found in part (b).			
	A1	$\frac{181}{128}$ or any equivalent fraction, eg: $\frac{362}{256}$ or $\frac{543}{384}$ . Also allow $\frac{256}{181}$ or any equivalent fraction.			
	Note	Also allow A1 for $p = 181$ , $q = 128$ or $p = 181\lambda$ , $q = 128\lambda$			
		or $p = 256, q = 181$ or $p = 256\lambda, q = 181\lambda$ , where $\lambda \in \mathbb{Z}^+$			
	Note Note Note	You can recover work for part (c) in part (b). You cannot recover part (b) work in part (c). Candidates are allowed to restart and gain all 2 marks in part (c) from an incorrect part (b). Award M1 A1 for the correct answer from no working.			

<b>1.</b> (a)	Alternative methods for part (a)					
	$\frac{\text{Altern}}{\left\{(4+1)\right\}}$	$\frac{\text{ative method 1:}}{5x)^{\frac{1}{2}}} = (4)^{\frac{1}{2}} + (\frac{1}{2})(4)^{-\frac{1}{2}}(5x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(4)^{-\frac{3}{2}}(5x)^{2}$	ne binomial expansion.			
	B1 M1 A1 A1	$(4)^{\frac{1}{2}}$ or 2 Any two of three (un-simplified) terms correct. All three (un-simplified) terms correct. $2 + \frac{5}{4}x$ (simplified fractions) or allow $2 + 1.25x$ or $2 - \frac{5}{4}x$	$+1\frac{1}{4}x$			
	A1	Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$				
	Note The terms in C need to be evaluated. So $\frac{1}{2}C_0(4)^{\frac{1}{2}} + \frac{1}{2}C_1(4)^{-\frac{1}{2}}(5x); + \frac{1}{2}C_2(4)^{-\frac{3}{2}}(5x)^2$ without further working is B0M0A0.					
	Altern	<b>ative Method 2: Maclaurin Expansion</b> $f(x) = (4+5x)^{\frac{1}{2}}$				
	f"(x)=	$= -\frac{25}{4}(4+5x)^{-\frac{3}{2}}$	Correct $f''(x)$	B1		
	f'(x) =	$\frac{1}{(4+5x)^{\frac{1}{2}}}$ (5)	$\pm a(4+5x)^{-\frac{1}{2}}; a \neq \pm 1$	M1		
	1 ( <i>x</i> )-	$\frac{1}{2}(4+5x)^{-1}(5)$	$\frac{1}{2}(4+5x)^{-\frac{1}{2}}(5)$	A1 oe		
	$\left\{ \therefore f(0) \right\}$	$f(0) = 2$ , $f'(0) = \frac{5}{4}$ and $f''(0) = -\frac{25}{32}$				
	So, f(.	$x) = 2 + \frac{5}{4}x; -\frac{25}{64}x^2 + \dots$		A1; A1		

Question	Scheme		Marks
Number	$x^2 - 2xy - 4x^2 + 64 = 0$		Marks
2.	$\begin{pmatrix} x - 3xy - 4y + 04 - 0 \\ (\lambda y) & (\lambda y) \\ (\lambda$		M1 Δ 1
(a)	$\left\{\frac{4x}{4x} \times \right\}  \underline{2x} - \left(\underline{3y + 3x\frac{dy}{dx}}\right) - \underline{8y\frac{dy}{dx}} = \underline{0}$		<u>M1</u>
	$2x - 3y + (-3x - 8y)\frac{dy}{dx} = 0$		dM1
	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$	o.e.	A1 cso
			[5]
(b)	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow\right\} 2x - 3y = 0$		M1
	$y = \frac{2}{3}x \qquad \qquad x = \frac{3}{2}y$		A1ft
	$x^{2} - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^{2} + 64 = 0 \qquad \qquad \left(\frac{3}{2}y\right)^{2} - 3\left(\frac{3}{2}y\right)y - 4y^{2} + 64 = 0$		dM1
	$x^{2} - 2x^{2} - \frac{16}{9}x^{2} + 64 = 0 \implies -\frac{25}{9}x^{2} + 64 = 0 \qquad \frac{9}{4}y^{2} - \frac{9}{2}y^{2} - 4y^{2} + 64 = 0 \implies -\frac{25}{4}y^{2} + 64 = 0$	0	
	$\left\{ \Rightarrow x^2 = \frac{576}{25} \Rightarrow \right\}  x = \frac{24}{5} \text{ or } -\frac{24}{5} \qquad \left\{ \Rightarrow y^2 = \frac{256}{25} \Rightarrow \right\}  y = \frac{16}{5} \text{ or } -\frac{16}{5}$		A1 <b>cso</b>
	When $x = \pm \frac{24}{5}$ , $y = \frac{2}{3} \left( \frac{24}{5} \right)$ and $-\frac{2}{3} \left( \frac{24}{5} \right)$ When $y = \pm \frac{16}{5}$ , $x = \frac{3}{2} \left( \frac{16}{5} \right)$ and $-\frac{3}{2} \left( \frac{16}{5} \right)$		
	$\begin{pmatrix} 24 & 16 \end{pmatrix}$ and $\begin{pmatrix} 24 & 16 \end{pmatrix}$ on $\mu = 24$ $\mu = 16$ and $\mu = 24$ $\mu = 16$		ddM1
	$\left(\frac{5}{5}, \frac{5}{5}\right)$ and $\left(-\frac{5}{5}, -\frac{5}{5}\right)$ or $x = \frac{5}{5}, y = \frac{5}{5}$ and $x = -\frac{5}{5}, y = -\frac{5}{5}$ cso		A1
			[6] 11
	Alternative method for part (a)		
(a)	$\left\{\frac{\partial \mathbf{x}}{\partial \mathbf{x}} \times\right\}  \underline{2x \frac{dx}{dy}} - \left(\underline{3y \frac{dx}{dy} + 3x}\right) = \underline{0}$		M1 <u>A1</u> <u>M1</u>
	$(2x-3y)\frac{\mathrm{d}x}{\mathrm{d}y} - 3x - 8y = 0$		dM1
	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y} \text{ or } \frac{3y - 2x}{-3x - 8y}$	o.e.	A1 cso
			[5]
2 (a)	Question 2 Notes		
<b>General</b>	<b>Note</b> Writing down $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ from <b>no working</b> is full marks		
	<b>Note</b> Writing down $\frac{dy}{dx} = \frac{2x - 3y}{-3x - 8y}$ or $\frac{3y - 2x}{3x + 8y}$ from <b>no working</b> is M1A0B1M1A0		
	<b>Note</b> Few candidates will write $2x dx - 3y dx - 3x dy - 8y dy = 0$ leading to $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ ,	, o.e.	
	This should get full marks.		

<b>2.</b> (a)	M1	Differentiates implicitly to include either $\pm 3x \frac{dy}{dx}$ or $-4y^2 \rightarrow \pm ky \frac{dy}{dx}$ . (Ignore $\left(\frac{dy}{dx}=\right)$ ).			
	A1	Both $x^2 \rightarrow \underline{2x}$ and $\dots -4y^2 + 64 = 0 \rightarrow -8y\frac{dy}{dx} = 0$			
	Note	If an extra term appears then award A0.			
	M1	$-3xy \rightarrow -3x\frac{dy}{dx} - 3y$ or $-3x\frac{dy}{dx} + 3y$ or $3x\frac{dy}{dx} - 3y$ or $3x\frac{dy}{dx} + 3y$			
	Note	$2x - 3y - 3x\frac{dy}{dx} - 8y\frac{dy}{dx} \rightarrow 2x - 3y = 3x\frac{dy}{dx} + 8y\frac{dy}{dx}$			
	JN // 1	will get 1 <sup>st</sup> A1 (implied) as the "= $0$ " can be implied by the rearrangement of their equation.			
	alvii	dependent on the FIKS1 method mark being awarded.			
		An attempt to factorise out <b>all the terms in</b> $\frac{dy}{dx}$ as long as there are <i>at least two terms</i> in $\frac{dy}{dx}$ .			
		i.e + $(-3x - 8y)\frac{dy}{dx} =$ or = $(3x + 8y)\frac{dy}{dx}$ . (Allow combining in 1 variable).			
	A1	$\frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ or equivalent.			
	Note Note	<b>cso</b> If the candidate's solution is not completely correct, then do not give this mark. You cannot recover work for part (a) in part (b).			
<b>2.</b> (b)	M1	Sets their numerator of their $\frac{dy}{dx}$ equal to zero (or the denominator of their $\frac{dx}{dy}$ equal to zero) o.e.			
	Note	1 <sup>st</sup> M1 can also be gained by setting $\frac{dy}{dx}$ equal to zero in their $(2x - 3y - 3x\frac{dy}{dx} - 8y\frac{dy}{dx} = 0)$			
	Note Note	If their numerator involves one variable only then only the 1 <sup>st</sup> M1 mark is possible in part (b). If their numerator is a constant then no marks are available in part (b)			
	Note	If their numerator is in the form $\pm ax^2 \pm by = 0$ or $\pm ax \pm by^2 = 0$ then the first 3 marks are			
		possible in part (b).			
Note $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y} = 0$ is not sufficient for M1.					
	A1ft	Either			
		• Sets $2x - 3y$ to zero and obtains either $y = \frac{2}{3}x$ or $x = \frac{3}{2}y$			
		• the follow through result of making either y or x the subject from setting their numerator of their $\frac{dy}{dx}$ equal to zero			
		dx			
	dM1	dependent on the first method mark being awarded.			
		Substitutes <i>either</i> their $y = \frac{2}{3}x$ or their $x = \frac{3}{2}y$ into the original equation to give an equation in one variable only.			
-		$\frac{24}{24} = \frac{24}{16} = \frac{16}{16}$			
	A1	Obtains either $x = \frac{24}{5}$ or $-\frac{24}{5}$ or $y = \frac{10}{5}$ or $-\frac{10}{5}$ , (or equivalent) by correct solution only.			
		i.e. You can allow for example $x = \frac{40}{10}$ or 4.8, etc.			
	Note	$x = \sqrt{\frac{576}{25}}$ (not simplified) or $y = \sqrt{\frac{256}{25}}$ (not simplified) is not sufficient for A1.			

<b>2.</b> (b)	ddM1	dependent on both previous method marks being awarded in this part.
ctd		Method 1
		Either:
		• substitutes their x into their $y = \frac{2}{3}x$ or substitutes their y into their $x = \frac{3}{2}y$ , or
		• substitutes <i>the other of</i> their $y = \frac{2}{3}x$ or their $x = \frac{3}{2}y$ into the original equation,
		and achieves either:
		• exactly two sets of two coordinates <b>or</b>
		• exactly two distinct values for x and exactly two distinct values for y.
		Method 2 Either:
		• substitutes their first x-value, $x_1$ into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one y-value, $y_1$ and
		substitutes their second x-value, $x_2$ into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain 1 y-value $y_2$ or
		• substitutes their first y-value, $y_1$ into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one x-value $x_1$ and
		substitutes their second y-value, $y_2$ into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one x-value $x_2$ .
	Note	Three or more sets of coordinates given (without identification of two sets of coordinates) is ddM0.
	A1	Both $\left(\frac{24}{5}, \frac{16}{5}\right)$ and $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ , only by cso. Note that decimal equivalents are fine.
	Note	Also allow $x = \frac{24}{5}$ , $y = \frac{16}{5}$ and $x = -\frac{24}{5}$ , $y = -\frac{16}{5}$ all seen in their working to part (b).
<b>Note</b> Allow $x = \pm \frac{24}{5}$ , $y = \pm \frac{16}{5}$ for 3 <sup>rd</sup> A1.		Allow $x = \pm \frac{24}{5}$ , $y = \pm \frac{16}{5}$ for 3 <sup>rd</sup> A1.
	Note $x = \pm \frac{24}{5}$ , $y = \pm \frac{16}{5}$ followed by eg. $\left(\frac{16}{5}, \frac{24}{5}\right)$ and $\left(-\frac{16}{5}, -\frac{24}{5}\right)$	
		(eg. coordinates stated the wrong way round) is 3 <sup>rd</sup> A0.
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator
		for $\frac{dy}{dx}$ ) to gain all 6 marks in part (b).
	Note	Decimal equivalents to fractions are fine in part (b). i.e. $(4.8, 3.2)$ and $(-4.8, -3.2)$ .
		$(24 \ 16) (24 \ 16)$
	Note	$\left(\frac{24}{5}, \frac{10}{5}\right)$ and $\left(-\frac{24}{5}, -\frac{10}{5}\right)$ from no working is M0A0M0A0M0A0.
	Note	Candidates could potentially lose the final 2 marks for setting both their numerator and denominator
	Note	No credit in this part can be gained by only setting the denominator to zero.
		1

Question Number		Scheme	Marks	
3.	y = 4x -	$-xe^{\frac{1}{2}x}, x \ge 0$		
(a)	$\begin{cases} y=0 = 0 \end{cases}$	$\Rightarrow 4x - xe^{\frac{1}{2}x} = 0 \Rightarrow x(4 - e^{\frac{1}{2}x}) = 0 \Rightarrow$		
	$e^{\frac{1}{2}}$	Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x =$ in terms of $\pm \lambda \ln \mu$ where $\mu > 0$	M1	
		$4\ln 2 \operatorname{cao} (\operatorname{Ignore} x = 0)$	A1 [2]	
(b)	$\int \int r e^{\frac{1}{2}x}$	$\alpha x e^{\frac{1}{2}x} - \beta \int e^{\frac{1}{2}x} \{dx\}, \alpha > 0, \beta > 0$	M1	
(0)		$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x} \{dx\}, \text{ with or without } dx$	A1 (M1 on ePEN)	
		$= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} \{+c\}$ $2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$ o.e. with or without $+c$	A1	
(c)	$\left\{\int 4x dx\right\}$	$x = 2x^2$ $4x \rightarrow 2x^2$ or $\frac{4x^2}{2}$ o.e.	B1	
	( • 4ln2	2		
	$\left\{ \int_{0}^{0} \right\} $	$4x - xe^{\frac{1}{2}x} dx = \left[ 2x^2 - \left( 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} \right) \right]_0$		
	$= \left(2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 4e^{\frac{1}{2}(4\ln 2)}\right) - \left(2(0)^2 - 2(0)e^{\frac{1}{2}(0)} + 4e^{\frac{1}{2}(0)}\right)$ See notes			
	$= (32(\ln 2)^2 - 32(\ln 2) + 16) - (4)$			
	$= 32(\ln 2)$	$(2)^2 - 32(\ln 2) + 12$ $(2)^2 - 32(\ln 2) + 12$ , see notes	A1 [3]	
	Ouestion 3 Notes		8	
<b>3.</b> (a)	M1	Attempts to solve $e^{\frac{1}{2}x} - 4$ giving $x - 1$ in terms of $\pm 2 \ln y$ where $y > 0$		
(	A1	4ln 2 <b>cao</b> stated in part (a) only (Ignore $x = 0$ )		
(b)	NOT F	Part (b) appears as M1M1A1 on ePEN, but is now marked as M1A1A1.		
	M1	Integration by parts is applied in the form $\alpha x e^{\frac{1}{2}x} - \beta \int e^{\frac{1}{2}x} \{ dx \}$ , where $\alpha > 0, \beta > 0$ .		
		(must be in this form) with or without $dx$		
	A1	A1 $2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x} \{dx\}$ or equivalent, with or without $dx$ . Can be un-simplified.		
	A1	$2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$ or equivalent with or without + c. Can be un-simplified.		
	Note	You can also allow $2e^{\frac{1}{2}x}(x-2)$ or $e^{\frac{1}{2}x}(2x-4)$ for the final A1.		
	isw	You can ignore subsequent working following on from a correct solution.		
	SC	<b><u>SPECIAL CASE</u></b> : A candidate who uses $u = x$ , $\frac{dv}{dx} = e^{\frac{1}{2}x}$ , writes down the correct "b	y parts"	
		formula, but makes only one error when applying it can be awarded Special Case M1. (Applying their <i>v</i> counts for one consistent error.)		

<b>3.</b> (c)	B1	$4x \rightarrow 2x^2 \text{ or } \frac{4x^2}{2} \text{ oe}$	
	M1	<b>Complete</b> method of applying limits of their $x_A$ and 0 to all terms of an expression of the form	
		$\pm Ax^2 \pm Bxe^{\frac{1}{2}x} \pm Ce^{\frac{1}{2}x}$ (where $A \neq 0, B \neq 0$ and $C \neq 0$ ) and subtracting the correct way round.	
	Note	vidence of a proper consideration of the limit of 0 is needed for M1.	
	Note	So subtracting 0 is M0. $\ln 16$ or $2\ln 4$ or equivalent is fine as an upper limit	
	A1	A correct three term exact quadratic expression in $\ln 2$ .	
		For example allow for A1	
		• $32(\ln 2)^2 - 32(\ln 2) + 12$	
		• $8(2\ln 2)^2 - 8(4\ln 2) + 12$	
		• $2(4\ln 2)^2 - 32(\ln 2) + 12$	
		• $2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 12$	
	Note	Note that the constant term of 12 needs to be combined from $4e^{\frac{1}{2}(4\ln 2)} - 4e^{\frac{1}{2}(0)}$ o.e.	
	Note	Also allow $32 \ln 2(\ln 2 - 1) + 12$ or $32 \ln 2 \left( \ln 2 - 1 + \frac{12}{32 \ln 2} \right)$ for A1.	
	Note	Do not apply "ignore subsequent working" for incorrect simplification.	
		Eg: $32(\ln 2)^2 - 32(\ln 2) + 12 \rightarrow 64(\ln 2) - 32(\ln 2) + 12$ or $32(\ln 4) - 32(\ln 2) + 12$	
	Note	<b>Bracketing error:</b> $32 \ln 2^2 - 32(\ln 2) + 12$ , unless recovered is final A0.	
	Note	<b>Notation:</b> Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.	
	Note	5.19378 without seeing $32(\ln 2)^2 - 32(\ln 2) + 12$ is A0.	
	Note	5.19378 following from a correct $2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)$ is M1A0.	
	Note	5.19378 from no working is M0A0.	

Question Number	Scheme		Marks
4.	$l_1: \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix},  l_2: \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}. \text{ Let } \theta$ Note: You can mark parts (a) and (b) together.	= acute angle between $l_1$ and $l_2$ .	
(a)	$\{l_1 = l_2 \Rightarrow \mathbf{i}:\} 5 = 8 + 3\mu \Rightarrow \mu = -1$	Finds $\mu$ and substitutes their $\mu$ into $l_2$	M1
	So, $\left\{\overrightarrow{OA}\right\} = \begin{pmatrix} 8\\5\\-2 \end{pmatrix} - 1 \begin{pmatrix} 3\\4\\-5 \end{pmatrix} = \begin{pmatrix} 5\\1\\3 \end{pmatrix}$	$5\mathbf{i} + \mathbf{j} + 3\mathbf{k} \text{ or } \begin{pmatrix} 5\\1\\3 \end{pmatrix} \text{ or } (5, 1, 3)$	A1
(b)	$\{\mathbf{j}: -3 + \lambda = 5 + 4\mu \Longrightarrow\} -3 + \lambda = 5 + 4(-1) \Longrightarrow \lambda = 4$	Equates <b>j</b> components, substitutes their $\mu$ and solves to give $\lambda =$	[2] M1
	$\mathbf{k}: p - 3\lambda = -2 - 5\mu \Rightarrow \qquad \text{Equates } \mathbf{k} \text{ constrained}$ $p - 3(4) = -2 - 5(-1) \Rightarrow \underline{p = 15}$	pomponents, substitutes their $\lambda$ and their $\mu$ and solves to give $p = \dots$ or equates <b>k</b> components to give	M1
	or $\mathbf{k}: p - 3\lambda = 3 \Rightarrow$ their " $p - 3\lambda$ substi-	$3\lambda$ = the <b>k</b> value of <i>A</i> found in part (a)", itutes their $\lambda$ and solves to give $p =$	1711
	$p - 3(4) = 3 \Longrightarrow \underline{p = 15}$	<i>p</i> = 15	A1
(c)	$\mathbf{d}_1 = \begin{pmatrix} 0\\1\\-3 \end{pmatrix}, \ \mathbf{d}_2 = \begin{pmatrix} 3\\4\\-5 \end{pmatrix} \implies \begin{pmatrix} 0\\1\\-3 \end{pmatrix} \bullet \begin{pmatrix} 3\\4\\-5 \end{pmatrix}$	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$ .	[3] M1
	$\cos \theta = \pm K \left( \frac{0(3) + (1)(4) + (-3)(-5)}{\sqrt{(0)^2 + (1)^2 + (-3)^2}} \sqrt{(3)^2 + (4)^2 + (-5)^2} \right)$	An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$ .	dM1 (A1 on ePEN)
	$\cos \theta = \frac{19}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116 = 31.82 \ (2 \text{ dp})$	anything that rounds to 31.82	A1
(d)	$\overrightarrow{OB} = \begin{pmatrix} 11\\9\\-7 \end{pmatrix}; \ \overrightarrow{AB} = \begin{pmatrix} 11\\9\\-7 \end{pmatrix} - \begin{pmatrix} 5\\1\\3 \end{pmatrix} = \begin{pmatrix} 6\\8\\-10 \end{pmatrix} \text{ or } \overrightarrow{AB}$ $\left  \overrightarrow{AB} \right  = \sqrt{6^2 + 8^2 + (-10)^2} \left\{ = 10\sqrt{2} \right\}$	$\overline{AB} = 2 \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ See notes	[ <b>3</b> ] M1
	$\frac{d}{10\sqrt{2}} = \sin\theta$ Writes down a contract the shortest	orrect trigonometric equation involving distance, d. Eg: $\frac{d}{\text{their } AB} = \sin \theta$ , oe.	dM1
	$\left\{ d = 10\sqrt{2}\sin 31.82 \Rightarrow \right\} d = 7.456540753 = 7.46$ (3sf	anything that rounds to 7.46	A1
			[3] 11

<b>4.</b> (b)	Alternative method for part (b)				
	$\begin{cases} 3 \times \mathbf{j}: -9 + 3\lambda = 15 + 12\mu \\ \mathbf{k}: p - 3\lambda = -2 + 5\mu \end{cases}  p - 9 = 13 + 7\mu$		Eliı	minates $\lambda$ to write down an equation in <i>p</i> and $\mu$	M1
	$p-9=13+7(-1) \implies p=15$	Su	ubstitute	s their $\mu$ and solves to give $p = \dots$	M1
				p = 15	A1
<b>4.</b> (d)	<u>Alternative Methods for part (d)</u> Let X be the foot of	the pe	erpendic	cular from <i>B</i> onto $l_1$	1
	$\mathbf{d}_{1} = \begin{pmatrix} 0\\1\\-3 \end{pmatrix},  \overrightarrow{OX} = \begin{pmatrix} 5\\-3\\15 \end{pmatrix} + \lambda \begin{pmatrix} 0\\1\\-3 \end{pmatrix} = \begin{pmatrix} 5\\-3+\lambda\\15-3\lambda \end{pmatrix}$				
	$\overrightarrow{BX} = \begin{pmatrix} 5 \\ -3+\lambda \\ 15-3\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} = \begin{pmatrix} -6 \\ -12+\lambda \\ 22-3\lambda \end{pmatrix}$				
	Method 1				
	$\begin{pmatrix} -6 \end{pmatrix} \begin{pmatrix} 0 \end{pmatrix}$			(Allow a sign slip in	
	$\overrightarrow{BX} \bullet \mathbf{d}_1 = 0 \implies -12 + \lambda \bullet 1 = -12 + \lambda - \epsilon$	56 + 9	$\partial \lambda = 0$	$\operatorname{copying} \mathbf{d}_1$	
	$\left[\begin{array}{c} 22-3\lambda \end{array}\right] \left[\begin{array}{c} -3 \end{array}\right]$			Applies $BX \bullet \mathbf{d}_1 = 0$ and	
	39			solves the resulting	M1
	leading to $10\lambda - 78 = 0 \implies \lambda = \frac{1}{5}$			a value for $\lambda$ .	
	$\overline{BX} = \begin{pmatrix} -6 \\ -12 + \frac{39}{5} \\ 22 - 3\left(\frac{39}{5}\right) \end{pmatrix} = \begin{pmatrix} -6 \\ -\frac{21}{5} \\ -\frac{7}{5} \end{pmatrix}$			Substitutes their value of $\lambda$ into their $\overline{BX}$ . Note: This mark is dependent upon the previous M1 mark .	dM1
	$d = BX = \sqrt{\left(-6\right)^2 + \left(-\frac{21}{5}\right)^2 + \left(-\frac{7}{5}\right)^2} = 7.456540753$			awrt 7.46	A1
	Method 2			-	I
	Let $\beta = \left  \overrightarrow{BX} \right ^2 = 36 + 144 - 24\lambda + \lambda^2 + 484 - 132\lambda + 9$	$\lambda^2$	Fin	nds $\beta = \left  \overrightarrow{BX} \right ^2$ in terms of $\lambda$ ,	
	$= 10\lambda^2 - 156\lambda + 664$		f	inde $\frac{d\beta}{d\beta}$ and sets this result	M1
	So $\frac{\mathrm{d}\beta}{\mathrm{d}\lambda} = 20\lambda - 156 = 0 \Rightarrow \lambda = \frac{39}{5}$		equ	had sets this result had sets this result had to 0 and finds a value for $\lambda$ .	IVI I
	$\left \overline{BX}\right ^2 = 10\left(\frac{39}{5}\right)^2 - 156\left(\frac{39}{5}\right) + 664 = \frac{278}{5}$ Sub	ostitute Note	es their v e: This	value of $\lambda$ into their $\left  \overrightarrow{BX} \right ^2$ . mark is dependent upon the previous M1 mark.	dM1
	$d = BX = \sqrt{\frac{278}{5}} = 7.456540753$			awrt 7.46	A1

	Question 4 Notes				
<b>4.</b> (a)	M1	Finds $\mu$ and substitutes their $\mu$ into $l_2$			
		(5)			
	A1	Point of intersection of $5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ . Allow 1 or (5,	1, 3).		
		(3)			
	Note	You cannot recover the answer for part (a) in part (c) or	part (d).		
(b)	M1	Equates <b>j</b> components, substitutes their $\mu$ and solves to	give $\lambda = \dots$		
	M1	Equates <b>k</b> components, substitutes their $\lambda$ and their $\mu$ a	and solves to give $p = \dots$		
		or equates <b>k</b> components to give their " $p - 3\lambda$ = the <b>k</b> v	alue of <i>A</i> " found in part (b).		
	A1	<i>p</i> = 15			
(c)	NOTE	Part (c) appears as M1A1A1 on ePEN, but now is ma	rked as M1M1A1.		
	M1	Realisation that the dot product is required between $\pm A$	$\mathbf{d}_1$ and $\pm B\mathbf{d}_2$ .		
	Note	Allow one slip in candidates copying down their direction	on vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$ .		
	dM1	dependent on the FIRST method mark being awarde	d.		
		An attempt to apply the dot product formula between $\pm A$	$\mathbf{d}_1$ and $\pm B\mathbf{d}_2$ .		
	A1	anything that rounds to 31.82. This can also be achieved	1  by  180 - 148.1796 = a  wrt  31.	.82	
	Note	$\theta = 0.5553^{\circ}$ is A0.			
		0-16-60	-76		
	Note	M1A1 for $\cos \theta = \left( \frac{\sqrt{(0)^2 + (4)^2 + (-12)^2}}{\sqrt{(-3)^2 + (-4)^2}} \right)^{-1}$	$\left \frac{1}{100}\right ^{2} + (5)^{2}$ = $\frac{1}{\sqrt{160}}$		
	Alternat	ive Method: Vector Cross Product			
	Only ap	ply this scheme if it is clear that a candidate is applying	a vector cross product method	I.	
		$\begin{pmatrix} 0 \end{pmatrix} \begin{pmatrix} 3 \end{pmatrix} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \end{bmatrix}$	Paglication that the vector		
	<b>d</b> . × <b>d</b> . =	$= 1 \times 4 = 0 \times -3 = 7i - 9i - 3k$	cross product is required	M1	
	-1 -2		between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$ .	<u>1011</u>	
		$\sqrt{(7)^2 + (-9)^2 + (3)^2}$	An attempt to apply the	dM1	
		$\sin \theta = \frac{1}{\sqrt{(0)^2 + (1)^2 + (-3)^2}} \sqrt{(3)^2 + (4)^2 + (-5)^2}$	vector cross product formula	(A1 on ePEN)	
		$\sqrt{120}$			
	$\sin \theta =$	$\frac{\sqrt{139}}{\sqrt{10}\sqrt{50}} \Rightarrow \theta = 31.8203116 = 31.82 (2 dp)$	anything that rounds to 31.82	A1	
(1)	M1 F				
(u)		full method for finding <i>B</i> and for finding the magnitude of	AB or the magnitude of BA.		
		<b>1</b> dependent on the first method mark being awarded. Writes down correct trigonometric equation involving the shortest distance d			
		d d			
	E	Eg: $\frac{1}{\text{their } AB} = \sin\theta$ or $\frac{1}{\text{their } AB} = \cos(90 - \theta)$ , o.e., wh	ere "their $AB$ " is a value.		
	a	nd $\theta$ = "their $\theta$ " or stated as $\theta$			
	A1 a	nything that rounds to 7.46			

Question	Scheme	Marks
5.	Note: You can mark parts (a) and (b) together.	
(a)	$x = 4t + 3,  y = 4t + 8 + \frac{5}{2t}$	
	$\frac{dx}{dt} = 4$ , $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ Both $\frac{dx}{dt} = 4$ or $\frac{dt}{dx} = \frac{1}{4}$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$	B1
	So, $\frac{dy}{dx} = \frac{4 - \frac{5}{2}t^{-2}}{4} \left\{ = 1 - \frac{5}{8}t^{-2} = 1 - \frac{5}{8t^2} \right\}$ Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$	M1 o.e.
	{When $t = 2$ , } $\frac{dy}{dx} = \frac{27}{32}$	A1
	Way 2. Contasian Mathed	[3]
	$\frac{dy}{dx} = 1 - \frac{10}{(x-3)^2}$ , simplified or un-simplified.	B1
	$\frac{dy}{dx} = \pm \lambda \pm \frac{\mu}{(x-3)^2}, \ \lambda \neq 0, \mu \neq 0$	M1
	{When $t = 2, x = 11$ } $\frac{dy}{dx} = \frac{27}{32}$ $\frac{27}{32}$ or 0.84375 cao	A1
		[3]
	Way 3: Cartesian Method $\frac{1}{2}$ (2) (2) (2) (2) (2) (2) (2) (3) (4) (4) (4) (4) (4) (4) (4) (4) (4) (4	
	$\frac{dy}{dx} = \frac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$ Correct expression for $\frac{dy}{dx}$ , simplified or un-simplified.	B1
	$\left\{ = \frac{x^2 - 6x - 1}{(x - 3)^2} \right\}$ where $f(x) = \text{their } "x^2 + ax + b", g(x) = x - 3$	M1
	$\left\{ \text{When } t = 2, x = 11 \right\} \frac{dy}{dx} = \frac{27}{32} \qquad \qquad \frac{27}{32} \text{ or } 0.84375 \text{ cao}$	A1 [3]
(b)	$\left\{t = \frac{x-3}{4} \implies \right\} \ y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$ Eliminates <i>t</i> to achieve an equation in only <i>x</i> and <i>y</i>	M1
	$y = x - 3 + 8 + \frac{10}{x - 3}$	
	$y = \frac{(x-3)(x-3) + 8(x-3) + 10}{x-3}  or  y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ or $y = \frac{(x+5)(x-3) + 10}{x-3}  or  y = \frac{(x+5)(x-3)}{x-3} + \frac{10}{x-3}$ See notes	dM1
	$\Rightarrow y = \frac{x^2 + 2x - 5}{x - 3},  \{a = 2 \text{ and } b = -5\}$ $y = \frac{x^2 + 2x - 5}{x - 3}  \text{or } a = 2 \text{ and } b = -5$	A1 cso
		[3] 6

Questier		
Number	Scheme	Marks
<b>5.</b> (b)	Alternative Method 1 of Equating Coefficients	
	$y = \frac{x^2 + ax + b}{x - 3} \implies y(x - 3) = x^2 + ax + b$	
	$y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2 + 32t + 10$	
	$x^{2} + ax + b = (4t + 3)^{2} + a(4t + 3) + b$	
	$(4t+3)^{2} + a(4t+3) + b = 16t^{2} + 32t + 10$ Correct method of obtaining an equation in only <i>t</i> , <i>a</i> and <i>b</i>	M1
	t: $24+4a=32 \implies a=2$ Equates their coefficients in t and finds both $a=-$ and $b=-$	dM1
	constant: $9 + 3a + b = 10 \implies b = -5$	A1
		[3]
<b>5.</b> (b)	Alternative Method 2 of Equating Coefficients	
	$\left\{t = \frac{x-3}{4} \Rightarrow \right\} \ y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$ Eliminates <i>t</i> to achieve an equation in only <i>x</i> and <i>y</i>	M1
	$y = x - 3 + 8 + \frac{10}{x - 3} \implies y = x + 5 + \frac{10}{(x - 3)}$	
	$\underline{y(x-3)} = (x+5)(x-3) + 10 \implies x^2 + ax + b = \underline{(x+5)(x-3) + 10}$	dM1
	$r^2 + 2r = 5$ Correct algebra leading to	
	$\Rightarrow y = \frac{x + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$ $y = \frac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$	A1
	x - 3	cso
		[3]

		Question 5 Notes	
<b>5.</b> (a)	B1	$\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ or $\frac{dy}{dt} = \frac{8t^2 - 5}{2t^2}$ or $\frac{dy}{dt} = 4 - 5(2t)^{-2}(2)$ , etc.	
	Note	$\frac{dy}{dt}$ can be simplified or un-simplified.	
	Note	You can imply the B1 mark by later working.	
	M1	Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$ or $\frac{dy}{dt}$ multiplied by a candidate's $\frac{dt}{dx}$	
	Note	M1 can be also be obtained by substituting $t = 2$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then	
		dividing their values the correct way round.	
	A1	$\frac{27}{32}$ or 0.84375 <b>cao</b>	
(b)	<u>M1</u>	Eliminates <i>t</i> to achieve an equation in only <i>x</i> and <i>y</i> .	
	dM1	dependent on the first method mark being awarded.	
		Either: (ignoring sign slips or constant slips, noting that k can be 1) - (10)	
		• Combining all three parts of their $\underline{x-3} + \overline{8} + \left(\frac{10}{x-3}\right)$ to form a single fraction with a	
		common denominator of $\pm k(x-3)$ . Accept three separate fractions with the same denominator.	
		• Combining both parts of their $\underline{x+5} + (\frac{10}{x-3})$ , (where $\underline{x+5}$ is their $4(\frac{x-3}{4}) + 8$ ),	
		to form a single fraction with a common denominator of $\pm k(x-3)$ . Accept two separate fractions with the same denominator.	
		• Multiplies both sides of their $y = \underline{x-3} + \overline{8} + \left(\frac{10}{x-3}\right)$ or their $y = \underline{x+5} + \left(\frac{10}{x-3}\right)$ by	
		$\pm k(x-3)$ . Note that all terms in their equation must be multiplied by $\pm k(x-3)$ .	
	Note	Condone "invisible" brackets for dM1.	
	A1	Correct algebra with no incorrect working leading to $y = \frac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$	
	Note	Some examples for the award of dM1 in (b):	
		<b>dM0</b> for $y = x - 3 + 8 + \frac{10}{x - 3} \rightarrow y = \frac{(x - 3)(x - 3) + 8 + 10}{x - 3}$ . Should be + 8(x - 3) +	
		<b>dM0</b> for $y = x - 3 + \frac{10}{x - 3} \rightarrow y = \frac{(x - 3)(x - 3) + 10}{x - 3}$ . The "8" part has been omitted.	
		<b>dM0</b> for $y = x + 5 + \frac{10}{x - 3} \rightarrow y = \frac{x(x - 3) + 5 + 10}{x - 3}$ . Should be + 5(x - 3) +	
		<b>dM0</b> for $y = x + 5 + \frac{10}{x - 3} \rightarrow y(x - 3) = x(x - 3) + 5(x - 3) + 10(x - 3)$ . Should be just 10.	
	Note	$y = x + 5 + \frac{10}{x - 3} \rightarrow y = \frac{x^2 + 2x - 5}{x - 3}$ with no intermediate working is dM1A1.	
	Note	$dM0 \text{ for } y = x - 3 + \frac{10}{x - 3} \rightarrow y = \frac{(x - 3)(x - 3) + 10}{x - 3}.$ The "8" part has been omitted. $dM0 \text{ for } y = x + 5 + \frac{10}{x - 3} \rightarrow y = \frac{x(x - 3) + 5 + 10}{x - 3}.$ Should be+5(x - 3) + $dM0 \text{ for } y = x + 5 + \frac{10}{x - 3} \rightarrow y(x - 3) = x(x - 3) + 5(x - 3) + 10(x - 3).$ Should be just 10. $y = x + 5 + \frac{10}{x - 3} \rightarrow y = \frac{x^2 + 2x - 5}{x - 3}$ with no intermediate working is dM1A1.	

Question Number	Scheme	Marks
<b>6.</b> (a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}  \mathrm{d}x \ , \ x = 1 + 2\sin\theta$	
	$\frac{dx}{d\theta} = 2\cos\theta \qquad \qquad \frac{dx}{d\theta} = 2\cos\theta \text{ or } 2\cos\theta \text{ used correctly}$ in their working. Can be implied	B1
	$\left\{ \int \sqrt{(3-x)(x+1)}  \mathrm{d}x \text{ or } \int \sqrt{(3+2x-x^2)}  \mathrm{d}x \right\}$	
	$= \int \sqrt{(3 - (1 + 2\sin\theta))((1 + 2\sin\theta) + 1)} 2\cos\theta \{d\theta\}$ Substitutes for both x and dx, where $dx \neq \lambda d\theta$ . Ignore $d\theta$	M1
	$= \int \sqrt{(2 - 2\sin\theta)(2 + 2\sin\theta)} 2\cos\theta \{d\theta\}$ $\int \sqrt{(4 - 4\sin^2\theta)(2 + 2\sin\theta)} 2\cos\theta \{d\theta\}$	
	$= \int \sqrt{(4 - 4\sin^2\theta)} 2\cos\theta \left\{ d\theta \right\}$	
	$= \int \sqrt{\left(4 - 4(1 - \cos^2\theta) 2\cos\theta \left\{ d\theta \right\}}  \text{or}  \int \sqrt{4\cos^2\theta} 2\cos\theta \left\{ d\theta \right\} \qquad \text{Applies } \cos^2\theta = 1 - \sin^2\theta \text{ see notes}$	M1
	$= 4 \int \cos^2 \theta  d\theta, \ \{k = 4\}$ $4 \int \cos^2 \theta  d\theta \text{ or } \int 4 \cos^2 \theta  d\theta$ Note: $d\theta$ is required here	A1
	$0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2} \Rightarrow \theta = -\frac{\pi}{6}$	
	and $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$ See notes	B1
		[5]
(b)	$\left\{k\int\cos^2\theta\left\{d\theta\right\}\right\} = \left\{k\right\}\int\left(\frac{1+\cos 2\theta}{2}\right)\left\{d\theta\right\} $ Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \{k\} \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$ Integrates to give $\pm \alpha \theta \pm \beta \sin 2\theta$ , $\alpha \neq 0$ , $\beta \neq 0$ or $k(\pm \alpha \theta \pm \beta \sin 2\theta)$	M1 (A1 on ePEN)
	$\left\{ \operatorname{So} 4 \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2 \theta  \mathrm{d}\theta = \left[ 2\theta + \sin 2\theta \right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \right\}$	
	$= \left(2\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right)\right) - \left(2\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right)\right)$	
	$\left\{ = \left( \pi \right) - \left( -\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \right\} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2} \qquad \qquad$	A1 cao cso
	$\frac{1}{6}\left(8\pi+3\sqrt{3}\right)$	
		[3] 8

		Question 6 Notes
<b>6.</b> (a)	B1	$\frac{dx}{d\theta} = 2\cos\theta$ . Also allow $dx = 2\cos\theta d\theta$ . This mark can be implied by later working.
	Note	You can give B1 for $2\cos\theta$ used correctly in their working.
	M1	Substitutes $x = 1 + 2\sin\theta$ and their $dx \left( \text{from their rearranged} \frac{dx}{d\theta} \right)$ into $\sqrt{(3-x)(x+1)} dx$ .
	Note Note	Condone bracketing errors here. $dx \neq \lambda d\theta$ . For example $dx \neq d\theta$ .
	Note	Condone substituting $dx = \cos\theta$ for the 1 <sup>st</sup> M1 after a correct $\frac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta d\theta$
	M1	Applies either • $1 - \sin^2 \theta = \cos^2 \theta$ • $\lambda - \lambda \sin^2 \theta$ or $\lambda (1 - \sin^2 \theta) = \lambda \cos^2 \theta$ • $4 - 4\sin^2 \theta = 4 + 2\cos 2\theta - 2 = 2 + 2\cos 2\theta = 4\cos^2 \theta$ to their expression where $\lambda$ is a numerical value.
	A1	Correctly proves that $\int \sqrt{(3-x)(x+1)}  dx$ is equal to $4 \int \cos^2 \theta  d\theta$ or $\int 4 \cos^2 \theta  d\theta$
	All three previous marks must have been awarded before A1 can be awarded. Their final answer must include $d\theta$ . You can ignore limits for the final A1 mark.	
	B1	Evidence of a correct equation in $\sin\theta$ or $\sin^{-1}\theta$ for both x-values leading to both $\theta$ values. Eg:
		• $0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2}$ which then leads to $\theta = -\frac{\pi}{6}$ , and
		• $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1$ which then leads to $\theta = \frac{\pi}{2}$
	Note	Allow B1 for $x = 1 + 2\sin\left(-\frac{\pi}{6}\right) = 0$ and $x = 1 + 2\sin\left(\frac{\pi}{2}\right) = 3$
	Note	Allow B1 for $\sin \theta = \left(\frac{x-1}{2}\right)$ or $\theta = \sin^{-1}\left(\frac{x-1}{2}\right)$ followed by $x = 0, \ \theta = -\frac{\pi}{6}; \ x = 3, \ \theta = \frac{\pi}{2}$
(b)	NOTE	Part (b) appears as M1A1A1 on ePEN, but is now marked as M1M1A1.
	M1	Writes down a correct equation involving $\cos 2\theta$ and $\cos^2 \theta$
		Eg: $\cos 2\theta = 2\cos^2 \theta - 1$ or $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$ or $\lambda \cos^2 \theta = \lambda \left(\frac{1 + \cos 2\theta}{2}\right)$
	M1	and <i>applies</i> it to their integral. Note: Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral. Integrates to give an expression of the form $\pm \alpha \theta \pm \beta \sin 2\theta$ or $k(\pm \alpha \theta \pm \beta \sin 2\theta)$ , $\alpha \neq 0$ , $\beta \neq 0$
	A1	(can be simplified or un-simplified). A <i>correct solution in part (b)</i> leading to a "two term" exact answer.
	N-4-	Eg: $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ or $\frac{8\pi}{6} + \frac{\sqrt{3}}{2}$ or $\frac{1}{6} \left( 8\pi + 3\sqrt{3} \right)$
	Note	Candidates can work in terms of k (note that k is not given in (a)) for the M1M1 marks in part (b).
	Note	If they incorrectly obtain $4\int_{-\pi}^{\frac{\pi}{2}} \cos^2\theta d\theta$ in part (a) (or guess $k = 4$ ) then the final A1 is available
		for a correct solution in part (b) only.

Question Number	Scheme	Marks
<b>7.</b> (a)	$\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$	
	$2 \equiv A(P-2) + BP$ Can be implied.	M1
	A = -1, B = 1 Either one.	A1
	giving $\frac{1}{(P-2)} - \frac{1}{P}$ See notes. cao, aef	A1
(b)	$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{2}P(P-2)\cos 2t$	[3]
	$\int \frac{2}{P(P-2)} dP = \int \cos 2t  dt \qquad \text{can be implied by later working}$	B1 oe
	$\pm \lambda \ln(P-2) \pm \mu \ln P,$ $\lambda \neq 0,  \mu \neq 0$	M1
	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t \ (+c)$ $\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$	A1
	$\{t = 0, P = 3 \Rightarrow\}  \ln 1 - \ln 3 = 0 + c  \{\Rightarrow c = -\ln 3 \text{ or } \ln(\frac{1}{3})\}$ See notes	M1
	$\ln (P-2) - \ln P = \frac{1}{2} \sin 2t - \ln 3$	
	$\ln\left(\frac{3(P-2)}{P}\right) = \frac{1}{2}\sin 2t$	
	Starting from an equation of the form + $2 \ln (P - R) + \mu \ln P = + K \sin \delta t + c$	
	$3(P-2) \qquad \qquad$	
	$\frac{1}{P} = e^2$ eliminate their logarithms.	MI
	Must have a constant of integration that need	
	not be evaluated (see note)	-
	$3(P-2) = Pe^{z \sin 2t} \Rightarrow 3P-6 = Pe^{z \sin 2t}$ A complete method of rearranging to make P the subject.	D (1
	gives $3P - Pe^{\frac{1}{2}\sin 2t} = 6 \implies P(3 - e^{\frac{1}{2}\sin 2t}) = 6$ Must have a constant of integration	dM1
	$P = \frac{6}{4} * \frac{1}{2}$ that need not be evaluated (see note)	
	$(3 - e^{\frac{1}{2}\sin 2t})$ Correct proof.	A1 * cso
		[7]
(c)	{population = $4000 \Rightarrow$ } $P = 4$ States $P = 4$ or applies $P = 4$	M1
	1 Obtains $\pm \lambda \sin 2t = \ln k$ or $\pm \lambda \sin t = \ln k$ ,	
	$\left  \frac{-\sin 2t}{2} = \ln \left( \frac{-1}{4} \right) \right  = \ln \left( \frac{-1}{2} \right) $ $\lambda \neq 0, k > 0$ where $\lambda$ and k are numerical	M1
	values and $\lambda$ can be 1	
	t = 0.4728700467 anything that rounds to $0.473$	A1
	Do not apply isw here	[3] 13
		15

Question Number		Scheme	Marks
Tumber	Method	<u>2 for Q7(b)</u>	
<b>7.</b> (b)	ln ( <i>I</i>	$P-2) - \ln P = \frac{1}{2}\sin 2t \ (+c)$ As before for	B1M1A1
	lı	$n\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$	
	$\frac{(P-P)}{P}$	$\frac{2}{2} = e^{\frac{1}{2}\sin 2t + c} \text{ or } \frac{(P-2)}{P} = Ae^{\frac{1}{2}\sin 2t}$ Starting from an equation of the form $\pm \lambda \ln(P-\beta) \pm \mu \ln P = \pm K \sin \delta t + c,$ $\lambda, \mu, \beta, K, \delta \neq 0,$ applies a fully correct method to eliminate their logarithms. <b>Must have a constant of integration</b> <b>that need not be evaluated (see note)</b>	3 <sup>rd</sup> M1
	$(P-2)$ $\Rightarrow P(1)$	$APe^{\frac{1}{2}\sin 2t} \Rightarrow P - APe^{\frac{1}{2}\sin 2t} = 2$ $-Ae^{\frac{1}{2}\sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2t})}$ A complete method of rearranging to make P the subject. Condone sign slips or constant errors. <b>Must have a</b> <b>constant of integration that need</b> <b>not be evaluated (see note)</b>	4 <sup>th</sup> dM1
	${t=0, t}$	$P = 3 \implies 3 = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2(0)})}$ (Allocate this mark as the 2 <sup>nd</sup> M1 mark on ePEN).	2 <sup>nd</sup> M1
	$\left\{ \Rightarrow 3 = \right.$	$=\frac{2}{(1-A)} \Rightarrow A = \frac{1}{3}$	
	$\Rightarrow P =$	$\frac{2}{\left(1-\frac{1}{3}e^{\frac{1}{2}\sin 2t}\right)} \Rightarrow P = \frac{6}{\left(3-e^{\frac{1}{2}\sin 2t}\right)}*$ Correct proof.	A1 * cso
		Question 7 Notes	
<b>7.</b> (a)	M1	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\frac{2}{P(P-2)} = \frac{A}{P} + \frac{A}{P}$	$\frac{B}{(P-2)}$
	Note A1	A and B are not referred to in question. Either one of $A = -1$ or $B = 1$ .	
	A1	$\frac{1}{(P-2)} - \frac{1}{P}$ or any equivalent form. This answer <i>cannot</i> be recovered from part (b	).
	Note	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\frac{A}{P}$	$+ + \frac{B}{(P-2)}$
		is seen in their working.	
	Note	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)} - \frac{1}{P}$ , so as to gain all three	e marks.
	Note	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A + B = 2, -2A = 2 \Longrightarrow A = -1$ ,	B = 1

<b>7.</b> (b)	B1	Separates variables as shown on the Mark Scheme. $dP$ and $dt$ should be in the correct positions,
		though this mark can be implied by later working. Ignore the integral signs. 2 $1$ $1$ $1$
	Note	Eg: $\int \frac{2}{P^2 - 2P} dP = \int \cos 2t  dt \text{ or } \int \frac{1}{P(P-2)} dP = \frac{1}{2} \int \cos 2t  dt \text{ o.e. are also fine for B1.}$
	1 <sup>st</sup> M1	$\pm \lambda \ln(P-2) \pm \mu \ln P, \ \lambda \neq 0, \ \mu \neq 0.$ Also allow $\pm \lambda \ln(M(P-2)) \pm \mu \ln NP; \ M, N$ can be 1.
	Note	Condone $2\ln(P-2) + 2\ln P$ or $2\ln(P(P-2))$ or $2\ln(P^2-2P)$ or $\ln(P^2-2P)$
	1 <sup>st</sup> A1	Correct result of $\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$ or $2\ln(P-2) - 2\ln P = \sin 2t$
		o.e. with or without $+c$
	2 <sup>nu</sup> M1	Some evidence of using both $t = 0$ and $P = 3$ in an integrated equation containing a constant of integration. Eg: c or A, etc.
	3 <sup>rd</sup> M1	Starting from an equation of the form $\pm \lambda \ln(P - \beta) \pm \mu \ln P = \pm K \sin \delta t + c, \ \lambda, \mu, \beta, K, \delta \neq 0,$
	4 <sup>th</sup> M1	applies a fully correct method to eliminate their logarithms. dependent on the third method mark being awarded.
	Note	For the $3^{rd}$ M1 and $4^{th}$ M1 marks, a candidate needs to have included a constant of integration,
		in their working. eg. c, A, ln A or an evaluated constant of integration.
	2 <sup>nd</sup> A1	Correct proof of $P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ . Note: This answer is given in the question.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c \text{ followed by } \frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t} + e^c \text{ is } 3^{\text{rd}} \text{ M0, } 4^{\text{th}} \text{ M0, } 2^{\text{rd}} \text{ A0.}$
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c \rightarrow \frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c} \rightarrow \frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t} + e^{c} \text{ is final M1M0A0}$
	<u>4<sup>th</sup> M1</u>	for making <i>P</i> the subject
	Note the <i>P</i> the su	ere are three type of manipulations here which are considered acceptable for making biect.
	(1) M1	for $\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P-6 = Pe^{\frac{1}{2}\sin 2t} \Rightarrow P(3-e^{\frac{1}{2}\sin 2t}) = 6$
		$\Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$
	( <b>2</b> ) M1	for $\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3 - \frac{6}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3 - e^{\frac{1}{2}\sin 2t} = \frac{6}{P} \Rightarrow \Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$
	( <b>3</b> ) M1	for $\left\{ \ln(P-2) + \ln P = \frac{1}{2}\sin 2t + \ln 3 \Rightarrow \right\} P(P-2) = 3e^{\frac{1}{2}\sin 2t} \Rightarrow P^2 - 2P = 3e^{\frac{1}{2}\sin 2t}$
		$\Rightarrow (P-1)^2 - 1 = 3e^{\frac{1}{2}\sin 2t} \text{ leading to } P =$
(c)	M1	States $P = 4$ or applies $P = 4$
	M1	Obtains $\pm \lambda \sin 2t = \ln k$ or $\pm \lambda \sin t = \ln k$ , where $\lambda$ and k are numerical values and $\lambda$ can be 1
	A1	anything that rounds to 0.473. (Do not apply isw here)
	Note	Do not apply ignore subsequent working for A1. (Eg: 0.473 followed by 473 years is A0.)
	Note	<u>Use of <math>P = 4000</math></u> : Without the mention of $P = 4$ , $\frac{1}{2}\sin 2t = \ln 2.9985$ or $\sin 2t = 2\ln 2.9985$
		or $\sin 2t = 2.1912$ will usually imply MOM1A0
	Note	<u>Use of Degrees:</u> $t = awrt 2/.1$ will usually imply M1M1A0

Question Number	Scheme		Marks
<b>8.</b> (a)	$\left\{ y = 3^x \Longrightarrow \right\} \frac{\mathrm{d}y}{\mathrm{d}x} = 3^x \ln 3$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3^x \ln 3 \text{ or } \ln 3 \left( \mathrm{e}^{x \ln 3} \right) \text{ or } y \ln 3$	B1
	<b>Either T</b> : $y - 9 = 3^2 \ln 3(x - 2)$	Saa notas	M1
	or T: $y = (3^2 \ln 3)x + 9 - 18 \ln 3$ , where $9 = (3^2 \ln 3)x + 9 - 18 \ln 3$	3)(2) + c	IVI I
	$\{\text{Cuts } x\text{-axis} \Rightarrow y = 0 \Rightarrow \}$	Sets $y = 0$ in their tensors equation	
	$-9 = 9\ln 3(x-2)$ or $0 = (3^2 \ln 3)x + 9 - 18\ln 3$ ,	Sets $y = 0$ in their tangent equation and progresses to $x =$	M1
	So, $x = 2 - \frac{1}{\ln 3}$	$2 - \frac{1}{\ln 3}$ or $\frac{2\ln 3 - 1}{\ln 3}$ o.e.	A1 <b>cso</b>
			[4]
(b)	$V = \pi \int \left(3^x\right)^2 \left\{dx\right\} \text{ or } \pi \int 3^{2x} \left\{dx\right\} \text{ or } \pi \int 9^x \left\{dx\right\}$	$V = \pi \int (3^x) \text{ with or without } dx,$ which can be implied	B1 o.e.
		Eg: either $3^{2x} \rightarrow \frac{3^{2x}}{\pm \alpha (\ln 3)}$ or $\pm \alpha (\ln 3) 3^{2x}$	
	$= \left\{\pi\right\} \left(\frac{3^{2x}}{2\ln 3}\right)  \text{or}  = \left\{\pi\right\} \left(\frac{9^x}{\ln 9}\right)$	or $9^x \to \frac{9^x}{\pm \alpha (\ln 9)}$ or $\pm \alpha (\ln 9)9^x$ , $\underline{\alpha \in }$	M1
	$3^{2x} \rightarrow \frac{3}{21}$	$\frac{2x}{\ln 3}$ or $9^x \rightarrow \frac{9^x}{\ln 9}$ or $e^{2x\ln 3} \rightarrow \frac{1}{2\ln 3} (e^{2x\ln 3})$	A1 o.e.
	$\left\{ V = \pi \int_0^2 3^{2x}  \mathrm{d}x = \left\{ \pi \right\} \left[ \frac{3^{2x}}{2\ln 3} \right]_0^2 \right\} = \left\{ \pi \right\} \left( \frac{3^4}{2\ln 3} - \frac{1}{2\ln 3} \right)$	$ \left\{ = \frac{40\pi}{\ln 3} \right\} $ <b>Dependent on the previous</b> <b>method mark.</b> Substitutes x = 2 and $x = 0$ and subtracts the correct way round	dM1
	$V_{\text{cone}} = \frac{1}{3}\pi(9)^2 \left(\frac{1}{\ln 3}\right) \left\{ = \frac{27\pi}{\ln 3} \right\}$	$V_{\text{cone}} = \frac{1}{3}\pi (9)^2 (2 - \text{their } (a)). \text{ See notes.}$	B1ft
	$\left\{ \text{Vol}(S) = \frac{40\pi}{\ln 3} - \frac{27\pi}{\ln 3} \right\} = \frac{13\pi}{\ln 3}$	$\frac{13\pi}{\ln 3}$ or $\frac{26\pi}{\ln 9}$ or $\frac{26\pi}{2\ln 3}$ etc., isw	A1 o.e.
		$\{ \text{Eg: } p = 13\pi, \ q = \ln 3 \}$	[6]
			10
(b)	Alternative Method 1: Use of a substitution $f(x) = \frac{1}{2}$		
	$V = \pi \int (3^x) \{ dx \}$		B1 o.e.
	$\left\{ u = 3^x \Longrightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 3^x \ln 3 = u \ln 3 \right\}  V = \left\{ \pi \right\} \int \frac{u^2}{u \ln 3} \left\{ \mathrm{d}u \right\}$	$= \left\{\pi\right\} \int \frac{u}{\ln 3} \left\{du\right\}$	
	$(3^x)$	$\int^2 \rightarrow \frac{u^2}{\pm \alpha (\ln 3)}$ or $\pm \alpha (\ln 3)u^2$ , where $u = 3^x$	M1
	$-\left\{ n\right\} \left( \frac{2\ln 3}{2\ln 3} \right)$	$(3^x)^2 \rightarrow \frac{u^2}{2(\ln 3)}$ , where $u = 3^x$	A1
	$\left\{ V = \pi \int_{0}^{2} (3^{x})^{2} dx = \left\{ \pi \right\} \left[ \frac{u^{2}}{2 \ln 3} \right]_{1}^{9} \right\} = \left\{ \pi \right\} \left( \frac{9^{2}}{2 \ln 3} - \frac{1}{2 \ln 3} \right)$	$\frac{1}{3} \begin{cases} = \frac{40\pi}{\ln 3} \end{cases}$ Substitutes limits of 9 and 1 in <i>u</i> (or 2 and 0 in <i>x</i> ) and subtracts the correct way round.	dM1
	then apply the main scheme.		

		Question 8 Notes
<b>8.</b> (a)	B1	$\frac{dy}{dx} = 3^x \ln 3$ or $\ln 3(e^{x \ln 3})$ or $y \ln 3$ . Can be implied by later working.
	M1	Substitutes either $x = 2$ or $y = 9$ into their $\frac{dy}{dx}$ which is a function of x or y to find $m_T$ and
		• either applies $y - 9 = (\text{their } m_T)(x - 2)$ , where $m_T$ is a numerical value.
		• or applies $y = (\text{their } m_T)x + \text{their } c$ , where $m_T$ is a numerical value and $c$ is found
		by solving $9 = (\text{their } m_T)(2) + c$
	Note	The first M1 mark can be implied from later working.
	M1	Sets $y = 0$ in their <i>tangent</i> equation, where $m_T$ is a numerical value, (seen or implied)
		and progresses to $x = \dots$
	A1	An exact value of $2 - \frac{1}{\ln 3}$ or $\frac{2\ln 3 - 1}{\ln 3}$ or $\frac{\ln 9 - 1}{\ln 3}$ by a correct solution only.
	Note	Allow A1 for $2 - \frac{\lambda}{\lambda \ln 3}$ or $\frac{\lambda(2\ln 3 - 1)}{\lambda \ln 3}$ or $\frac{\lambda(\ln 9 - 1)}{\lambda \ln 3}$ or $2 - \frac{\lambda}{\lambda \ln 3}$ , where $\lambda$ is an integer, and ignore subsequent working.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$ ) is M0 M0 in part (a).
	Note	Candidates who invent a value for $m_T$ (which bears no resemblance to their gradient function)
	Note	<b>cannot</b> gain the 1 <sup>st</sup> M1 and 2 <sup>nd</sup> M1 mark in part (a). A decimal answer of 1.089760773 (without a correct <b>exact</b> answer) is A0.
<b>8.</b> (b)	<b>B1</b>	A correct expression for the volume with or without $dx$
	Note	<b>Eg:</b> Allow B1 for $\pi \left[ \left( 3^x \right)^2 \left\{ dx \right\} \text{ or } \pi \left[ 3^{2x} \left\{ dx \right\} \text{ or } \pi \left[ 9^x \left\{ dx \right\} \text{ or } \pi \left[ \left( e^{x \ln 3} \right)^2 \left\{ dx \right\} \right] \right] \right]$
		or $\pi \int (e^{2x \ln 3}) \{ dx \}$ or $\pi \int e^{x \ln 9} \{ dx \}$ with or without $dx$
	M1	Either $3^{2x} \rightarrow \frac{3^{2x}}{\pm \alpha (\ln 3)}$ or $\pm \alpha (\ln 3) 3^{2x}$ or $9^x \rightarrow \frac{9^x}{\pm \alpha (\ln 9)}$ or $\pm \alpha (\ln 9) 9^x$
		$e^{2x\ln 3} \rightarrow \frac{e^{2x\ln 3}}{\pm \alpha(\ln 3)}$ or $\pm \alpha(\ln 3)e^{2x\ln 3}$ or $e^{x\ln 9} \rightarrow \frac{e^{x\ln 9}}{\pm \alpha(\ln 9)}$ or $\pm \alpha(\ln 9)e^{x\ln 9}$ , etc where $\alpha \in \mathbb{C}$
	Note	$3^{2x} \rightarrow \frac{3^{2x+1}}{\pm \alpha (\ln 3)}$ or $9^x \rightarrow \frac{9^{x+1}}{\pm \alpha (\ln 3)}$ are allowed for M1
	Note	$3^{2x} \to \frac{3^{2x+1}}{2x+1}$ or $9^x \to \frac{9^{x+1}}{x+1}$ are both M0
	Note	M1 can be given for $9^{2x} \rightarrow \frac{9^{2x}}{\pm \alpha (\ln 9)}$ or $\pm \alpha (\ln 9)9^{2x}$
	A1	Correct integration of $3^{2x}$ . Eg: $3^{2x} \rightarrow \frac{3^{2x}}{2\ln 3}$ or $\frac{3^{2x}}{\ln 9}$ or $9^x \rightarrow \frac{9^x}{\ln 9}$ or $e^{2x\ln 3} \rightarrow \frac{1}{2\ln 3} (e^{2x\ln 3})$
	dM1	dependent on the previous method mark being awarded. Attempts to apply $y = 2$ and $y = 0$ to integrated expression and whether the second states $y = 1$
	Note	Autompts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way found. Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0
	THUR	Evidence of a proper consideration of the mint of 0 is needed for M1. So subtracting 0 is M0.

	dM1	dependent on the previous method mark being awarded. Attempts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way round
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	B1ft	$V_{\text{cone}} = \frac{1}{3}\pi (9)^2 (2 - \text{their answer to part } (a)).$
		Sight of $\frac{27\pi}{\ln 3}$ implies the B1 mark.
	Note	Alternatively they can apply the volume formula to the line segment. They need to achieve the result highlighted by **** on either page 29 or page 30 in order to obtain the B1ft mark.
	A1	$\frac{13\pi}{\ln 3}$ or $\frac{26\pi}{\ln 9}$ or $\frac{26\pi}{2\ln 3}$ , etc., where their answer is in the form $\frac{p}{q}$
	Note Note	The $\pi$ in the volume formula is only needed for the 1 <sup>st</sup> B1 mark and the final A1 mark. A decimal answer of 37.17481128 (without a correct <b>exact</b> answer) is A0.
	Note	A candidate who applies $\int 3^x dx$ will either get B0 M0 A0 M0 <b>B0</b> A0 or B0 M0 A0 M0 <b>B1</b> A0
	Note	$\pi \int 3^{x^2} dx$ unless recovered is B0.
	Note	Be careful! A correct answer may follow from incorrect working
		$V = \pi \int_{0}^{2} 3^{x^{2}} dx - \frac{1}{3} \pi (9)^{2} \left(\frac{1}{\ln 3}\right) = \pi \left[\frac{3^{x^{2}}}{2\ln 3}\right]_{0}^{2} - \frac{27\pi}{\ln 3} = \frac{\pi 3^{4}}{2\ln 3} - \frac{\pi}{2\ln 3} - \frac{27\pi}{\ln 3} = \frac{13\pi}{\ln 3}$
		would score B0 M0 A0 dM0 M1 A0.
<b>8.</b> (b)	2 <sup>nd</sup> B1ft	mark for finding the Volume of a Cone
	$V_{\rm cone} = \pi$	$\int_{2-\frac{1}{\ln 3}}^{2} (9x\ln 3 - 18\ln 3 + 9)^2 dx$
	ſ	$\left[\left(9x\ln 3 - 18\ln 3 + 9\right)^3\right]^2$ Award B1ft here where their
	$=\pi$	$\frac{(1 + 1 + 1 + 2)}{27 \ln 3}$ = $\frac{1}{2 - \frac{1}{\ln 3}}$ or their part (a) answer
		$\left( \left( \left( \left( 1 \right) \right)^{3} \right)^{3} \right) \right)$
	$=\pi$	$\left(\frac{\left(18\ln 3 - 18\ln 3 + 9\right)^{3}}{27\ln 3}\right) - \left(\frac{\left(9\left(2 - \frac{1}{\ln 3}\right)\ln 3 - 18\ln 3 + 9\right)}{27\ln 3}\right)\right)$
	$=\pi$	$\left(\left(\frac{729}{27\ln 3}\right) - \left(\frac{\left(18\ln 3 - 9 - 18\ln 3 + 9\right)^3}{27\ln 3}\right)\right)$
	$=\frac{27}{\ln n}$	$\frac{\pi}{3}$

<b>9</b> (1-)	2 <sup>nd</sup> B1ft mark for finding the Volume of a Cone
<b>ð.</b> (D)	Alternative method 2:
	$V_{\text{cone}} = \pi \int_{2 - \frac{1}{\ln 3}} (9x \ln 3 - 18 \ln 3 + 9)^2  \mathrm{d}x$
	$= \pi \int_{2^{-1} \ln 3}^{2} \left( 81x^{2} \left( \ln 3 \right)^{2} - 324x \left( \ln 3 \right)^{2} + 162x \ln 3 - 324 \ln 3 + 324 (\ln 3)^{2} + 81 \right) dx$
	$=\pi \left[ 27x^{3}(\ln 3)^{2} - 162x^{2}(\ln 3)^{2} + 81x^{2}\ln 3 - 324x\ln 3 + 324x(\ln 3)^{2} + 81x \right]_{2-\frac{1}{\ln 3}}^{2}$ Award B1ft here where their lower limit is $2-\frac{1}{\ln 2}$
	In 3 **** or their part (a) answer.
	$\left(\left(216(\ln 3)^2 - 648(\ln 3)^2 + 324\ln 3 - 648\ln 3 + 648(\ln 3)^2 + 162\right)\right)$
	$= \pi \left[ 27 \left( 2 - \frac{1}{\ln 3} \right)^3 \left( \ln 3 \right)^2 - 162 \left( 2 - \frac{1}{\ln 3} \right)^2 \left( \ln 3 \right)^2 + 81 \left( 2 - \frac{1}{\ln 3} \right)^2 \ln 3 \right] \right]$
	$\left( -324\left(2 - \frac{1}{\ln 3}\right)\ln 3 + 324\left(2 - \frac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2 - \frac{1}{\ln 3}\right) \right) \right)$
	$\left(27\left(8-\frac{12}{\ln 3}+\frac{6}{(\ln 3)^2}-\frac{1}{(\ln 3)^3}\right)(\ln 3)^2-162\left(4-\frac{4}{\ln 3}+\frac{1}{(\ln 3)^2}\right)(\ln 3)^2\right)\right)$
	$=\pi \left  \left( 216(\ln 3)^2 - 324\ln 3 + 162 \right) - \right  + 81 \left( 4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2} \right) \ln 3 - 324 \left( 2 - \frac{1}{\ln 3} \right) \ln 3$
	$\left( + 324\left(2 - \frac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2 - \frac{1}{\ln 3}\right) \right) \right)$
	$\left(216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3} - 648(\ln 3)^2 + 648\ln 3 - 162\right)\right)$
	$= \pi \left[ \left( 216(\ln 3)^2 - 324\ln 3 + 162 \right) - \right] + 324\ln 3 - 324 + \frac{81}{\ln 3} - 648\ln 3 + 324 \right]$
	$\left( + 648(\ln 3)^2 - 324\ln 3 + 162 - \frac{81}{\ln 3} \right) \right)$
	$=\pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3}\right)\right)$
	$=\frac{27\pi}{\ln 3}$
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