Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Monday 16 January 2017 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

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1. $f(x) = 2^x - 10 \sin x - 2$, where x is measured in radians

(a) Show that $f(x) = 0$ has a root, α, between 2 and 3

(2)

(b) Use linear interpolation once on the interval [2, 3] to find an approximation to α. Give your answer to 3 decimal places.

(3)

Leave blank

Question 1 continued

Q1

(Total 5 marks)

Leave blank

2. The quadratic equation

$$2x^2 - x + 3 = 0$$

has roots α and β.

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$

(2)

(c) find a quadratic equation which has roots

$$\left(2\alpha - \frac{1}{\beta}\right) \text{ and } \left(2\beta - \frac{1}{\alpha}\right)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers.

(4)

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Question 2 continued

Q2

(Total 7 marks)

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3. $f(x) = x^4 + 2x^3 + 26x^2 + 32x + 160$

Given that $x = -1 + 3i$ is a root of the equation $f(x) = 0$, use algebra to find the three other roots of $f(x) = 0$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

Leave blank

Question 3 continued

Q3

(Total 7 marks)

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4. (a) Use the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(2r+1)(3r+1) = \frac{1}{6}n(n+1)\left(an^2 + bn + c\right)$$

where a, b and c are integers to be determined.

(5)

(b) Hence find the value of

$$\sum_{r=10}^{20} r(2r+1)(3r+1)$$

(2)

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Question 4 continued

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Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 7 marks)

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5. The complex number z is given by

$$z = -7 + 3\mathrm{i}$$

Find

(a) $|z|$

(1)

(b) arg z, giving your answer in radians to 2 decimal places.

(2)

Given that $\frac{z}{1+\mathrm{i}} + w = 3 - 6\mathrm{i}$

(c) find the complex number w, giving your answer in the form $a + b\mathrm{i}$, where a and b are real numbers. You must show all your working.

(3)

(d) Show the points representing z and w on a single Argand diagram.

(2)

Leave blank

Question 5 continued

Q5

(Total 8 marks)

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6. $$f(x) = x^3 - \frac{1}{2x} + x^{\frac{3}{2}}, \qquad x > 0$$

The root α of the equation $f(x) = 0$ lies in the interval $[0.6, 0.7]$.

(a) Taking 0.6 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(5)

(b) Show that your answer to part (a) is correct to 3 decimal places.

(2)

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Question 6 continued

Q6

(Total 7 marks)

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7. (i)

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents a stretch, scale factor 3, parallel to the x-axis.

(b) Find the matrix **B**.

(2)

(ii)

$$\mathbf{M} = \begin{pmatrix} -4 & 3 \\ -3 & -4 \end{pmatrix}$$

The matrix **M** represents an enlargement with scale factor k and centre (0, 0), where $k > 0$, followed by a rotation anticlockwise through an angle θ about (0, 0).

(a) Find the value of k.

(2)

(b) Find the value of θ, giving your answer in radians to 2 decimal places.

(2)

(c) Find $\mathbf{M}^{-1}$

(2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 10 marks)

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8. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(at^2, 2at)$ lies on C.

(a) Using calculus, show that the normal to C at P has equation

$$y + tx = at^3 + 2at$$

(5)

The point S is the focus of the parabola C.

The point B lies on the positive x-axis and $OB = 5OS$, where O is the origin.

(b) Write down, in terms of a, the coordinates of the point B.

(1)

A circle has centre B and touches the parabola C at two distinct points Q and R.

Given that $t \neq 0$,

(c) find the coordinates of the points Q and R.

(4)

(d) Hence find, in terms of a, the area of triangle BQR.

(2)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

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Question 8 continued

Q8

(Total 12 marks)

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9. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n}\left(4r^3 - 3r^2 + r\right) = n^3(n+1)$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 5^{2n} + 3n - 1$$

is divisible by 9

(6)

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Question 9 continued

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END