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Other names

Pearson Edexcel
International
Advanced Level

Centre Number

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Candidate Number

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Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Friday 19 May 2017 – Morning

Time: 1 hour 30 minutes

Paper Reference

WFM01/01**You must have:**

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1. The quadratic equation

$$3x^2 - 5x + 1 = 0$$

has roots α and β .

Without solving the quadratic equation, find the exact value of

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

(4)

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Question 1 continued

Q1

(Total 4 marks)



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2. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & 4 \\ -k & 2k \\ 3 & 0 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) find the matrix \mathbf{AB} , (2)

(b) find the exact value of k for which $\det(\mathbf{AB}) = 0$ (2)

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Question 2 continued

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(Total 4 marks)

Q2

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3. Prove by induction that for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^n \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$

(5)

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Question 3 continued

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(Total 5 marks)

Q3

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4. The rectangular hyperbola H has parametric equations

$$x = 4t, \quad y = \frac{4}{t}$$

The straight line with equation $3y - 2x = 10$ intersects H at the points A and B .

Given that the point A is above the x -axis,

- (a) find the coordinates of the point A and the coordinates of the point B .

(5)

- (b) Find the coordinates of the midpoint of AB .

(2)

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Question 4 continued

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(Total 7 marks)

Q4

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$$f(x) = 30 + \frac{7}{\sqrt{x}} - x^5, \quad x > 0$$

(a) Starting with the interval $[2, 2.1]$, use interval bisection twice to find an interval of width 0.025 that contains α .

(4)

(b) Taking 2 as a first approximation to α , apply the Newton-Raphson process once to $f(x)$ to find a second approximation to α , giving your answer to 2 decimal places.

(5)



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Question 5 continued

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(Total 9 marks)

Q5

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6. (a) Use the standard results for $\sum_{r=1}^n r^2$ and for $\sum_{r=1}^n r^3$ to show that, for all positive integers n ,

$$\sum_{r=1}^n r^2(r+1) = \frac{n}{a}(n+1)(n+2)(3n+b)$$

where a and b are integers to be found.

(4)

- (b) Hence find the value of

$$\sum_{r=25}^{49} (r^2(r+1) + 2)$$

(4)

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Question 6 continued

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Q6

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7.

$$f(z) = z^4 + 4z^3 + 6z^2 + 4z + a$$

where a is a real constant.

Given that $1 + 2i$ is a complex root of the equation $f(z) = 0$

(a) write down another complex root of this equation.

(1)

(b) (i) Hence, find the other roots of the equation $f(z) = 0$

(ii) State the value of a .

(7)

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Question 7 continued

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(Total 8 marks)

Q7

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- (a) Using calculus, show that an equation of the tangent to C at P is

This tangent cuts the directrix of C at the point $A(-a, 6)$, where a is a constant.

- (d) Hence find the exact coordinates of the point P , giving each coordinate as a simplified surd. (3)



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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

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$$z = \frac{1}{5} - \frac{2}{5}i$$

- (3)

$$zw = \lambda i$$

(3)

- (i) find $\frac{4}{3}(z + w)$,

- (4)



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Question 9 continued

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Question 9 continued

Q9

(Total 10 marks)



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10. In your answers to this question, the elements of each matrix should be expressed in exact form in surds where necessary.

The transformation U , represented by the 2×2 matrix \mathbf{P} , is a rotation through 45° anticlockwise about the origin.

- (a) Write down the matrix \mathbf{P} . (1)

The transformation V , represented by the 2×2 matrix \mathbf{Q} , is a rotation through 60° anticlockwise about the origin.

- (b) Write down the matrix \mathbf{Q} . (1)

The transformation U followed by the transformation V is the transformation T . The transformation T is represented by the matrix \mathbf{R} .

- (c) Use your matrices from parts (a) and (b) to find the matrix \mathbf{R} . (3)

- (d) Give a full geometric description of T as a single transformation. (2)

- (e) Deduce from your answers to parts (c) and (d) that $\sin 75^\circ = \frac{1 + \sqrt{3}}{2\sqrt{2}}$ and find the exact value of $\cos 75^\circ$, explaining your answers fully. (2)

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Question 10 continued

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Question 10 continued

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(Total 9 marks)

Q10

TOTAL FOR PAPER: 75 MARKS

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