Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Monday 15 January 2018 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. $$f(x) = 3x^2 - \frac{5}{3\sqrt{x}} - 6, \quad x > 0$$

The single root α of the equation $f(x) = 0$ lies in the interval [1.5, 1.6].

(a) Taking 1.5 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.
(4)

(b) Use linear interpolation once on the interval [1.5, 1.6] to find another approximation to α. Give your answer to 3 decimal places.
(3)

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Question 1 continued

Q1

(Total 7 marks)

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2. $$f(z) = z^4 - 6z^3 + 38z^2 - 94z + 221$$

(a) Given that $z = 2 + 3i$ is a root of the equation $f(z) = 0$, use algebra to find the three other roots of $f(z) = 0$

(7)

(b) Show the four roots of $f(z) = 0$ on a single Argand diagram.

(2)

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Question 2 continued

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Question 2 continued

Q2

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3. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(an+b)$$

where a and b are integers to be determined.

(4)

(b) Given that

$$\sum_{r=5}^{25} r^2(r+1) + \sum_{r=1}^{k} 3^r = 140543$$

find the value of the integer k.

(4)

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Question 3 continued

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Question 3 continued

Q3

(Total 8 marks)

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4. The quadratic equation

$$3x^2 + 2x + 5 = 0$$

has roots α and β.

Without solving the equation,

(a) find the value of $\alpha^2 + \beta^2$

(2)

(b) show that $\alpha^3 + \beta^3 = \dfrac{82}{27}$

(2)

(c) find a quadratic equation which has roots

$$\left(\alpha + \frac{\alpha}{\beta^2}\right) \text{ and } \left(\beta + \frac{\beta}{\alpha^2}\right)$$

giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers.

(4)

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Question 4 continued

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Question 4 continued

Q4

(Total 8 marks)

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5. (i) Given that

$$\frac{2z+3}{z+5-2\mathrm{i}} = 1+\mathrm{i}$$

find z, giving your answer in the form $a + b\mathrm{i}$, where a and b are real constants.

(5)

(ii) Given that

$$w = (3+\lambda\mathrm{i})(2+\mathrm{i})$$

where λ is a real constant, and that

$$|w| = 15$$

find the possible values of λ.

(4)

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Question 5 continued

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Question 5 continued

Q5

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6. The parabola C has equation $y^2 = 32x$ and the point S is the focus of this parabola. The point $P(2, 8)$ lies on C and the point T lies on the directrix of C. The line segment PT is parallel to the x-axis.

(a) Write down the coordinates of S.

(1)

(b) Find the length of PT.

(1)

(c) Using calculus, show that the tangent to C at the point P has equation

$$y = 2x + 4$$

(4)

The hyperbola H has equation $xy = 4$. The tangent to C at P meets H at the points L and M.

(d) Find the exact coordinates of the points L and M, giving your answers in their simplest form.

(6)

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Question 6 continued

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Q6

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7. (i)

$$\mathbf{A} = \begin{pmatrix} 6 & k \\ -3 & -4 \end{pmatrix}, \text{ where } k \text{ is a real constant, } k \neq 8$$

Find, in terms of k,

(a) $\mathbf{A}^{-1}$

(3)

(b) $\mathbf{A}^2$

(1)

Given that $\mathbf{A}^2 + 3\mathbf{A}^{-1} = \begin{pmatrix} 5 & 9 \\ -3 & -5 \end{pmatrix}$

(c) find the value of k.

(3)

(ii)

$$\mathbf{M} = \begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix}$$

The matrix **M** represents a one way stretch, parallel to the y-axis, scale factor p, where $p > 0$, followed by a rotation anticlockwise through an angle θ about (0, 0).

(a) Find the value of p.

(2)

(b) Find the value of θ.

(2)

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Question 7 continued

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Q7

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8. (i) A sequence of numbers is defined by

$$u_1 = 3$$

$$u_{n+1} = u_n + 3n - 2 \qquad n \geqslant 1$$

Prove by induction that, for all positive integers n,

$$u_n = \frac{3}{2}n^2 - \frac{7}{2}n + 5$$

(5)

(ii) Prove by induction that, for all positive integers n,

$$\mathrm{f}(n) = 3^{2n+3} + 40n - 27 \text{ is divisible by } 64$$

(6)

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Q8

(Total 11 marks)

TOTAL FOR PAPER 75 MARKS

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