Mark Scheme (Results)

Summer 2014

Pearson Edexcel International A Level in Further Pure Mathematics F1 (WFM01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper or ag- answer given
- ⎿ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\text{ where}|pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q),\text{ where}|pq|=|c|\text{ and }|mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n\rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n\rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

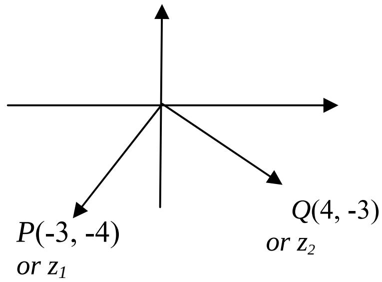
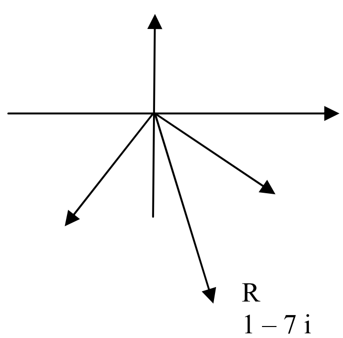
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme		Marks
1.	$(r+1)(r-1)=r^2-1$	Correct expansion. Allow $r^2-r+r-1$	B1
	$\sum_{r=1}^{200} r^2 = \frac{1}{6}200(201)(401)$	Use of $\frac{1}{6}n(n+1)(2n+1)$ with $n=200$	M1
	$\sum_{r=1}^{200} -1 = -200$	Cao (May be implied by their work)	B1
	$\sum_{r=1}^{200}(r^2-1) = 2686700 - 200 = 2686500$	2686500	A1
	Note use of $\sum_{r=1}^{200} -1 = -1$ gives a sum of 2686699 and usually scores B1M1B0A0		
			Total 4

Question Number	Scheme		Marks
	Mark (a) and (b) together		
2.(a)	$-2-3\text{i}$	cao	B1
			(1)
	Way 1		
(b)	$p = -\text{sum of roots} = -(-2+3\text{i}-2-3\text{i})$ **or** $q = \text{product of roots} = (-2+3\text{i})(-2-3\text{i})$	A correct approach for **either** p **or** q	M1
	$p = 4,\ q = 13$	1st A1: One value correct 2nd A1: Both values correct **Can be implied by a correct equation or expression e.g** $z^2+4z+13$	A1A1
			(3)
			Total 4
	(b) Way 2		
	$(z-(-2+3\text{i}))(z-(-2-3\text{i}))$	$z-(-2+3\text{i})$ **and** $z-(-2-3\text{i})$ and attempt to expand (condone invisible brackets)	M1
	Equation is $z^2+4z+13(=0)$ **or** $p = 4,\ q = 13$	1st A1: One value correct 2nd A1: Both values correct Condone use of x instead of z	A1 A1
	(b) Way 3		
	$(-2+3\text{i})^2 + p(-2+3\text{i})+q=0 \Rightarrow (3p-12)\text{i}+q-2p-5=0$		
	$(3p-12)\text{i}+q-2p-5=0$ $\Rightarrow 3p-12=0,\ q-2p-5$ $\Rightarrow p=\ldots$ or $q=\ldots$	Substitutes – 2 + 3i or – 2 – 3i into the given equation, compares real and imaginary parts and obtains a real value for p or a real value for q	M1
	$p = 4,\ q = 13$	1st A1: One value correct 2nd A1: Both values correct	A1A1
	(b) Way 4		
	$z^2+pz+q=0 \Rightarrow z=\frac{-p\pm\sqrt{p^2-4q}}{2}$		
	$\frac{-p\pm\sqrt{p^2-4q}}{2} = -2\pm 3\text{i} \Rightarrow -\frac{p}{2}=-2 \Rightarrow p=\ldots$		M1
	Correct method to find a value for p		
	$p=4$		A1
	$p^2-4q=-36 \Rightarrow q=13$	Correct value for q	A1
	(b) Way 5		
	$(-2+3\text{i})^2+p(-2+3\text{i})+q=0$ and $(-2-3\text{i})^2+p(-2-3\text{i})+q=0$		
	$\Rightarrow 24i-6pi=0 \Rightarrow p=\ldots$ M1: Substitutes both roots into the given equation and attempts to solve simultaneously to obtain a **real** value for p or a **real** value q		M1
	$p = 4,\ q = 13$	1st A1: One value correct 2nd A1: Both values correct	A1A1

Question Number	Scheme		Marks
3.(a)	$\det\mathbf{A} = 4\times -3 - a\times -2 (= 2a-12)$	Any correct form (possibly un-simplified) of the determinant	B1
	$\text{adj}\mathbf{A} = \begin{pmatrix} -3 & 2 \\ -a & 4 \end{pmatrix}$	Correct attempt at swapping elements in the major diagonal and changing signs in the minor diagonal. Three or four of the numbers in the matrix should be correct e.g. allow one slip	M1
	$\mathbf{A}^{-1} = \frac{1}{2a-12}\begin{pmatrix} -3 & 2 \\ -a & 4 \end{pmatrix}$	Correct inverse	A1
			(3)
(b)	$\begin{pmatrix} 4 & -2 \\ a & -3 \end{pmatrix} + \frac{2}{2a-12}\begin{pmatrix} -3 & 2 \\ -a & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Correct statement for **A**, **"**their" inverse and use of the correct identity matrix	M1
	$\Rightarrow \begin{pmatrix} 4-\frac{6}{2a-12} & -2+\frac{4}{2a-12} \\ a-\frac{2a}{2a-12} & -3+\frac{8}{2a-12} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$		
	So e.g. $4-\frac{6}{2a-12} = 1 \Rightarrow a = ...$	**Adds** their **A** and 2 x their$\mathbf{A}^{-1}$ and compares corresponding elements to form an equation in a and attempts to solve as far as $a =$ … **or adds** an element of their **A** and the corresponding element of 2 x their$\mathbf{A}^{-1}$ to form an equation in a and attempts to solve as far as $a =$ …	M1
	$a = 7$ only	Cao (from a **correct** equation i.e. their $\mathbf{A}^{-1}$ might be incorrect) **If they solve a second equation and get a different value for *a*, this mark can be withheld.**	A1
			(3)
			Total 6
	(b) Way 2 (does not use the inverse)		
	$\mathbf{A}+2\mathbf{A}^{-1} = \mathbf{I} \Rightarrow \mathbf{A}^2 + 2\mathbf{I} = \mathbf{A}$ $\begin{pmatrix} 16-2a & -2 \\ a & 9-2a \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ a & -3 \end{pmatrix}$	Correct statement for $\mathbf{A}^2 + 2\mathbf{I} = \mathbf{A}$ using **A**, **"**their" $\mathbf{A}^2$ and use of the correct identity matrix	M1
	$\Rightarrow \begin{pmatrix} 16-2a+2 & -2 \\ a & 9-2a+2 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ a & -3 \end{pmatrix}$		
	So $16-2a+2 = 4$ or $11-2a = -3$	**Adds** their $\mathbf{A}^2$ and $2\mathbf{I}$, compares elements, forms an equation in a and attempts to solve as far as $a =$ …	M1
	$a = 7$	cao	A1

Question Number	Scheme		Marks
4.(a)	$f(4) = ...$ **and** $f(5) = ...$	Attempt to evaluate both f(4) and f(5) NB f(5) = $2\sqrt{5}-3$ but this must be evaluated to score the A1	M1
	$f(4) = -1$, $f(5) = 1.472...$ Sign change (and $f(x)$ is continuous) therefore a root α exists between $x = 4$ and $x = 5$	Both values correct $f(4) = -1$, and $f(5) = 1.472...$(awrt 1.5) , sign change (or equivalent) and conclusion E.g. $f(4) = -1 < 0$ and f(5) = 1.472 > 0 so $4 < \alpha < 5$ scores M1A1	A1
			(2)
(b)	$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{1}{2}}$	M1: $x^n \to x^{n-1}$ A1: Either $\frac{3}{2}x^{\frac{1}{2}}$ or $-\frac{3}{2}x^{-\frac{1}{2}}$ A1: Correct derivative	M1A1A1
	$x_1 = 4.5 - \frac{f(4.5)}{f'(4.5)} = 4.5 - \frac{0.1819805153..}{2.474873734...}$	Correct attempt at Newton-Raphson Can be implied by a correct answer or their working provided a correct derivative is seen or implied.	M1
	$= 4.426$	Cao (Ignore any subsequent applications)	A1
	Correct derivative followed by correct answer scores full marks in (b) **Correct answer with <u>no</u> working scores no marks in (b)**		
			(5)
(c)	$\frac{5-\alpha}{1.472} = \frac{\alpha - 4}{1}$ or $\frac{\alpha-4}{1} = \frac{5-4}{1.472+1}$	A **correct** statement for α or 5 - α or α - 4	M1
	$\alpha(1.472+1) = 5 + 4 \times 1.472$ so $\alpha = ...$	Attempt to make "α" the subject (allow poor manipulation). **Dependent** on the previous M1.	**d**M1
	$\alpha = 4.405$	cao	A1
	There are no marks for interval bisection		
			(3)
			Total 10

Question	Scheme		Marks
5. (a)		M1: One point in third quadrant and one in the fourth quadrant. Can be vectors, points or even lines. A1: The points representing the complex numbers plotted correctly. The points must be indicated by a scale (could be ticks on axes) **or** labelled with coordinates or as complex numbers.	M1A1
			(2)
(b)	M1 requires a correct strategy e.g.		
	1. Gradient OP = $\frac{4}{3}$, Gradient OQ = $\frac{-3}{4}$ $\quad \frac{4}{3}\times -\frac{3}{4} = \ldots\ldots\ldots\ldots$		M1
	2. Angles with Im axis are $\tan^{-1}\frac{3}{4}$ and $\tan^{-1}\frac{4}{3}$. $\quad \tan^{-1}\frac{3}{4}+\tan^{-1}\frac{4}{3}=\ldots\ldots$		
	3. Angles with Re axis are $\tan^{-1}\frac{4}{3}$ and $\tan^{-1}\frac{3}{4}$. $180-\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{4}{3}=\right)\ldots\ldots$		
	4. $OP^2=3^2+4^2$, $OQ^2=3^2+4^2$, $PQ^2=1^2+7^2$ $OP^2+OQ^2=\ldots\ldots$		
	5. $\overrightarrow{OP}=\begin{pmatrix}-3\\-4\end{pmatrix}, \overrightarrow{OQ}=\begin{pmatrix}4\\-3\end{pmatrix} \Rightarrow \overrightarrow{OP}.\overrightarrow{OQ}=\ldots\ldots$		
	$\frac{4}{3}\times-\frac{3}{4}=-1$ so right angle or 53.1 + 26.9 = 90 (accept 53 + 27) or radians or $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{4}{3}=\frac{\pi}{2}$ $OP^2+OQ^2=PQ^2=50$ so right angle or $\overrightarrow{OP}.\overrightarrow{OQ}=0$ so right angle **Correct work with no slips and conclusion**		A1
			(2)
(c)	(c) $z_1 + z_2 = 1 - 7\text{i}$ 	New point as shown. It must be the point 1 – 7 i and it must be correctly plotted. The point must be indicated by a scale (could be ticks on axes) **or** labelled with coordinates or as a complex number. **May be on its own axes.**	B1
			(1)
(d)	Writes down another fact about OPQR other than OP being perpendicular to OQ: e.g. OP = OQ, OP is parallel to QR, QR = PR, QR is perpendicular to PR		B1
	Sufficient justification that OPQR is a square and conclusion If their explanation could relate to something other than a square score B0		B1
			(2)
			Total 7

Question Number	Scheme		Marks
6.(a)	$\alpha+\beta=-\frac{5}{3}$ **and** $\alpha\beta=-\frac{1}{3}$	**Both** correct statements	B1
	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=......$	**Use** of a **correct** identity for $\alpha^3+\beta^3$ (may be implied by their work)	M1
	$\alpha^3+\beta^3=\left(-\frac{5}{3}\right)^3+\left(-\frac{5}{3}\right)=-\frac{170}{27}$	Correct value (allow exact equivalent – even the correct recurring decimal - 6.296296......)	A1
	Special Case – but must be a complete method – generally there are no marks for finding the roots explicitly $\alpha=\frac{-5+\sqrt{37}}{6},\ \beta=\frac{-5-\sqrt{37}}{6}\Rightarrow\alpha^3+\beta^3=\left(\frac{-5+\sqrt{37}}{6}\right)^3+\left(\frac{-5-\sqrt{37}}{6}\right)^3=-\frac{170}{27}$ Could score 3/3 in (a) B1: Both correct roots M1: Cube and add A1: Correct value		
			(3)
(b)	$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{-\frac{170}{27}}{-\frac{1}{3}}=\frac{170}{9}$	M1: Uses the identity $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$ A1: Correct sum (or equivalent)	M1 A1
	$\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=-\frac{1}{3}$	Correct product	B1
	$x^2-\frac{170}{9}x-\frac{1}{3}$	Uses $x^2-(\textit{their sum})x+(\textit{their product})$ (= 0 not needed here)	M1
	$9x^2-170x-3=0$	This equation or any integer multiple including = 0. **Follow through their sum and product.**	A1ft
			(5)
			Total 8
	(b) Alternative using explicit roots		
	$\alpha=\frac{-5+\sqrt{37}}{6},\ \beta=\frac{-5-\sqrt{37}}{6}$		
	$\frac{\alpha^2}{\beta}=\frac{85-14\sqrt{37}}{9},\ \frac{\beta^2}{\alpha}=\frac{85+14\sqrt{37}}{9}$		
	$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{170}{9}$	M1: Adds their $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ A1: Correct sum	M1A1
	$\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=-\frac{1}{3}$	Correct product	B1
	$x^2-\frac{170}{9}x-\frac{1}{3}$	Uses $x^2-(\textit{their sum})x+(\textit{their product})$ (= 0 not needed here)	M1
	$9x^2-170x-3=0$	This equation or any integer multiple including = 0. **Follow through their sum and product.**	A1ft

Question Number	Scheme		Marks
7. (a)	**Rotation**, **30 degrees** (anticlockwise), **about *O*** Allow $\frac{\pi}{6}$ (radians) for 30 degrees. Anticlockwise may be omitted but do not allow -30 degrees or 30 degrees clockwise **B1: Rotation B1: 30 degrees B1: About O**		B1, B1, B1
			(3)
(b)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Correct matrix	B1
			(1)
(c)	$\mathbf{R} = \mathbf{PQ} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Multiplies **P** by their **Q** This statement is sufficient in correct order	M1
	$= \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Correct matrix	A1
			(2)
(d)	$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$	$\mathbf{R} \begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$ A correct statement but allow poor notation provided there is an indication that the candidate understands that the point (1, *k*) is mapped onto itself. This Method mark could be implied by a correct equation or correct follow through equation below.	M1
	$\frac{\sqrt{3}}{2} + \frac{k}{2} = 1$ or $\frac{1}{2} - \frac{k\sqrt{3}}{2} = k$	One correct equation (not a matrix equation)	A1
	$\frac{\sqrt{3}}{2} + \frac{k}{2} = 1$ or $\frac{1}{2} - \frac{k\sqrt{3}}{2} = k \Rightarrow k = ...$	Attempts to solve their equation for *k*. **Dependent on the first M.**	**d**M1
	$k = 2 - \sqrt{3}$	cao	A1
	Solves both $\frac{\sqrt{3}}{2} + \frac{k}{2} = 1$ and $\frac{1}{2} - \frac{k\sqrt{3}}{2} = k$ or checks other component	Solves both equations **explicitly** to obtain the same correct value for *k* or clearly verifies that $k = 2 - \sqrt{3}$ is valid for the other equation	B1
			(5)
			Total 11

Question	Scheme		Marks
8.(a)	$8t^2 \times 16t = 16$ **or** $\left(\frac{16}{x}\right)^2 = 32x$ **or** $y^2 = 32\times\left(\frac{16}{y}\right)$	Attempts to obtains an equation in one variable *x*, *y* or *t*	M1
	$t = \frac{1}{2}$ or $x = 2$ or $y = 8$	A correct value for *t*, *x* or *y*	A1
	(2, 8)	Correct coordinates following correct work with no other points	B1
			(3)
(b)	$\left(y = \frac{16}{x} \Rightarrow\right)\frac{dy}{dx} = -16x^{-2}$ **or** $\left(y + x\frac{dy}{dx} = 0 \Rightarrow\right)\frac{dy}{dx} = -\frac{y}{x}$ **or** $\left(\dot{x} = 4,\ \dot{y} = -\frac{4}{t^2} \Rightarrow\right)\frac{dy}{dx} = -\frac{1}{t^2}$	Correct derivative in terms of *x*, *y* and *x*, or *t*	B1
	at (8, 2) $\frac{dy}{dx} = -\frac{16}{(8)^2} = -\frac{1}{4}$	Uses $x = 8$, $x = 8$ and $y = 2$, or $t = 2$	M1
	gradient of normal is 4	Correct normal gradient	A1
	$y - 2 = 4(x - 8)$ **or** $y = 4x + c$ and uses $x = 8$ and $y = 2$ to find *c*	Correct straight line method using the point (8, 2) and a numerical gradient from their $\frac{dy}{dx}$ which is not the tangent gradient.	M1
	$y = 4x - 30$	Correct equation	A1
			(5)
(c)	$16t = 32t^2 - 30$ **or** $y = \frac{y^2}{8} - 30$ **or** $\frac{16\sqrt{x}}{\sqrt{8}} = 4x - 30$	Uses their straight line from part (b) and the parabola to obtain an equation in one variable (*x*, *y* or *t*)	M1
	$(4t + 3)(4t - 5) = 0 \Rightarrow t = ...$ $(y - 20)(y + 12) = 0 \Rightarrow y = ...$ $(2x - 25)(2x - 9) = 0 \Rightarrow x = ...$	Attempts to solve three term quadratic (see general guidance) to obtain *t* = ... or *y* = ... or *x* = ... **Dependent on the previous M**	**d**M1
	Note if they solve the **tangent** with the parabola this gives $x^2 - 544x + 256 = 0$ which has roots *x* = 543.53... and 0.47...... (Seeing these values would imply a correct attempt to solve their 3TQ)		
	$t = -\frac{3}{4}$ and $\frac{5}{4}$ **or** $y = 20$ and -12 **or** $x = \frac{25}{2}$ and $\frac{9}{2}$	Correct values for *t* or *y* or *x*	A1
	$t = -\frac{3}{4} \Rightarrow (x, y) =$ or $t = \frac{5}{4} \Rightarrow (x, y) =$ $y = 20 \Rightarrow x =$ or $y = -12 \Rightarrow x =$ $x = \frac{25}{2} \Rightarrow y =$ or $x = \frac{9}{2} \Rightarrow y =$	Uses their values of *t* to find at least one point **or** uses their values of *y* to find at least one *x* **or** uses their values of *x* to find at least one *y*. **Not dependent on previous method marks.**	M1
	$\left(\frac{25}{2}, 20\right)$, $\left(\frac{9}{2}, -12\right)$	A1: One correct pair of coordinates A1: Both pairs correct	A1 A1
			(6)
			Total 14

Question	Scheme		Marks
9.(i)	$\sum_{r=1}^{n} r(r+1)(r+2)=6$ **and** $\frac{n(n+1)(n+2)(n+3)}{4}=\frac{1\times2\times3\times4}{4}=6$ **Minimum: lhs = rhs = 6**		B1
	Assume result true for n = "k" so $\sum_{r=1}^{k} r(r+1)(r+2)=\frac{k(k+1)(k+2)(k+3)}{4}$ Minimum **Assume result true**		M1
	$\sum_{r=1}^{k+1} r(r+1)(r+2)=\frac{k(k+1)(k+2)(k+3)}{4}+(k+1)(k+2)(k+3)$ Adds the (k + 1)th term to the given result		M1
	$=\frac{1}{4}(k+1)(k+2)(k+3)(k+4)$	Achieves this result with no errors **Note this may be written down directly from the line above.**	A1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n.</u>		A1cso
			(5)
(ii)	$f(1)=4^1+6\times1+8=18$	f(1) = 18 is the minimum	B1
	$f(k+1)-f(k)=4^{k+1}+6(k+1)+8-(4^k+6k+8)$	M1: Attempts f(k + 1) – f(k)	M1
	$=3\times4^k+6=3(4^k+6k+8)-18k-18$	A1: $3(4^k+6k+8)$ or $3f(k)$ A1: $-18-18k$ or $-18(k+1)$	A1A1
	$f(k+1)=3(4^k+6k+8)-18(k+1)+f(k)$	Makes f(k + 1) the subject	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n</u>.		A1cso
			(6)
			Total 11
(ii) ALT 1	$f(1)=4^1+6\times1+8=18$		B1
	$f(k+1)-4f(k)=4^{k+1}+6(k+1)+8-4(4^k+6k+8)$	M1: Attempts f(k + 1) – 4f(k)	M1
	$=-18k-18$	A1: – 18k A1: – 18	A1A1
	$f(k+1)=4f(k)-18(k+1)$	Makes f(k + 1) the subject	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n.</u>		A1cso
(ii) ALT 2	$f(1)=4^1+6\times1+8=18$		B1
	$f(k+1)=4^{k+1}+6(k+1)+8$	M1: Attempts f(k + 1)	M1
	$=4(4^k+6k+8)-18k-18$	A1: $4(4^k+6k+8)$ or $4f(k)$ A1: $-18-18k$ or $-18(k+1)$	A1A1
	$f(k+1)=4f(k)-18(k+1)$	Makes f(k + 1) the subject (implicit with first M)	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n.</u>		A1cso

	See general case below for f(k) – mf(k)		

	f(k) – mf(k)		
(ii)	$\mathrm{f}(1)=4^1+6\times1+8=18$		B1
	$\mathrm{f}(k+1)-m\mathrm{f}(k)=4^{k+1}+6(k+1)+8-m\left(4^k+6k+8\right)$	M1: Attempts f(k + 1) – mf(k)	M1
	$=(4-m)\left(4^k+6k+8\right)-18k-18$	A1: $(4-m)\left(4^k+6k+8\right)$ or $(4-m)\mathrm{f}(k)$ A1: $-18-18k$ or $-18(k+1)$	A1A1
	$\mathrm{f}(k+1)=4\mathrm{f}(k)-18(k+1)$	Makes f(k + 1) the subject	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.		A1cso

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