Mark Scheme (Results)

January 2014

Pearson Edexcel International Advanced Level

Further Pure Mathematics 1 (WFM01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x =

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x =

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0,$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

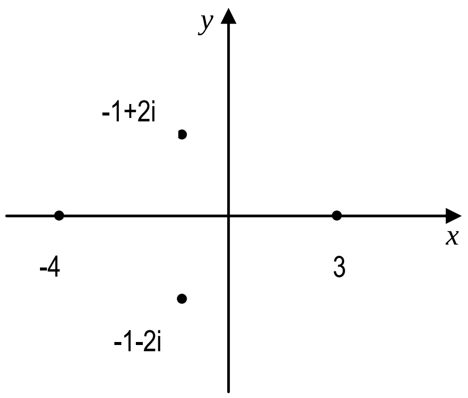
Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme		Marks
1. (a)	$f(x) = 6\sqrt{x} - x^2 - \frac{1}{2x}$		
	$f(3) = 1.225638179...$ $f(4) = -4.125\left(-\frac{33}{8}\right)$	Either any one of $f(3) =$ awrt 1.2 or $f(4) =$ awrt -4.1	M1
	Sign change (and $f(x)$ is continuous) therefore a root α exists between $x = 3$ and $x = 4$	both values correct, sign change (or equivalent) and conclusion	A1
			[2]
(b)	$f'(x) = 3x^{-\frac{1}{2}} - 2x + \frac{1}{2x^2}$	$x^n \to x^{n-1}$ on at least one term At least two terms differentiated correctly (May be un-simplified) Correct differentiation (May be un-simplified)	M1 A1 A1
	$\{f'(3) = -4.212393637...\}$		
	$\alpha = 3 - \frac{f(3)}{f'(3)} = 3 - \left(\frac{"1.225638179..."}{"-4.212393637"}\right)$	Correct application of Newton-Raphson using their values of $f(3)$ and $f'(3)$. May be implied by a correct answer.	M1
	$= 3.29096003...\ \{= 3.291\ (3\text{dp})\}$	awrt 3.291	A1
	Ignore any further applications of N-R		
			[5]
(c)	$\frac{\alpha - 3}{"1.225638179..."} = \frac{4-\alpha}{"4.125"}$ or $\frac{\alpha - 3}{"1.225638179..."} = \frac{1}{"1.225638179..." - "-4.125"}$	This mark can be implied. Do not allow if any 'negative lengths' are used or if either fraction is the wrong way up	M1
	$\alpha = 3 + \left(\frac{"1.225638179..."}{"1.225638179..." + "4.125"}\right)1$	Attempt to make α the subject	M1
	$\alpha = \frac{3\times"4.125" + 4\times"1.225638179..."}{"1.225638179..." + "4.125"}$ would score both method marks		
	$= 3.229063924...$ $= 3.229\ (3\text{dp})$	awrt 3.229	A1
			[3]
			10
	NB if -4.125 is used this gives 2.577273119….		

Question Number	Scheme		Marks
2. (a)	$5x^2 - 4x + 2 = 0$ has roots α and β		
	$\alpha + \beta = \frac{4}{5}$, $\alpha\beta = \frac{2}{5}$	At least one of $\alpha + \beta$ or $\alpha\beta$ correct Both $\alpha + \beta$ and $\alpha\beta$ correct	B1 B1
			[2]
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $\left\{= \left(\frac{4}{5}\right)^2 - 2\left(\frac{2}{5}\right)\right\}$	Writes down or applies the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1
	$= -\frac{4}{25} (-0.16)$	$-\frac{4}{25}$	A1**cso**
			[2]
Note 1	**cso** so: $\alpha + \beta = -\frac{4}{5}$, $\alpha\beta = \frac{2}{5}$ scores B1B0 in (a) and $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $\left\{= \left(-\frac{4}{5}\right)^2 - 2\left(\frac{2}{5}\right)\right\}$ $= -\frac{4}{25}$ M1A0 in (b) **But allow recovery of marks in (c)**		
Note 2	$\alpha + \beta = 4$, $\alpha\beta = 2$ is quite common and gives $\alpha^2 + \beta^2 = 12$, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 3$, $\frac{1}{\alpha^2\beta^2} = \frac{1}{4}$, and $4x^2 - 12x + 1 = 0$. This scores a maximum of 4/8		
(c)	A quadratic equation with roots of $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$		
	Sum of roots $\left\{= \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2} = \right\} = \frac{-\frac{4}{25}}{\frac{4}{25}}$ $\{= -1\}$	Applies $\frac{\text{their } (\alpha^2 + \beta^2)}{\text{their } (\alpha\beta)^2}$	M1
	Product of roots $\left\{= \frac{1}{\alpha^2\beta^2} = \right\} = \frac{1}{\left(\frac{4}{25}\right)}$ $\left\{= \frac{25}{4}\right\}$	Applies $\frac{1}{\text{their } (\alpha\beta)^2}$	M1
	So, $x^2 - (-1)x + \frac{25}{4} (= 0)$	Applies $x^2 - (\text{their sum})x + (\text{their product}) (= 0)$ **Dependent on at least one of the previous M's having been scored.**	dM1
	$4x^2 + 4x + 25 = 0$	$4x^2 + 4x + 25 = 0$ or any integer multiple	A1
			[4]
			8
	Alternative to part (c) 1^{st} M1: $\left(x - \frac{1}{\alpha^2}\right)\left(x - \frac{1}{\beta^2}\right) = 0$ 2^{nd} M1: $(\alpha^2\beta^2)x^2 - (\alpha^2 + \beta^2)x + 1 = 0$ 3^{rd} M1: $\frac{4x^2}{25} + \frac{4x}{25} + 1 = 0$ 4^{th} A1: $4x^2 + 4x + 25 = 0$		

Question Number	Scheme		Marks
3. (a)	$\mathbf{A} = \begin{pmatrix} 6 & 4 \\ 1 & 1 \end{pmatrix}$, Area$(R) = 10$, $\mathbf{B} = \mathbf{A}^4$		
	$\det(\mathbf{A}) = 6(1) - 4(1)$	Correct attempt at the determinant	M1
	$\det(\mathbf{A}) \neq 0$ (so **A** is non-singular)	$\det(\mathbf{A}) = 2$ or $6 - 4$ **and** some reference to zero e.g. $2 \neq 0$ is sufficient	A1
			[2]
(b)	Area$(S) = 2(10)$; $= 20$	(their $\det(\mathbf{A})) \times (10)$ 20	M1; A1
	$(10) \div$ (their $\det(\mathbf{A})$) is M0		
			[2]
(c)	Area$(T) = 2^4(10)$; $= 160$	(their $\det(\mathbf{A}))^4 \times (10)$ 160	M1 ; A1
	$(10) \div$ (their $\det(\mathbf{A}))^4$ is M0		
	$\mathbf{A}^2 = \begin{pmatrix} 40 & 28 \\ 7 & 5 \end{pmatrix} \Rightarrow \lvert\mathbf{A}^2\rvert = 4 \Rightarrow$ Area$(T) = 4^2(10)$; $= 160$ Is acceptable (their $\det(\mathbf{A}^2))^2 \times (10)$; M1 160; A1		
	BUT there must be no attempt to evaluate $\mathbf{A}^4$ to give $\det(\mathbf{A}) = 16$		
			[2]
			6
Note 1	If they think $\det(\mathbf{A}) = \dfrac{1}{\det(\mathbf{A})}$ then no marks in (a) but allow M's in (b) and (c).		
	NB $\mathbf{A}^4 = \begin{pmatrix} 6 & 4 \\ 1 & 1 \end{pmatrix}^4 = \begin{pmatrix} 1796 & 1260 \\ 315 & 221 \end{pmatrix}$		

Question Number	Scheme		Marks
4. (a)	$\text{f}(x) = x^4 + 3x^3 - 5x^2 - 19x - 60$		
	Quadratic factor: $(x+4)(x-3)\ \{= x^2 + x - 12\}$	$(x \pm 4)(x \pm 3)$ or $x^2 \pm x \pm 12$ $(x+4)(x-3)$ or $x^2 + x - 12$	M1 A1
	$\text{f}(x) = \{x^2 + x - 12\}(x^2 + 2x + 5)$	Attempt to find the other quadratic factor of the form $(x^2 + bx + c)$ $(x^2 + 2x + 5)$	M1 A1
	$x = \dfrac{-2 \pm \sqrt{4-20}}{2}$ or $(x+1)^2 - 1 + 5 = 0$, $x = ...$	Solving a 3-term quadratic by formula or completion of the square	M1
	$= -1 + 2\text{i}$ and $-1 - 2\text{i}$	Allow $-1 \pm 2\text{i}$ (-4 and 3 are not needed for this mark)	A1 A1ft
			[7]
(b)		Note that the points are $(-4, 0)$, $(3, 0)$, $(-1, 2)$ and $(-1, -2)$. The points $(-4, 0)$ and $(3, 0)$ plotted on the Argand diagram with -4 and 3 indicated. They could be labelled as e.g. x_1 and x_2 and referred to elsewhere. The distinct points representing the other two **complex** roots plotted correctly and symmetrically about the x-axis. The points must be indicated by a scale (could be ticks on axes) **or** labelled with coordinates or as complex numbers. They could be labelled as e.g. x_3 and x_4 and referred to elsewhere. If there is any contradiction in position in an otherwise correct diagram (e.g. -1 + 2i further to the left than -4, deduct one mark.	B1 B1ft
			[2]
			9
	Alternative by long division		
	1[st] M1: for attempting to divide $\text{f}(x)$ by $(x \pm 3)$ or $(x \pm 4)$. 1[st] A1: $\dfrac{\text{f}(x)}{(x-3)} = x^3 + 6x^2 + 13x + 20$ or $\dfrac{\text{f}(x)}{(x+4)} = x^3 - x^2 - x - 15$ 2[nd] M1: Attempt quadratic factor $\dfrac{x^3 + 6x^2 + 13x + 20}{(x+4)}$ or $\dfrac{x^3 - x^2 - x - 15}{(x-3)}$ 2[nd] A1: $(x^2 + 2x + 5)$		
	Alternative by comparing coefficients		
	$\text{f}(x) = (x^2 + x - 12)(ax^2 + bx + c) = x^4 + 3x^3 - 5x^2 - 19x - 60$ $\Rightarrow a = 1,\ c = 5,\ b + a = 3$ or $c + b - 12a = -5 \Rightarrow b = 2$ M1: Compares coefficients to obtain values for a, b and c A1: $a = 1$, $b = 2$ and $c = 5$		

Question Number	Scheme		Marks
5. (a)	$\sum_{r=1}^{n}\left(9r^2-4r\right)$		
	$=\frac{9}{6}n(n+1)(2n+1)-\frac{4}{2}n(n+1)$	An attempt to use at least one of the standard formulae correctly. Correct expression.	M1 A1
	$=\frac{3}{2}n(n+1)(2n+1)-2n(n+1)$		
	$=\frac{1}{2}n(n+1)\left(3(2n+1)-4\right)$	An attempt to factorise out at least $n(n+1)$. May not come until their last line.	M1
	$=\frac{1}{2}n(n+1)\left(6n+3-4\right)$		
	$=\frac{1}{2}n(n+1)(6n-1)$ (*)	Achieves the correct answer with no errors	A1 *
	There are no marks for proof by induction		
			[4]
	$\sum_{r=1}^{12}\left(9r^2-4r+k(2^r)\right)=6630$		
	$\sum_{r=1}^{12}\left(9r^2-4r\right)=\frac{1}{2}(12)(13)(71)\ \{=5538\}$	Attempt to evaluate $\sum_{r=1}^{12}\left(9r^2-4r\right)$ May be implied by 5538	M1
	$\sum_{r=1}^{12}(2^r)=\frac{2(1-2^{12})}{1-2}\ \{=8190\}$	Attempt to apply the sum to n terms of a GP $\frac{2(1-2^{12})}{1-2}$	M1 A1
	So, $5538+8190k=6630\Rightarrow 8190k=1092$ giving, $k=\frac{2}{15}$ oe		A1
			[4]
			8
(b)	2^{nd} M1 1^{st} A1: These two marks can be implied by seeing 8190 or 8190k $\sum_{r=1}^{12}(2^r)=2^{12}=4096$ is common and gives $k=\frac{273}{1024}$ (0.2666...) (Usually scores M1M0A0A0)		

Question Number	Scheme		Marks
6. (i) (a)	$\mathbf{B}^{-1} = -\frac{1}{2}\begin{pmatrix}-4 & -2\\-3 & -1\end{pmatrix}\left(=\frac{1}{2}\begin{pmatrix}4 & 2\\3 & 1\end{pmatrix}\right)\left(=\begin{pmatrix}2 & 1\\\frac{3}{2} & \frac{1}{2}\end{pmatrix}\right)$	Either $-\frac{1}{2}$ or $\begin{pmatrix}-4 & -2\\-3 & -1\end{pmatrix}$	M1
		Correct matrix	A1
			[2]
(b)	$\mathbf{Y} = \mathbf{AB} \Rightarrow \mathbf{YB}^{-1} = \mathbf{ABB}^{-1} \Rightarrow \mathbf{YB}^{-1} = \mathbf{A}$		
	$\mathbf{A} = \begin{pmatrix}4 & -2\\1 & 0\end{pmatrix}.-\frac{1}{2}\begin{pmatrix}-4 & -2\\-3 & -1\end{pmatrix}$	Multiplies their **Y** by $\mathbf{B}^{-1}$ This statement is sufficient	M1
	$= -\frac{1}{2}\begin{pmatrix}-10 & -6\\-4 & -2\end{pmatrix}$ or $\begin{pmatrix}5 & 3\\2 & 1\end{pmatrix}$	Correct matrix	A1
	NB $\mathbf{B}^{-1}\mathbf{Y} = \begin{pmatrix}9 & -4\\\frac{13}{2} & -3\end{pmatrix}$		
			[2]
(ii) (a)	$k = \sqrt{3-(-1)};= 2$	Applies $\sqrt{(\text{their det}\mathbf{M})}$ 2 (Accept correct answer only)	M1 A1
			[2]
(b)	$\cos\theta = -\frac{\sqrt{3}}{2},\ \sin\theta = \frac{1}{2},\ \tan\theta = -\frac{1}{\sqrt{3}}$	Writes down a correct trigonometric ratio Or a correct expression for the required angle e.g. $180 - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (This mark can be implied by a correct answer)	M1
	$\theta = 150^\circ$ or $\frac{5\pi}{6}$	150° or $\frac{5\pi}{6}$ (Accept correct answer only)	A1
			[2]
			8
(i)(b)	***Alternative method for (i)(b)*** $\mathbf{AB} = \mathbf{Y} \Rightarrow \begin{pmatrix}p & q\\r & s\end{pmatrix}\begin{pmatrix}-1 & 2\\3 & -4\end{pmatrix} = \begin{pmatrix}4 & -2\\1 & 0\end{pmatrix}$ $\begin{Bmatrix}-p+3q=4 & -r+3s=1\\2p-4q=-2 & 2r-4s=0\end{Bmatrix}$ leading to $\mathbf{A} = \begin{pmatrix}5 & 3\\2 & 1\end{pmatrix}$	Applies the matrix equation $\mathbf{AB} = \mathbf{Y}$ for an unknown **A.** This statement is sufficient Correct matrix	M1 A1 **[2]**
	Alternative method for (ii)(b)- Marks likely to come in the order (b), (a) $\begin{pmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}k & 0\\0 & k\end{pmatrix} \Rightarrow k\cos\theta = -\sqrt{3},\ k\sin\theta = 1,\ \tan\theta = -\frac{1}{\sqrt{3}} \Rightarrow \theta = 150^\circ$ or $\frac{5\pi}{6}$ M1: Writes down a correct trigonometric ratio. A1: 150° or $\frac{5\pi}{6}$ $k\sin\theta = 1 \Rightarrow \frac{1}{2}k = 1 \Rightarrow k = 2$ **(from correct** θ**)** M1: Uses their value of θ to obtain an equation in k. A1: $k = 2$		

Question Number	Scheme		Marks
7. (i) **Way 1**	$\frac{2w-3}{10} = \frac{4+7\text{i}}{4-3\text{i}}$		
	$\frac{2w-3}{10} = \frac{(4+7\text{i})}{(4-3\text{i})} \times \frac{(4+3\text{i})}{(4+3\text{i})}$	Multiplies by $\frac{(4+3\text{i})}{(4+3\text{i})}$	M1
	$= \frac{(16+12\text{i}+28\text{i}-21)}{16+9}$	Simplifies realising that a real number is needed in the denominator and applies $\text{i}^2 = -1$ on their numerator expression and denominator	M1
	$\left\{= \frac{1}{25}(-5+40\text{i})\right\}$		
	So $w = \frac{\frac{10}{25}(-5+40\text{i})+3}{2} = \frac{-2+16\text{i}+3}{2}$	Rearranges to $w = ...$	ddM1
	and $w = \frac{1}{2}+8\text{i}$	$\frac{1}{2}+8\text{i}$ Do not allow $\frac{1+16\text{i}}{2}$	A1
			[4]
(ii)	$(2+\lambda\text{i})(5+\text{i}) = 10+2\text{i}+5\lambda\text{i}-\lambda$	Multiplies out to give a four term expression and applies $\text{i}^2 = -1$ Correct expression	M1 A1
	$= (10-\lambda)+(2+5\lambda)\text{i}$ $\left\{\arg z = \frac{\pi}{4} \Rightarrow\right\}$ $\frac{2+5\lambda}{10-\lambda} = \tan\left(\frac{\pi}{4}\right)$	$\frac{\text{their combined imaginary part}}{\text{their combined real part}} = \tan\left(\frac{\pi}{4}\right)$ or sets real part = imaginary part	M1 oe
	$\{10-\lambda = 2+5\lambda \Rightarrow 8 = 6\lambda \Rightarrow\}$ $\lambda = \frac{4}{3}$	$\frac{4}{3}$ oe or awrt 1.33	A1
			[4]
			8
Way 2	***Alternative method for part (i)*** $2w = \frac{10(4+7\text{i})}{(4-3\text{i})}+3 = \frac{40+70\text{i}+12-9\text{i}}{(4-3\text{i})}$		
	$2w = \frac{(52+61\text{i})}{(4-3\text{i})} \times \frac{(4+3\text{i})}{(4+3\text{i})}$	Multiplies by $\frac{\text{their}(4-3\text{i})^*}{\text{their}(4-3\text{i})^*}$	M1
	$= \frac{(208+156\text{i}+244\text{i}-183)}{16+9}$ $= \frac{1}{25}(25+400\text{i}) = 1+16\text{i}$	Simplifies realising that a real number is needed in the denominator and applies $\text{i}^2 = -1$ on their numerator expression and denominator.	M1
	So, $w = \frac{1+16\text{i}}{2}$	Rearranges to $w = ...$ If w is made the subject as a first step only award this mark if the previous two M's are scored.	ddM1
	and $w = \frac{1}{2}+8\text{i}$	$\frac{1}{2}+8\text{i}$	A1

Question Number	Scheme		Marks
8.(a)	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \sqrt{a}\,x^{-\frac{1}{2}}$	$\dfrac{dy}{dx} = \pm k x^{-\frac{1}{2}}$	
	or (implicitly) $2y\dfrac{dy}{dx} = 4a$	or $ky\dfrac{dy}{dx} = c$	M1
	or (chain rule) $\dfrac{dy}{dx} = 2a \times \dfrac{1}{2ap}$	or $\dfrac{\text{their } \frac{dy}{dt}}{\text{their } \frac{dx}{dt}}$	
	$x = ap^2,\ m_T = \dfrac{dy}{dx} = \dfrac{\sqrt{a}}{\sqrt{ap^2}} = \dfrac{\sqrt{a}}{\sqrt{a}\,p} = \dfrac{1}{p}$ or $m_T = \dfrac{dy}{dx} = \dfrac{4a}{2(2ap)} = \dfrac{1}{p}$	$\dfrac{dy}{dx} = \dfrac{1}{p}$	A1
	T: $y - 2ap = \dfrac{1}{p}\left(x - ap^2\right)$	Applies $y - 2ap = (\text{their } m_T)\left(x - ap^2\right)$ Where $(\text{their } m_T)$ is a function of p and has come from calculus.	M1
	T: $py - 2ap^2 = x - ap^2$		
	T: $py = x + ap^2$	Correct solution.	A1 **cso** *
			[4]
(b)	$B\left(-a, \tfrac{5}{6}a\right) \Rightarrow p\left(\tfrac{5}{6}a\right) = -a + ap^2$ or $p\left(\tfrac{5}{6}a\right) = x + ap^2$ or $py = -a + ap^2$	Substitutes $x = -a$ or $y = \frac{5}{6}a$ or both into **T** **(**or their rearranged **T)**	M1
	$p\left(\tfrac{5}{6}a\right) = -a + ap^2 \quad (6p^2 - 5p - 6 = 0)$	Correct equation **in any form** with $x = -a$ and $y = \frac{5}{6}a$	A1
	$\Rightarrow (3p+2)(2p-3) = 0$ leading to $p = ...$	Attempts to solve their 3TQ in p having substituted both $x = -a$ and $y = \frac{5}{6}a$ into **T**	M1
	$\Rightarrow \left\{p = -\dfrac{2}{3}\ (\text{reject})\right\}\ p = \dfrac{3}{2}$	$p = \dfrac{3}{2}$ (Can just be stated from a correct quadratic)	A1
	So, $0 = x + a\left(\dfrac{3}{2}\right)^2$	Substitutes "$p = \frac{3}{2}$" and $y = 0$ in **T**	M1
	giving, $x = -\dfrac{9a}{4}$	$x = -\dfrac{9a}{4}$	A1
			[6]
(c)	When $p = \dfrac{3}{2},\ y_P = 2a\left(\dfrac{3}{2}\right) = 3a$		
	$\text{Area}(OAD) = \dfrac{1}{2}\left(\dfrac{9a}{4}\right)(3a) = \dfrac{27a^2}{8}$ Or $\text{Area}(OAD) = \dfrac{1}{2}\begin{vmatrix} 0 & \frac{9a}{4} & -\frac{9a}{4} & 0 \\ 0 & 3a & 0 & 0 \end{vmatrix} = \dfrac{1}{2} \times 3a \times \dfrac{9a}{4}$	Applies $\dfrac{1}{2}(\text{their } \lvert OD \rvert)(\text{their } y_P)$ Allow if $OD < 0$ and a correct method in terms of a and p e.g. $\dfrac{1}{2} \times -ap^2 \times 2ap$ $\dfrac{27a^2}{8}$	M1 A1
	Do not allow $\dfrac{1}{2} \times 2ap \times \left(\dfrac{5ap}{6} - ap^2\right)$ as this implies that $y = 0$ has not been used for D		
			[2]
			12

Question Number	Scheme		Marks
	$\mathrm{f}(n) = 7^n - 2^n$ is divisible by 5		
9.	$\mathrm{f}(1) = 7^1 - 2^1 = 5$	Shows or states that $\mathrm{f}(1) = 5$	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 7^k - 2^k$ is divisible by 5 for $k \in \mathbb{Z}^+$.		
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 7^{k+1} - 2^{k+1} - (7^k - 2^k)$	Applies $\mathrm{f}(k+1) - \mathrm{f}(k)$	M1
	$= 7(7^k) - 2(2^k) - (7^k - 2^k)$	Achieves an expression in 7^k and 2^k. Correct expression in 7^k and 2^k	M1 A1
	$= 6(7^k) - 2^k$ $= 6(7^k - 2^k) + 5(2^k)$ $= 6\mathrm{f}(k) + 5(2^k)$	Or $(7^k - 2^k) + 5(7^k)$ Or $\mathrm{f}(k) + 5(7^k)$	
	$\therefore \mathrm{f}(k+1) = 7\mathrm{f}(k) + 5(2^k)$ or $2\mathrm{f}(k) + 5(7^k)$	$\mathrm{f}(k+1) = 7\mathrm{f}(k) + 5(2^k)$ or $\mathrm{f}(k+1) = 2\mathrm{f}(k) + 5(7^k)$ or e.g. $\mathrm{f}(k+1) = \mathrm{f}(k) + 5(7^k) + 7^k - 2^k$ Correctly achieves f(k + 1) that is clearly a multiple of 5	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion with all previous marks scored.	A1 cso
			[6]
			6

Appendix

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
- ft denotes "follow through"
- cao denotes "correct answer only"
- oe denotes "or equivalent"

Other Possible Solutions

Question Number	Scheme		Marks
2.	$5x^2 - 4x + 2 = 0$ has roots α and β		
***Aliter* Way 2**	$x = \frac{4 \pm \sqrt{-24}}{10} = \frac{2}{5} \pm \frac{\sqrt{6}}{5}\text{i}$. Hence let, say $\alpha = \frac{2}{5} + \frac{\sqrt{6}}{5}\text{i}$ and $\beta = \frac{2}{5} - \frac{\sqrt{6}}{5}\text{i}$		
(a)	$\alpha + \beta = \frac{4}{5}, \ \alpha\beta = \frac{2}{5}$	At least one of $\alpha + \beta$ or $\alpha\beta$ correct Both $\alpha + \beta$ and $\alpha\beta$ correct	B1 B1
			[2]
(b)	$\alpha^2 = -\frac{2}{25} + \frac{4\sqrt{6}}{25}\text{i}, \ \beta^2 = -\frac{2}{25} - \frac{4\sqrt{6}}{25}\text{i}$	Uses their α and their β to find both α^2 and β^2	M1
	So, $\alpha^2 + \beta^2 = -\frac{4}{25}$	$-\frac{4}{25}$	A1
			[2]
(c)	A quadratic equation with roots of $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$		
	$\frac{1}{\alpha^2} = 25\left(\frac{1}{-2+4\sqrt{6}\text{i}}\right) = 25\left(\frac{-2+4\sqrt{6}\text{i}}{4+96}\right) = \frac{1}{2}\left(-1-2\sqrt{6}\text{i}\right) = -\frac{1}{2} - \sqrt{6}\text{i}$ Hence, $\frac{1}{\beta^2} = -\frac{1}{2} + \sqrt{6}\text{i}$	A valid attempt to find either $\frac{1}{\alpha^2}$ or $\frac{1}{\beta^2}$.	M1
	So, $\left(x - \left(-\frac{1}{2} - \sqrt{6}\text{i}\right)\right)\left(x - \left(-\frac{1}{2} + \sqrt{6}\text{i}\right)\right) = 0$	An attempt to form a quadratic equation using their $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.	M1
	So, $x^2 - (-1)x + \frac{25}{4} (= 0)$	... leading to a quadratic expression with integer coefficients.	M1
	leading to, $4x^2 + 4x + 25 = 0$	$4x^2 + 4x + 25 = 0$ or any integer multiple	A1
			[4]
			8

Question Number	Scheme		Marks
7(i) **Way 3**	$\frac{2(u+\text{i}v)-3}{10}=\frac{4+7\text{i}}{4-3\text{i}}$		
	$\Rightarrow (2(u+\text{i}v)-3)(4-3\text{i})=40+70\text{i}$	Replaces *w* with *u* + i*v* and eliminates fractions	M1
	$\therefore 8u+6v-12=40$ and $8v-6u+9=70$	Correct equations	A1
	$u=\frac{1}{2},\ \ v=8$	Solves simultaneously to at least *u* = or *v* = Correct values	M1 A1
			[4]

7(i) **Way 4**	$\frac{2w-3}{10}=\frac{4+7\text{i}}{4-3\text{i}}\Rightarrow\frac{2w-3}{10}-\frac{4+7\text{i}}{4-3\text{i}}=0$		
	$\Rightarrow\frac{(2w-3)(4-3\text{i})-10(4+7\text{i})}{10(4-3\text{i})}=0$		
	$8w-6\text{i}w=52+61\text{i}$		
	$w=\frac{52+61\text{i}}{8-6\text{i}}$		
	$w=\frac{52+61\text{i}}{8-6\text{i}}\times\frac{8+6\text{i}}{8+6\text{i}}$	Multiplies by $\frac{\text{their}(8-6\text{i})^*}{\text{their}(8-6\text{i})^*}$	M1
	$w=\frac{416+800\text{i}-366}{100}$	Simplifies realising that a real number is needed in the denominator and applies $\text{i}^2=-1$ on their numerator expression and denominator	M1
	$w=\frac{1}{2}+8\text{i}$	The ddM1 can be awarded now	ddM1 A1
	Cross multiplication essentially follows the same scheme		
			[4]

7(ii)	$z=(2+\lambda\text{i})(5+\text{i})\Rightarrow\arg z=\arg(2+\lambda\text{i})(5+\text{i})$		
	$\arg(2+\lambda\text{i})(5+\text{i})=\arg(2+\lambda\text{i})+\arg(5+\text{i})$	Use of $\arg z_1z_2=\arg z_1+\arg z_2$ $\arg z=\arg(2+\lambda\text{i})+\arg(5+\text{i})$	M1 A1
	$\frac{\pi}{4}=\arctan\left(\frac{\lambda}{2}\right)+\arctan\left(\frac{1}{5}\right)$		
	$1=\frac{\frac{\lambda}{2}+\frac{1}{5}}{1-\frac{\lambda}{2}\frac{1}{5}}$	Use of the **correct** addition formula $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$	M1
	$10-\lambda=5\lambda+2\Rightarrow\lambda=\frac{4}{3}$	$\frac{4}{3}$ oe	A1
			[4]

Question Number	Scheme		Marks
Aliter **9. Way 2**	$f(n) = 7^n - 2^n$ is divisible by 5		
	$f(1) = 7^1 - 2^1 = 5$	Shows or states $f(1) = 5$	B1
	Assume that for $n = k$, $f(k) = 7^k - 2^k$ is divisible by 5 for $k \in \mathbb{Z}^+$.		
	$f(k+1) = 7^{k+1} - 2^{k+1}$	Applies $f(k+1)$	M1
	$= 7(7^k) - 2(2^k)$	Achieves an expression in 7^k and 2^k Correct expression in 7^k and 2^k	M1 A1
	$= 7(7^k - 2^k) + 5(2^k)$ or $5(7^k) + 2(7^k - 2^k)$ $\therefore f(k+1) = 7f(k) + 5(2^k)$ or $5(7^k) + 2f(k)$	$f(k+1) = 7f(k) + 5(2^k)$ or $5(7^k) + 2f(k)$ Correctly achieves f(*k* + 1) that is clearly a multiple of 5	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion with all previous marks scored.	A1 cso
			[6]

Question Number	Scheme		Marks
Aliter **9. Way 3**	$f(n) = 7^n - 2^n$ is divisible by 5		
	$f(1) = 7^1 - 2^1 = 5$	Shows or states $f(1) = 5$	B1
	Assume that for $n = k$, $f(k) = 7^k - 2^k$ is divisible by 5 for $k \in \mathbb{Z}^+$.		
	$f(k+1) - 2f(k) = 7^{k+1} - 2^{k+1} - 2\left(7^k - 2^k\right)$	Applies $f(k+1) - 2f(k)$	M1
	$= 5(7^k)$	Achieves an expression in 7^k Correct expression in 7^k	M1 A1
	$\therefore f(k+1) = 5(7^k) + 2f(k)$	$5(7^k) + 2f(k)$	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion with all previous marks scored.	A1 cso
			[6]