Mark Scheme (Results)

January 2015

Pearson Edexcel International A Level in Further Pure Mathematics F1 (WFM01/01)

Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\sqsubset$ or d… The second mark is dependent on gaining the first mark

Question Number	Scheme	Notes	Marks
1.	$\mathrm{f}(x) = x^4 - x^3 - 9x^2 + 29x - 60$		
	1 – 2i is also a root	Seen anywhere	B1
	$x^2 - 2x + 5$	M1: Attempt to expand $(x-(1+2i))(x-(1-2i))$ or any valid method to establish the quadratic factor A1: $x^2 - 2x + 5$	M1A1
	$\mathrm{f}(x) = \left(x^2 - 2x + 5\right)\left(x^2 + x - 12\right)$	M1: Attempt other **quadratic** factor A1: $x^2 + x - 12$	M1A1
	$x^2 + x - 12 = (x+4)(x-3) \Rightarrow x = ...$	Attempt to solve their **other** quadratic factor.	M1
	$x = {}^-4$ and $x = 3$	Both values correct	A1
			(7)
			Total 7
	Alternative using Factor Theorem		
	$\mathrm{f}(3) = ...$ **or** $\mathrm{f}(-4) = ...$	M1: Attempts f(3) **or** f(-4)	M1
	$\mathrm{f}(3) = 0$ **or** $\mathrm{f}(-4) = 0$	A1: Shows or states f(3) = 0 **or** f(-4) = 0	A1
	$\mathrm{f}(3) = ...$ **and** $\mathrm{f}(-4) = ...$	M1: Attempts f(3) **and** f(-4) or f(3) and g(-4) where g(x) = f(x)/(x - 3) or f(-4) and h(3) where h(x) = f(x)/(x + 4)	M1
	$\mathrm{f}(3) = 0$ **and** $\mathrm{f}(-4) = 0$	A1: Shows or states f(3) = 0 **and** f(-4) = 0 or shows or states f(3) = 0 and g(-4) = 0 where g(x) = f(x)/(x - 3) or shows or states f(-4) = 0 and h(3) = 0 where h(x) = f(x)/(x + 4)	A1
	NB $\mathrm{g}(x) = x^3 + 2x^2 - 3x + 20,\ \mathrm{h}(x) = x^3 - 5x^2 + 11x - 15$		
	$x = 3$ **or** $x = {}^-4$	One of $x = 3$ **or** $x = {}^-4$ clearly stated as a **root**	M1
	$x = 3$ **and** $x = {}^-4$	Both $x = 3$ **and** $x = {}^-4$ clearly stated as **roots**	A1
	$x = 1 - 2i$		B1

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(x) = x^3 - 3x^2 + \dfrac{1}{2\sqrt{x^5}} + 2$		
(a)	$\mathrm{f}(2) = \ldots$ **and** $\mathrm{f}(3) = \ldots$	Attempts both f(2) and f(3)	M1
	$\mathrm{f}(2) = -1.9116..$, $\mathrm{f}(3) = 2.032...$ **Sign change** (and $\mathrm{f}(x)$ is continuous) therefore a **root** α exists between $x = 2$ and $x = 3$	Both values correct : $\mathrm{f}(2) = -1.9116..$ (awrt -1.9), and $\mathrm{f}(3) = 2.032...$(awrt 2.0 or e.g. $2 + \dfrac{\sqrt{3}}{54}$) , **sign change** (or equivalent) and **conclusion**	A1
			(2)
(b)	$\mathrm{f}'(x) = 3x^2 - 6x - \dfrac{5}{4}x^{-3.5}$	M1: $x^n \to x^{n-1}$ A1: $3x^2 - 6x$ A1: $-\dfrac{5}{4}x^{-3.5}$ or equivalent un-simplified and no other terms (+ c loses this mark)	M1A1A1
	$\alpha = 3 - \dfrac{2.032075015}{8.973270821}$	Correct attempt at Newton-Raphson using their values of $\mathrm{f}(3)$ and $\mathrm{f}'(3)$.	M1
	$\alpha = 2.774$	Cao (Ignore any subsequent applications)	A1
	Correct derivative followed by correct answer scores full marks in (b) **Correct answer with <u>no</u> working scores <u>no</u> marks in (b)**		
			(5)
	NB if the answer is incorrect it must be clear that both $\mathrm{f}(3)$ and $\mathrm{f}'(3)$ are being used in the Newton-Raphson process. So that just $3 - \dfrac{f(3)}{f'(3)}$ with an incorrect answer and no other evidence scores M0.		
			Total 7

Question Number	Scheme	Notes	Marks
3	$(z-2\text{i})(z^*-2\text{i})=21-12\text{i}$		
	$z^*=x-\text{i}y$		B1
	$(x+\text{i}y-2\text{i})(x-\text{i}y-2\text{i})=....$	Substitutes for z and their z^* and attempts to expand	M1
	$=x^2-x(y+2)\text{i}+x(y-2)\text{i}+y^2-4$		
	$=x^2+y^2-4-4x\text{i}$		
	$x^2+y^2-4=21$ and $4x=12$	Compares real and imaginary parts (allow sign errors only)	M1
	$4x=12\Rightarrow x=....$	Solves real and imaginary parts to obtain at least one value of x or y	M1
	$x=3,\ \ y=\pm 4$	$x=3$ cso $y=\pm 4$ cso	A1, A1
			(6)
			Total 6
Way 2	$(z-2\text{i})(z^*-2\text{i})=zz^*-2\text{i}(z+z^*)-4$	Attempt to expand	M1
	$=(x+\text{i}y)(x-\text{i}y)-2\text{i}(x+\text{i}y+x-\text{i}y)-4$	$z^*=x-\text{i}y$ (may be implied)	B1
	$=x^2+y^2-4x\text{i}-4$		
	$x^2+y^2-4=21$ and $4x=12$	Compares real and imaginary parts (allow sign errors only)	M1
	$4x=12\Rightarrow x=....$	Solves real and imaginary parts to obtain at least one value of x or y	M1
	$x=3,\ \ y=\pm 4$	$x=3$ cso $y=\pm 4$ cso	A1, A1
			Total 6
Way 3	$(z-2\text{i})(z^*-2\text{i})=zz^*-2\text{i}(z+z^*)-4$	Attempt to expand	M1
	$zz^*-2\text{i}(z+z^*)-4=21-12\text{i}$		
	$zz^*-4=21,\quad 2(z+z^*)=12$	Compares real and imaginary parts (allow sign errors only)	M1
	$z^2-6z+25=0\ \left(\text{or}\left(z^*\right)^2-6z^*+25=0\right)$	Correct quadratic	B1
	$z^2-6z+25=0\ \left(\text{or}\left(z^*\right)^2-6z^*+25=0\right)$ $\Rightarrow z=.....$ or $z^*=.....$	Solves to obtain at least one value of z or z^*	M1
	$z=3,\pm 4\text{i}$	$x=3$ cso $y=\pm 4$ cso	A1, A1
			Total 6

Question Number	Scheme	Notes	Marks
4(a)	$y^2=12x \Rightarrow y=\sqrt{12}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx}=\frac{1}{2}\sqrt{12}x^{-\frac{1}{2}}$	$\frac{dy}{dx}=kx^{-\frac{1}{2}}$	
	$y^2=12x \Rightarrow 2y\frac{dy}{dx}=12$	$\alpha y\frac{dy}{dx}=\beta$	M1
	$\frac{dy}{dx}=\frac{dy}{dp}\cdot\frac{dp}{dx}=6.\frac{1}{6p}$	their $\frac{dy}{dp}\times\left(\frac{1}{\text{their }\frac{dx}{dp}}\right)$	
	$\frac{dy}{dx}=\frac{1}{2}\sqrt{12}x^{-\frac{1}{2}}$ or $2y\frac{dy}{dx}=12$ or $\frac{dy}{dx}=6.\frac{1}{6p}$ or equivalent expressions	Correct differentiation	A1
	$m_T=\frac{1}{p}\Rightarrow m_N=-p$	Correct perpendicular gradient rule	M1
	$y-6p=-p\left(x-3p^2\right)$	$y-6p=$ their $m_N\left(x-3p^2\right)$ or $y=mx+c$ with their m_N and $(3p^2,6p)$ in an attempt to find 'c'. **Their m_N must have come from calculus and should be a function of p which is not their tangent gradient.**	M1
	$y+px=6p+3p^3$ *	Achieves printed answer with no errors	A1*
			(5)
(b)	$p=2\Rightarrow y+2x=12+24$	Substitutes the given value of p into the normal	M1
	$y+\frac{y^2}{6}=36$	Substitutes to obtain an equation in one variable (x, y or "q")	M1
	$y^2+6y-216=0$		
	$(y+18)(y-12)=0\Rightarrow y=$	Solves their 3TQ	M1
	$y=-18\Rightarrow x=27$	A1: One correct coordinate A1: Both coordinates correct	A1, A1
			(5)
(c)	Focus is (3, 0) or a = 3 or OS = 3	Must be seen or used in (c)	B1
	$y=0\Rightarrow x=18$		
	$A=\frac{1}{2}(18-3)(12)+\frac{1}{2}(18-3)(18)$	M1: Correct attempt at area A1: Correct expression	M1A1
	A = 225	Correct area	A1
			(4)
			Total 14

Question Number	Scheme	Notes	Marks
5(a)	$\alpha+\beta=-\frac{3}{4},\quad \alpha\beta=\frac{1}{4}$		B1, B1
			(2)
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{9}{16}-\frac{1}{2}=\frac{1}{16}$	M1:Use of $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ A1: $\frac{1}{16}$ cso (allow 0.0625)	M1 A1
			(2)
(c)	Sum $4\alpha-\beta+4\beta-\alpha=3(\alpha+\beta)=-\frac{9}{4}$	Attempt numerical sum	M1
	Product $(4\alpha-\beta)(4\beta-\alpha)=17\alpha\beta-4(\alpha^2+\beta^2)$ $=\frac{17}{4}-\frac{1}{4}=4$	Attempt numerical product	M1
	$x^2-\left(-\frac{9}{4}\right)x+4(=0)$	Uses x^2 – (sum)x + (prod) with sum, prod numerical (= 0 not reqd.)	M1
	$4x^2+9x+16=0$	Any multiple (including = 0)	A1
			(4)
			Total 8

	Alternative: Finds roots explicitly		
(a)	$x=-\frac{3}{8}\pm\frac{\sqrt{7}}{8}\mathrm{i}$		
	$\alpha+\beta=-\frac{3}{8}+\frac{\sqrt{7}}{8}\mathrm{i}-\frac{3}{8}-\frac{\sqrt{7}}{8}\mathrm{i}=-\frac{3}{4}$		B1
	$\alpha\beta=\left(-\frac{3}{8}+\frac{\sqrt{7}}{8}\mathrm{i}\right)\left(-\frac{3}{8}-\frac{\sqrt{7}}{8}\mathrm{i}\right)=\frac{1}{4}$		B1
			(2)
(b)	$\alpha^2+\beta^2=\left(-\frac{3}{8}+\frac{\sqrt{7}}{8}\mathrm{i}\right)^2+\left(-\frac{3}{8}-\frac{\sqrt{7}}{8}\mathrm{i}\right)^2=\frac{1}{16}$	M1: Substitutes their α and β and attempt to square and add both brackets A1: $\frac{1}{16}$ cso (allow 0.0625)	M1 A1
			(2)
(c)	$4\alpha-\beta=-\frac{9}{8}+\frac{5\sqrt{7}}{8}\mathrm{i},\ 4\beta-\alpha=-\frac{9}{8}-\frac{5\sqrt{7}}{8}\mathrm{i}$		
	$\mathrm{f}(x)=\left(x-\left(-\frac{9}{8}+\frac{5\sqrt{7}}{8}\mathrm{i}\right)\right)\left(x-\left(-\frac{9}{8}-\frac{5\sqrt{7}}{8}\mathrm{i}\right)\right)$	Uses $(x-(4\alpha-\beta))(x-(4\beta-\alpha))$ With numerical values (May expand first)	M1
	$\mathrm{f}(x)=x^2+x\left(-\frac{9}{8}-\frac{5\sqrt{7}}{8}\mathrm{i}\right)-x\left(-\frac{9}{8}+\frac{5\sqrt{7}}{8}\mathrm{i}\right)+\left(-\frac{9}{8}+\frac{5\sqrt{7}}{8}\mathrm{i}\right)\left(-\frac{9}{8}-\frac{5\sqrt{7}}{8}\mathrm{i}\right)$ Attempt to expand (may occur in terms of α and β but must be numerical for both M's)		M1
	$=x^2+\frac{9}{4}x+4(=0)$	Collects terms (= 0 not reqd.)	M1
	$4x^2+9x+16=0$	Any multiple (including = 0)	A1
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
6(i)(a)	**A**: Stretch scale factor 3 parallel to the x-axis	B1: **Stretch** B1: SF 3 parallel to (or along) x-axis Allow e.g. horizontal stretch SF 3 (Ignore any reference to the origin)	B1B1
			(2)
(b)	**B**: Rotation 210 degrees (anticlockwise) about (0, 0) or about O	B1: Rotation about (0, 0) B1: 210 degrees (anticlockwise) (or equivalent e.g. -150° or 150° clockwise). Allow equivalents in radians.	B1B1
			(2)
(c)	$\mathbf{C}=\mathbf{BA}=\begin{pmatrix}-\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & -\frac{\sqrt{3}}{2}\end{pmatrix}\begin{pmatrix}3 & 0\\ 0 & 1\end{pmatrix}$	Attempts **BA** (This statement is sufficient)	M1
	$=\begin{pmatrix}-\frac{3\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{3}{2} & -\frac{\sqrt{3}}{2}\end{pmatrix}$	Correct matrix	A1
			(2)
(ii)	det$\mathbf{M}$ = $(2k+5).k-1\times(-4)\left(=2k^2+5k+4\right)$	M1: Correct attempt at determinant A1: Correct determinant (allow un-simplified)	M1A1
	$b^2-4ac=25-32$	Attempts discriminant or uses quadratic formula	M1
	$b^2-4ac<0$ So **no real roots** so **det$\mathbf{M}\neq 0$**	Convincing explanation **and** conclusion with no previous errors	A1
			(4)
			Total 10
(ii) Way 2	$(2k+5).k-1\times(-4)\left(=2k^2+5k+4\right)$	M1: Correct attempt at determinant A1: Correct determinant (allow un-simplified)	M1A1
	$=2\left(k+\frac{5}{4}\right)^2+\frac{7}{8}$	Attempts to complete the square:	M1
	det$\mathbf{M}>0\ \forall\ k$ Therefore **det$\mathbf{M}\neq 0$**	Convincing explanation **and** conclusion with no previous errors	A1
(ii) Way 3	$(2k+5).k-1\times(-4)\left(=2k^2+5k+4\right)$	M1: Correct attempt at determinant A1: Correct determinant (allow un-simplified)	M1A1
	$\frac{\mathrm{d}(det\mathbf{M})}{\mathrm{d}k}=4k+5=0\Rightarrow k=-\frac{5}{4}$		
	$k=-\frac{5}{4}\Rightarrow det\mathbf{M}=\frac{7}{8}$	Attempts coordinates of turning point	M1
	Minimum $det\mathbf{M}$ is $\frac{7}{8}$ therefore **det$\mathbf{M}\neq 0$**	Convincing explanation **and** conclusion with no previous errors	A1

Question Number	Scheme	Notes	Marks
7	$\sum_{r=1}^{n}(r+a)(r+b)=\frac{1}{6}n(2n+11)(n-1)$		
(a)	$(r+a)(r+b)=r^2+ra+rb+ab$		B1
	$\sum_{r=1}^{n}(r+a)(r+b)=\frac{1}{6}n(n+1)(2n+1)+(a+b)\frac{1}{2}n(n+1)+abn$		M1A1B1
	M1: Attempt to use one of the standard formulae correctly A1: $\frac{1}{6}n(n+1)(2n+1)+(a+b)\frac{1}{2}n(n+1)$ B1: abn		
	$\frac{1}{6}n\left[(n+1)(2n+1)+3(a+b)(n+1)+6ab\right]=\frac{1}{6}n(2n+11)(n-1)$		
	$(n+1)(2n+1)+3(a+b)(n+1)+6ab=2n^2+9n-11$		
	$2n^2+3n+1+3(a+b)(n+1)+6ab=2n^2+9n-11$		
	$3+3a+3b=9,\ 3a+3b+1+6ab=-11$ $(a+b=2,\ ab=-3)$	M1: Compares coefficients to obtain at least one equation in a and b M1: One correct equation M1: Both equations correct	M1M1M1
	$b=-1,\ a=3$	Both values correct. This can be withheld if $b=3,\ a=-1$ is not rejected.	A1
			(8)
(b)	$\sum_{r=9}^{20}(r+a)(r+b)$		
	$\sum_{r=9}^{20}(r+a)(r+b)=\mathrm{f}(20)-\mathrm{f}(8\text{ or }9)$	**<u>Use</u>** of $\mathrm{f}(20)-\mathrm{f}(8\text{ or }9)$	M1
	$=\frac{1}{6}(20)(51)(19)-\frac{1}{6}(8)(27)(7)$	Correct (possibly un-simplified) numerical expression	A1
	=3230 – 252 = 2978	cao	A1
			(3)
			Total 11

Question Number	Scheme	Notes	Marks
8(i)	When $n = 1$ $u_1 = 2^1 + 3^1 = 5$ When $n = 2$ $u_2 = 2^2 + 3^2 = 13$	**Both**	B1
	True for $n = 1$ and $n = 2$		
	Assume $u_k = 2^k + 3^k$ and $u_{k+1} = 2^{k+1} + 3^{k+1}$		
	$u_{k+2} = 5u_{k+1} - 6u_k = 5\left(2^{k+1} + 3^{k+1}\right) - 6\left(2^k + 3^k\right)$	M1: Attempts u_{k+2} in terms of u_{k+1} and u_k A1: Correct expression	M1A1
	$= 5.2^{k+1} + 5.3^{k+1} - 6.2^k - 6.3^k$		
	$= 5.2^{k+1} - 3.2^{k+1} + 5.3^{k+1} - 2.3^{k+1}$	Attempt u_{k+2} in terms of $2^{f(k)}$ and $3^{f(k)}$ only	M1
	So $u_{k+2} = 2.2^{k+1} + 3.3^{k+1}$		
	$= 2^{(k+1)+1} + 3^{(k+1)+1}$ or $2^{k+2} + 3^{k+2}$	Correct expression with **no errors**	A1
	If true for k and $k + 1$ then shown true for $k + 2$ and as true for $n = 1$ and $n = 2$, true for $n \in \mathbb{Z}^+$	**Full conclusion** with all previous marks scored	A1
			(6)
(ii)	$f(2) = 7^4 - 48(2) - 1 = 2304$ So true for $n = 2$	Shows true for $n = 2$	B1
	Assume $f(k) = 7^{2k} - 48k - 1 = 2304p$ for some integer p		
	$f(k+1) - f(k) = 7^{2k+2} - 48(k+1) - 1 - \left(7^{2k} - 48k - 1\right)$	Attempt $f(k+1) - f(k)$	M1
	$= 7^{2k+2} - 7^{2k} - 48$		
	$= 7^{2k}(49-1) - 48$		
	$= 48f(k) + 48^2 k$	M1: Attempt rhs in terms of $f(k)$ or $7^{2k} - 48k - 1$ A1: Correct expression which is a multiple of 2304	M1A1
	$= 48 \times 2304p + 2304k$		
	$f(k+1) = 49 \times 2304p + 2304k$	Obtains $f(k+1)$ as a correct multiple of 2304 with **no errors**	A1
	If true for k then shown true for $k + 1$ and as true for $n = 2$, true for $n \geq 2$ $(n \in \mathbb{Z})$	**Full conclusion** with all previous marks scored	A1
			(6)
			Total 12