

4.

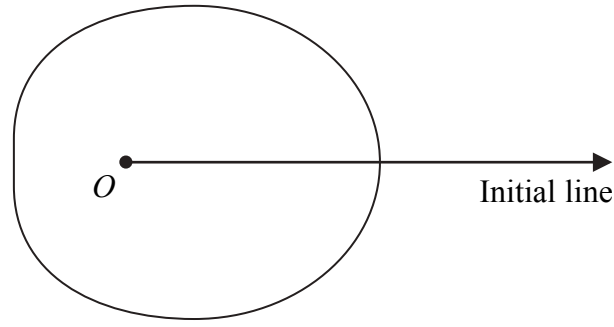


Figure 1

Figure 1 shows a sketch of the curve with polar equation

$$r = a + 3 \cos \theta, \quad a > 0, \quad 0 \leq \theta < 2\pi$$

The area enclosed by the curve is $\frac{107}{2} \pi$.

Find the value of a .

(8)

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Question 5 continued

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Q5

(Total 10 marks)



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6. A transformation T from the z -plane to the w -plane is given by

$$w = \frac{z}{z + i}, \quad z \neq -i$$

The circle with equation $|z|=3$ is mapped by T onto the curve C .

(a) Show that C is a circle and find its centre and radius.

(8)

The region $|z|<3$ in the z -plane is mapped by T onto the region R in the w -plane.

(b) Shade the region R on an Argand diagram.

(2)

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Question 6 continued

Lined area for writing answers.



M 3 5 1 4 4 A 0 1 7 2 8

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7. (a) Sketch the graph of $y = |x^2 - a^2|$, where $a > 1$, showing the coordinates of the points where the graph meets the axes. (2)

(b) Solve $|x^2 - a^2| = a^2 - x$, $a > 1$. (6)

(c) Find the set of values of x for which $|x^2 - a^2| > a^2 - x$, $a > 1$. (4)



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8.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}$$

Given that $x = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$,

(a) find x in terms of t .

(8)

The solution to part (a) is used to represent the motion of a particle P on the x -axis. At time t seconds, where $t > 0$, P is x metres from the origin O .

(b) Show that the maximum distance between O and P is $\frac{2\sqrt{3}}{9}$ m and justify that this distance is a maximum.

(7)



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Question 8 continued

Lined area for writing the answer to Question 8.



