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Question 1 continued

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(Total 7 marks)

Q1



N 3 5 3 8 8 A 0 3 2 4

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- 2. The displacement x metres of a particle at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + x + \cos x = 0$$

When $t = 0$, $x = 0$ and $\frac{dx}{dt} = \frac{1}{2}$.

Find a Taylor series solution for x in ascending powers of t , up to and including the term in t^3 .

(5)



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4.
$$z = -8 + (8\sqrt{3})i$$

- (a) Find the modulus of z and the argument of z . **(3)**

Using de Moivre's theorem,

- (b) find z^3 , **(2)**

- (c) find the values of w such that $w^4 = z$, giving your answers in the form $a + ib$, where $a, b \in \mathbb{R}$. **(5)**



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5.

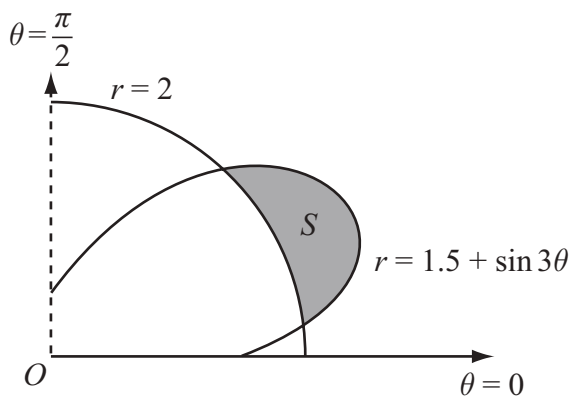


Figure 1

Figure 1 shows the curves given by the polar equations

$$r = 2, \quad 0 \leq \theta \leq \frac{\pi}{2},$$

and $r = 1.5 + \sin 3\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}.$

- (a) Find the coordinates of the points where the curves intersect. (3)

The region S , between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

- (b) Find, by integration, the area of the shaded region S , giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are simplified fractions. (7)



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Question 5 continued

Handwriting lines for the answer to Question 5.

(Total 10 marks)

Q5

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6. A complex number z is represented by the point P in the Argand diagram.

(a) Given that $|z - 6| = |z|$, sketch the locus of P . (2)

(b) Find the complex numbers z which satisfy both $|z - 6| = |z|$ and $|z - 3 - 4i| = 5$. (3)

The transformation T from the z -plane to the w -plane is given by $w = \frac{30}{z}$.

(c) Show that T maps $|z - 6| = |z|$ onto a circle in the w -plane and give the cartesian equation of this circle. (5)



7. (a) Show that the transformation $z = y^{\frac{1}{2}}$ transforms the differential equation

$$\frac{dy}{dx} - 4y \tan x = 2y^{\frac{1}{2}} \quad (\text{I})$$

into the differential equation

$$\frac{dz}{dx} - 2z \tan x = 1 \quad (\text{II}) \quad (\mathbf{5})$$

(b) Solve the differential equation (II) to find z as a function of x . (\mathbf{6})

(c) Hence obtain the general solution of the differential equation (I). (\mathbf{1})



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Question 7 continued

Lined area for writing the answer to Question 7.

Q7

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(Total 12 marks)



N 3 5 3 8 8 A 0 2 1 2 4

8. (a) Find the value of λ for which $y = \lambda x \sin 5x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3 \cos 5x \quad (4)$$

(b) Using your answer to part (a), find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3 \cos 5x \quad (3)$$

Given that at $x = 0$, $y = 0$ and $\frac{dy}{dx} = 5$,

(c) find the particular solution of this differential equation, giving your solution in the form $y = f(x)$. (5)

(d) Sketch the curve with equation $y = f(x)$ for $0 \leq x \leq \pi$. (2)



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Question 8 continued

Lined writing area for the question.

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

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