

Mark Scheme (Results)

June 2011

GCE Further Pure FP2 (6668) Paper 1

Edexcel is one of the leading examining and awarding bodies in the UK and throughout the world. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers.

Through a network of UK and overseas offices, Edexcel's centres receive the support they need to help them deliver their education and training programmes to learners.

For further information, please call our GCE line on 0844 576 0025 or visit our website at www.edexcel.com.

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our **Ask The Expert** email service helpful.

Ask The Expert can be accessed online at the following link:
<http://www.edexcel.com/Aboutus/contact-us/>

June 2011

Publications Code UA027968

All the material in this publication is copyright

© Edexcel Ltd 2011

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\frac{\Delta}{\Delta}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- \square The second mark is dependent on gaining the first mark

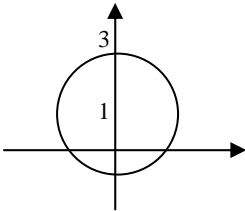
June 2011
Further Pure Mathematics FP 26668
Mark Scheme

Question Number	Scheme	Marks
1.	$3x = (x-4)(x+3) \quad x^2 - 4x - 12 = 0$ $x = -2, x = 6$ both Other critical values are $x = -3, x = 0$ $-3 < x < -2, \quad 0 < x < 6$	M1 A1 B1, B1 M1 A1 A1 (7) 7
	1 st M1 for $\pm(x^2 - 4x - 12) - '=0'$ not required. B marks can be awarded for values appearing in solution e.g. on sketch of graph or in final answer. 2 nd M1 for attempt at method using graph sketch or +/- If cvs correct but correct inequalities are not strict award A1A0.	

Question Number	Scheme	Marks
2. (a)	$\frac{d^3 y}{dx^3} = e^x \left(2y \frac{d^2 y}{dx^2} + 2 \left(\frac{dy}{dx} \right)^2 + 2y \frac{dy}{dx} \right) + e^x \left(2y \frac{dy}{dx} + y^2 + 1 \right)$ $\frac{d^3 y}{dx^3} = e^x \left(2y \frac{d^2 y}{dx^2} + 2 \left(\frac{dy}{dx} \right)^2 + 4y \frac{dy}{dx} + y^2 + 1 \right) \quad (k = 4)$	<p>M1 A1</p> <p>A1</p> <p>(3)</p>
(b)	$\left(\frac{d^2 y}{dx^2} \right)_0 = e^0 (4 + 1 + 1) = 6$ $\left(\frac{d^3 y}{dx^3} \right)_0 = e^0 (12 + 8 + 8 + 1 + 1) = 30$ $y = 1 + 2x + \frac{6x^2}{2} + \frac{30x^3}{6} = 1 + 2x + 3x^2 + 5x^3$	<p>B1</p> <p>B1</p> <p>M1 A1ft</p> <p>(4)</p> <p>7</p>
(a) (b)	<p>1st M1 for evidence of Product Rule</p> <p>1st A1 for completely correct expression or equivalent</p> <p>2nd A1 for correct expression or $k = 4$ stated</p> <p>2nd M1 require four terms and denominators of 2 and 6 (might be implied)</p> <p>A1 follow through from their values in the final answer.</p>	

Question Number	Scheme	Marks
3.	$\frac{dy}{dx} + 5\frac{y}{x} = \frac{\ln x}{x^2}$ <p>Integrating factor $e^{\int \frac{5}{x} dx}$</p> $e^{\int \frac{5}{x} dx} = e^{5 \ln x} = x^5$ $\int x^3 \ln x dx = \frac{x^4 \ln x}{4} - \int \frac{x^3}{4} dx$ $= \frac{x^4 \ln x}{4} - \frac{x^4}{16} (+C)$ $x^5 y = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C \quad y = \frac{\ln x}{4x} - \frac{1}{16x} + \frac{C}{x^5}$	<p>M1</p> <p>A1</p> <p>M1 M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>(8) 8</p>
	<p>1st M1 for attempt at correct Integrating Factor</p> <p>1st A1 for simplified IF</p> <p>2nd M1 for $\frac{\ln x}{x^2}$ times their IF to give their '$x^3 \ln x$'</p> <p>3rd M1 for attempt at correct Integration by Parts</p> <p>2nd A1 for both terms correct</p> <p>3rd A1 constant not required</p> <p>4th M1 $x^5 y = \text{their answer} + C$</p>	

Question Number	Scheme	Marks
4.		
(a)	$(2r+1)^3 = (2r)^3 + 3(2r)^2 + 3(2r) + 1$ $A = 8, B = 12, C = 6$	M1 A1 (2)
(b)	$(2r-1)^3 = (2r)^3 - 3(2r)^2 + 3(2r) - 1$ $(2r+1)^3 - (2r-1)^3 = 24r^2 + 2$	M1 A1cso (*) (2)
(c)	$r=1: \quad 3^3 - 1^3 = 24 \times 1^2 + 2$ $r=2: \quad 5^3 - 3^3 = 24 \times 2^2 + 2$ $\quad \quad \quad : \quad \quad \quad :$ $r=n: \quad (2n+1)^3 - (2n-1)^3 = 24 \times n^2 + 2$ <p>Summing: $(2n+1)^3 - 1 = 24 \sum r^2 + \left(\sum \right) 2$</p> $\left(\sum 2 \right) = 2n$ <p>Proceeding to $\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1)$</p>	M1 A1 M1 B1 A1cso (5) 9
(a) (b) (c)	1 st M1 require coefficients of 1,3,3,1 or equivalent 1 st M1 require 1,-3,3,-1 or equivalent 1 st M1 for attempt with at least 1,2 and n if summing expression incorrect. RHS of display not required at this stage. 1 st A1 for 1,2 and n correct. 2 nd M1 require cancelling and use of $24r^2 + 2$ Award B1 for correct kn for their approach 2 nd A1 is for correct solution only	

Question Number	Scheme	Marks
5.		
(a)	$x^2 + (y-1)^2 = 4$	M1 A1 (2)
(b)	 <p>M1: Sketch of circle A1: Evidence of correct centre and radius</p>	M1 A1 (2)
(c)	$w = \frac{(x+iy)+i}{3+i(x+iy)} = \frac{x+i(y+1)}{(3-y)+ix}$ $= \frac{[x+i(y+1)][(3-y)-ix]}{[(3-y)+ix][(3-y)-ix]}$ <p>On x-axis, so imaginary part = 0: $(y+1)(3-y) - x^2 = 0$ $(y+1)(3-y) - x^2 = 0 \Rightarrow x^2 + (y-1)^2 = 4$, so Q is on C</p>	M1 M1 M1 A1 A1cso (5) 9
Alt. (c)	<p>Let $w = u + iv$: $u = \frac{z+i}{3+iz}$ (since $v = 0$)</p> $z = \frac{3u-i}{1-ui}$ $z-i = \frac{3u-i-i-u}{1-ui} = \frac{2(u-i)}{1-ui}$ $ z-i = \frac{2\sqrt{u^2+1}}{\sqrt{u^2+1}} = 2$, so Q is on C	M1 dM1 M1 A1 A1cso
(a) (b) (c)	<p>M1 Use of $z = x + iy$ and find modulus Award A0 if circle doesn't intersect x - axis twice 1st M for subbing $z = x + iy$ and collecting real and imaginary parts 2nd M for multiply numerator and denominator by their complex conjugate 3rd M for equating imaginary parts of numerator to 0 Award A1 for equation matching part (a), statement not required.</p>	

Question Number	Scheme	Marks
6.	$2 + \cos \theta = \frac{5}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\frac{1}{2} \int (2 + \cos \theta)^2 d\theta = \frac{1}{2} \int (4 + 4 \cos \theta + \cos^2 \theta) d\theta$ $= \frac{1}{2} \left[4\theta + 4 \sin \theta + \frac{\sin 2\theta}{4} + \frac{\theta}{2} \right]$ <p>Substituting limits $\left(\frac{1}{2} \left[\frac{9\pi}{6} + 4 \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{8} \right] = \frac{1}{2} \left(\frac{3\pi}{2} + \frac{17\sqrt{3}}{8} \right) \right)$</p> $\text{Area of triangle} = \frac{1}{2} (r \cos \theta) (r \sin \theta) = \frac{1}{2} \times \frac{25}{4} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \left(= \frac{25\sqrt{3}}{32} \right)$ $\text{Area of } R = \frac{3\pi}{4} + \frac{17\sqrt{3}}{16} - \frac{25\sqrt{3}}{32} = \frac{3\pi}{4} + \frac{9\sqrt{3}}{32}$	B1 M1 M1 A1 M1 M1 A1 M1 A1 (9) 9
	<p>1st M1 for use of $\frac{1}{2} \int r^2 d\theta$ and correct attempt to expand</p> <p>2nd M1 for use of double angle formula - $\sin 2\theta$ required in square brackets</p> <p>3rd M1 for substituting their limits</p> <p>4th M1 for use of $\frac{1}{2}$ base x height</p> <p>5th M1 area of sector – area of triangle</p> <p>Please note there are no follow through marks on accuracy.</p>	

Question Number	Scheme	Marks
7. (a)	$\sin 5\theta = \text{Im}(\cos \theta + i \sin \theta)^5$ $5 \cos^4 \theta (i \sin \theta) + 10 \cos^2 \theta (i^3 \sin^3 \theta) + i^5 \sin^5 \theta$ $= i(5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta)$ $(\text{Im}(\cos \theta + i \sin \theta)^5) = 5 \sin \theta (1 - \sin^2 \theta)^2 - 10 \sin^3 \theta (1 - \sin^2 \theta) + \sin^5 \theta$ $\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta \quad (*)$	B1 M1 A1 M1 A1cso (5)
(b)	$16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta = 5(3 \sin \theta - 4 \sin^3 \theta)$ $16 \sin^5 \theta - 10 \sin \theta = 0$ $\sin^4 \theta = \frac{5}{8} \quad \theta = 1.095$ $\text{Inclusion of solutions from } \sin \theta = -\sqrt[4]{\frac{5}{8}}$ $\text{Other solutions: } \theta = 2.046, 4.237, 5.188$ $\sin \theta = 0 \Rightarrow \theta = 0, \theta = \pi (3.142)$	M1 M1 A1 M1 A1 B1 (6) 11
(a) (b)	Award B if solution considers Imaginary parts and equates to $\sin 5\theta$ 1 st M1 for correct attempt at expansion and collection of imaginary parts 2 nd M1 for substitution powers of $\cos \theta$ 1 st M for substituting correct expressions 2 nd M for attempting to form equation Imply 3 rd M if 4.237 or 5.188 seen. Award for their negative root. Ignore 2π but 2 nd A0 if other extra solutions given.	

Question Number	Scheme	Marks
8.		
(a)	$m^2 + 6m + 9 = 0 \quad m = -3$ C.F. $x = (A + Bt)e^{-3t}$ P.I. $x = P \cos 3t + Q \sin 3t$ $\dot{x} = -3P \sin 3t + 3Q \cos 3t$ $\ddot{x} = -9P \cos 3t - 9Q \sin 3t$ $(-9P \cos 3t - 9Q \sin 3t) + 6(-3P \sin 3t + 3Q \cos 3t) + 9(P \cos 3t + Q \sin 3t) = \cos 3t$ $-9P + 18Q + 9P = 1 \quad \text{and} \quad -9Q - 18P + 9Q = 0$ $P = 0 \quad \text{and} \quad Q = \frac{1}{18}$ $x = (A + Bt)e^{-3t} + \frac{1}{18} \sin 3t$	M1 A1 B1 M1 M1 M1 A1ft (8)
(b)	$t = 0: \quad x = A = \frac{1}{2}$ $\ddot{x} = -3(A + Bt)e^{-3t} + Be^{-3t} + \frac{3}{18} \cos 3t$ $t = 0: \quad \ddot{x} = -3A + B + \frac{1}{6} = 0 \quad B = \frac{4}{3}$ $x = \left(\frac{1}{2} + \frac{4t}{3}\right)e^{-3t} + \frac{1}{18} \sin 3t$	B1 M1 M1 A1 A1 (5)
(c)	$t \approx \frac{59\pi}{6} \quad (\approx 30.9)$ $x \approx -\frac{1}{18}$	B1 B1ft (2) 15
(a)	1 st M1 Form auxiliary equation and correct attempt to solve. Can be implied from correct exponential.	
	2 nd M1 for attempt to differentiate PI twice	
	3 rd M1 for substituting their expression into differential equation	
	4 th M1 for substitution of both boundary values	
(b)	1 st M1 for correct attempt to differentiate their answer to part (a)	
	2 nd M1 for substituting boundary value	

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467

Fax 01623 450481

Email publication.orders@edexcel.com

Order Code UA027968 June 2011

For more information on Edexcel qualifications, please visit
www.edexcel.com/quals

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Ofqual
■■■■■■■■■■



Llywodraeth Cynulliad Cymru
Welsh Assembly Government

