# Mark Scheme (Results)

# June 2011

## GCE Further Pure FP2 (6668) Paper 1

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# EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

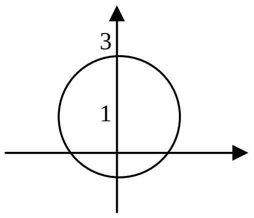
# June 2011
# Further Pure Mathematics FP 26668
# Mark Scheme

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** | $3x = (x-4)(x+3) \qquad x^2 - 4x - 12 = 0$<br>$x = -2,\ x = 6$<br>both<br>Other critical values are $x = -3,\ x = 0$<br>$-3 < x < -2, \qquad 0 < x < 6$ | M1<br>A1<br><br>B1, B1<br>M1 A1 A1<br>(7)<br>**7** |
| | 1st M1 for $\pm(x^2 - 4x - 12)$ – '=0' not required.<br>B marks can be awarded for values appearing in solution e.g. on sketch of graph or in final answer.<br>2nd M1 for attempt at method using graph sketch or +/-<br>If cvs correct but correct inequalities are not strict award A1A0. | |

| Question Number | Scheme | Marks |
|---|---|---|
| **2.** | | |
| **(a)** | $\frac{d^3y}{dx^3} = e^x\left(2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx}\right) + e^x\left(2y\frac{dy}{dx} + y^2 + 1\right)$ | M1 A1 |
| | $\frac{d^3y}{dx^3} = e^x\left(2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 4y\frac{dy}{dx} + y^2 + 1\right)$ $(k = 4)$ | A1 |
| | | (3) |
| **(b)** | $\left(\frac{d^2y}{dx^2}\right)_0 = e^0(4+1+1) = 6$ | B1 |
| | $\left(\frac{d^3y}{dx^3}\right)_0 = e^0(12+8+8+1+1) = 30$ | B1 |
| | $y = 1 + 2x + \frac{6x^2}{2} + \frac{30x^3}{6} = 1 + 2x + 3x^2 + 5x^3$ | M1 A1ft |
| | | (4) **7** |
| **(a)** | 1st M1 for evidence of Product Rule<br>1st A1 for completely correct expression or equivalent<br>2nd A1 for correct expression or $k = 4$ stated | |
| **(b)** | 2nd M1 require four terms and denominators of 2 and 6 (might be implied)<br>A1 follow through from their values in the final answer. | |

| Question Number | Scheme | Marks |
|---|---|---|
| **3.** | $\frac{dy}{dx}+5\frac{y}{x}=\frac{\ln x}{x^2}$ Integrating factor $e^{\int\frac{5}{x}}$ | M1 |
| | $e^{\int\frac{5}{x}}=e^{5\ln x}=x^5$ | A1 |
| | $\int x^3\ln x dx=\frac{x^4\ln x}{4}-\int\frac{x^3}{4}dx$ | M1 M1 A1 |
| | $=\frac{x^4\ln x}{4}-\frac{x^4}{16}\ (+C)$ | A1 |
| | $x^5y=\frac{x^4\ln x}{4}-\frac{x^4}{16}+C$ $y=\frac{\ln x}{4x}-\frac{1}{16x}+\frac{C}{x^5}$ | M1 A1 |
| | | (8) **8** |
| | 1st M1 for attempt at correct Integrating Factor<br>1st A1 for simplified IF<br>2nd M1 for $\frac{\ln x}{x^2}$ times their IF to give their '$x^3\ln x$'<br>3rd M1 for attempt at correct Integration by Parts<br>2nd A1 for both terms correct<br>3rd A1 constant not required<br>4th M1 $x^5y=$ their answer + $C$ | |

| Question Number | Scheme | Marks |
|---|---|---|
| **4.** **(a)** | $(2r+1)^3 = (2r)^3 + 3(2r)^2 + 3(2r) + 1$ | M1 |
| | $A = 8,\ B = 12,\ C = 6$ | A1 (2) |
| **(b)** | $(2r-1)^3 = (2r)^3 - 3(2r)^2 + 3(2r) - 1$ | M1 |
| | $(2r+1)^3 - (2r-1)^3 = 24r^2 + 2$ (*) | A1cso (2) |
| **(c)** | $r=1: \quad 3^3 - 1^3 \quad = \quad 24\times1^2 + 2$ | |
| | $r=2: \quad 5^3 - 3^3 \quad = \quad 24\times2^2 + 2$ | |
| | $\vdots \qquad\qquad \vdots$ | M1 A1 |
| | $r=n: \quad (2n+1)^3 - (2n-1)^3 \quad = \quad 24\times n^2 + 2$ | |
| | Summing: $(2n+1)^3 - 1 = 24\sum r^2 + \left(\sum\ \right)2$ | M1 |
| | $\left(\sum 2\right) = 2n$ | B1 |
| | Proceeding to $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ | A1cso |
| | | (5) **9** |
| **(a)** | 1st M1 require coefficients of 1,3,3,1 or equivalent | |
| **(b)** | 1st M1 require 1,-3,3,-1 or equivalent | |
| **(c)** | 1st M1 for attempt with at least 1,2 and $n$ if summing expression incorrect. RHS of display not required at this stage.<br>1st A1 for 1,2 and n correct.<br>2nd M1 require cancelling and use of $24r^2 + 2$<br>Award B1 for correct $kn$ for their approach<br>2nd A1 is for correct solution only | |

| Question Number | Scheme | Marks |
|---|---|---|
| **5.** **(a)** | $x^2+(y-1)^2=4$ | M1 A1 (2) |
| **(b)** |  M1: Sketch of circle<br>A1: Evidence of correct centre and radius | M1<br>A1 (2) |
| **(c)** | $w=\dfrac{(x+\text{i}y)+\text{i}}{3+\text{i}(x+\text{i}y)}=\dfrac{x+\text{i}(y+1)}{(3-y)+\text{i}x}$<br>$=\dfrac{\left[x+\text{i}(y+1)\right]\left[(3-y)-\text{i}x\right]}{\left[(3-y)+\text{i}x\right]\left[(3-y)-\text{i}x\right]}$<br>On $x$-axis, so imaginary part = 0: $(y+1)(3-y)-x^2=0$<br>$(y+1)(3-y)-x^2=0 \Rightarrow x^2+(y-1)^2=4$, so $Q$ is on $C$ | M1<br>M1<br>M1 A1<br>A1cso (5) **9** |
| **Alt. (c)** | Let $w=u+\text{i}v$: $u=\dfrac{z+\text{i}}{3+\text{i}z}$ (since $v=0$)<br>$z=\dfrac{3u-\text{i}}{1-u\text{i}}$<br>$z-\text{i}=\dfrac{3u-\text{i}-\text{i}-u}{1-u\text{i}}=\dfrac{2(u-\text{i})}{1-u\text{i}}$<br>$\lvert z-\text{i}\rvert=\dfrac{2\sqrt{u^2+1}}{\sqrt{u^2+1}}=2$, so $Q$ is on $C$ | M1<br>dM1<br>M1 A1<br>A1cso |
| **(a)**<br>**(b)**<br>**(c)** | M1 Use of $z=x+\text{i}y$ and find modulus<br>Award A0 if circle doesn't intersect $x$ - axis twice<br>1st M for subbing $z=x+\text{i}y$ and collecting real and imaginary parts<br>2nd M for multiply numerator and denominator by their complex conjugate<br>3rd M for equating imaginary parts of numerator to 0<br>Award A1 for equation matching part (a), statement not required. | |

| Question Number | Scheme | Marks |
|---|---|---|
| **6.** | $2+\cos\theta=\frac{5}{2}\Rightarrow\theta=\frac{\pi}{3}$ | B1 |
| | $\frac{1}{2}\int(2+\cos\theta)^2 d\theta=\frac{1}{2}\int(4+4\cos\theta+\cos^2\theta)d\theta$ | M1 |
| | $=\frac{1}{2}\left[4\theta+4\sin\theta+\frac{\sin 2\theta}{4}+\frac{\theta}{2}\right]$ | M1 A1 |
| | Substituting limits $\left(\frac{1}{2}\left[\frac{9\pi}{6}+4\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{8}\right]=\frac{1}{2}\left(\frac{3\pi}{2}+\frac{17\sqrt{3}}{8}\right)\right)$ | M1 |
| | Area of triangle = $\frac{1}{2}(r\cos\theta)(r\sin\theta)=\frac{1}{2}\times\frac{25}{4}\times\frac{1}{2}\times\frac{\sqrt{3}}{2}\left(=\frac{25\sqrt{3}}{32}\right)$ | M1 A1 |
| | Area of $R$ = $\frac{3\pi}{4}+\frac{17\sqrt{3}}{16}-\frac{25\sqrt{3}}{32}=\frac{3\pi}{4}+\frac{9\sqrt{3}}{32}$ | M1 A1 |
| | | (9) **9** |
| | 1st M1 for use of $\frac{1}{2}\int r^2\, \mathrm{d}\theta$ and correct attempt to expand<br>2nd M1 for use of double angle formula - $\sin 2\theta$ required in square brackets<br>3rd M1 for substituting their limits<br>4th M1 for use of $\frac{1}{2}$ base x height<br>5th M1 area of sector – area of triangle<br>Please note there are no follow through marks on accuracy. | |

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** | | |
| **(a)** | $\sin 5\theta = \text{Im}(\cos\theta + \text{i}\sin\theta)^5$ | B1 |
| | $5\cos^4\theta(\text{i}\sin\theta) + 10\cos^2\theta(\text{i}^3\sin^3\theta) + \text{i}^5\sin^5\theta$ | M1 |
| | $= \text{i}(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$ | A1 |
| | $\left(\text{Im}(\cos\theta + \text{i}\sin\theta)^5\right) = 5\sin\theta(1-\sin^2\theta)^2 - 10\sin^3\theta(1-\sin^2\theta) + \sin^5\theta$ | M1 |
| | $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta \quad (*)$ | A1cso |
| | | (5) |
| **(b)** | $16\sin^5\theta - 20\sin^3\theta + 5\sin\theta = 5\left(3\sin\theta - 4\sin^3\theta\right)$ | M1 |
| | $16\sin^5\theta - 10\sin\theta = 0$ | M1 |
| | $\sin^4\theta = \dfrac{5}{8} \qquad \theta = 1.095$ | A1 |
| | Inclusion of solutions from $\sin\theta = -\sqrt[4]{\dfrac{5}{8}}$ | M1 |
| | Other solutions: $\theta = 2.046,\ 4.237,\ 5.188$ | A1 |
| | $\sin\theta = 0 \Rightarrow \theta = 0,\ \theta = \pi\ (3.142)$ | B1 |
| | | (6) |
| | | **11** |
| **(a)** | Award B if solution considers Imaginary parts and equates to $\sin 5\theta$<br>1st M1 for correct attempt at expansion and collection of imaginary parts<br>2nd M1 for substitution powers of $\cos\theta$ | |
| **(b)** | 1st M for substituting correct expressions<br>2nd M for attempting to form equation<br>Imply 3rd M if 4.237 or 5.188 seen. Award for their negative root.<br>Ignore $2\pi$ but 2nd A0 if other extra solutions given. | |

| Question Number | Scheme | Marks |
|---|---|---|
| **8.** (a) | $m^2+6m+9=0 \qquad m=-3$ | M1 |
| | C.F. $x=(A+Bt)e^{-3t}$ | A1 |
| | P.I. $x=P\cos 3t+Q\sin 3t$ | B1 |
| | $\dot{x}=-3P\sin 3t+3Q\cos 3t$<br>$\ddot{x}=-9P\cos 3t-9Q\sin 3t$ | M1 |
| | $(-9P\cos 3t-9Q\sin 3t)+6(-3P\sin 3t+3Q\cos 3t)+9(P\cos 3t+Q\sin 3t)=\cos$ | M1 |
| | $-9P+18Q+9P=1$ and $-9Q-18P+9Q=0$ | M1 |
| | $P=0$ and $Q=\frac{1}{18}$ | A1 |
| | $x=(A+Bt)e^{-3t}+\frac{1}{18}\sin 3t$ | A1ft |
| | | (8) |
| (b) | $t=0: \quad x=A=\frac{1}{2}$ | B1 |
| | $\dot{x}=-3(A+Bt)\mathrm{e}^{-3t}+B\mathrm{e}^{-3t}+\frac{3}{18}\cos 3t$ | M1 |
| | $t=0: \quad \dot{x}=-3A+B+\frac{1}{6}=0 \qquad B=\frac{4}{3}$ | M1 A1 |
| | $x=\left(\frac{1}{2}+\frac{4t}{3}\right)\mathrm{e}^{-3t}+\frac{1}{18}\sin 3t$ | A1 |
| | | (5) |
| (c) | $t\approx\frac{59\pi}{6} \quad (\approx 30.9)$ | B1 |
| | $x\approx-\frac{1}{18}$ | B1ft |
| | | (2)<br>**15** |
| (a)<br><br><br><br><br>(b) | 1st M1 Form auxiliary equation and correct attempt to solve. Can be implied from correct exponential.<br>2nd M1 for attempt to differentiate PI twice<br>3rd M1 for substituting their expression into differential equation<br>4th M1 for substitution of both boundary values<br>1st M1 for correct attempt to differentiate their answer to part (a)<br>2nd M1 for substituting boundary value | |

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