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3. $\frac{d^2y}{dx^2} + 4y - \sin x = 0$

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = \frac{1}{8}$ at $x = 0$,

find a series expansion for y in terms of x , up to and including the term in x^3 .

(5)



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4. (a) Given that

$$z = r(\cos \theta + i \sin \theta), \quad r \in \mathbb{R}$$

prove, by induction, that $z^n = r^n(\cos n\theta + i \sin n\theta)$, $n \in \mathbb{Z}^+$

(5)

$$w = 3\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$$

(b) Find the exact value of w^5 , giving your answer in the form $a + ib$, where $a, b \in \mathbb{R}$.

(2)



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Question 4 continued

Lined area for writing the answer to Question 4.



P 4 3 1 4 9 A 0 9 2 8

5. (a) Find the general solution of the differential equation

$$x \frac{dy}{dx} + 2y = 4x^2 \tag{5}$$

(b) Find the particular solution for which $y = 5$ at $x = 1$, giving your answer in the form $y = f(x)$. (2)

(c) (i) Find the exact values of the coordinates of the turning points of the curve with equation $y = f(x)$, making your method clear.

(ii) Sketch the curve with equation $y = f(x)$, showing the coordinates of the turning points. (5)



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Question 5 continued

(Total 12 marks)

Q5

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Question 6 continued



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8.

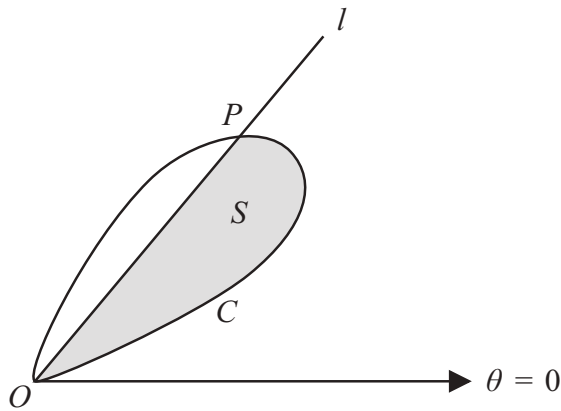


Figure 1

Figure 1 shows a curve C with polar equation $r = a \sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and a half-line l .

The half-line l meets C at the pole O and at the point P . The tangent to C at P is parallel to the initial line. The polar coordinates of P are (R, ϕ) .

(a) Show that $\cos \phi = \frac{1}{\sqrt{3}}$ (6)

(b) Find the exact value of R . (2)

The region S , shown shaded in Figure 1, is bounded by C and l .

(c) Use calculus to show that the exact area of S is

$$\frac{1}{36} a^2 \left(9 \arccos \left(\frac{1}{\sqrt{3}} \right) + \sqrt{2} \right) \quad (7)$$



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Question 8 continued

Lined area for writing answers.

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

Q8

