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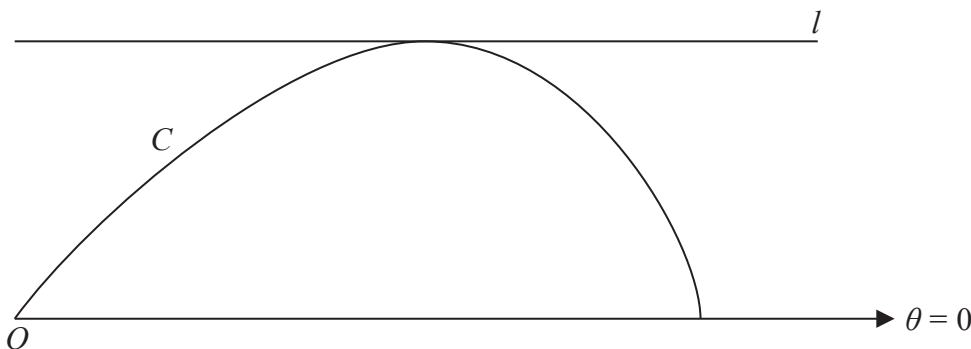


Figure 1

Figure 1 shows the curve  $C$  with polar equation

$$r = 2 \cos 2\theta, \quad 0 \leq \theta \leq \frac{\pi}{4}$$

The line  $l$  is parallel to the initial line and is a tangent to  $C$ .

Find an equation of  $l$ , giving your answer in the form  $r = f(\theta)$ .

(9)

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5. 
$$y \frac{d^2y}{dx^2} + 2 \left( \frac{dy}{dx} \right)^2 + 2y = 0$$

(a) Find an expression for  $\frac{d^3y}{dx^3}$  in terms of  $\frac{d^2y}{dx^2}$ ,  $\frac{dy}{dx}$  and  $y$ . (4)

Given that  $y = 2$  and  $\frac{dy}{dx} = 0.5$  at  $x = 0$ ,

(b) find a series solution for  $y$  in ascending powers of  $x$ , up to and including the term in  $x^3$ . (5)

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**Question 5 continued**

Lined area for writing the answer to Question 5.

**(Total 9 marks)**

**Q5**



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6. The transformation  $T$  maps points from the  $z$ -plane, where  $z = x + iy$ , to the  $w$ -plane, where  $w = u + iv$ .

The transformation  $T$  is given by

$$w = \frac{z}{iz + 1}, \quad z \neq i$$

The transformation  $T$  maps the line  $l$  in the  $z$ -plane onto the line with equation  $v = -1$  in the  $w$ -plane.

- (a) Find a cartesian equation of  $l$  in terms of  $x$  and  $y$ . (5)

The transformation  $T$  maps the line with equation  $y = \frac{1}{2}$  in the  $z$ -plane onto the curve  $C$  in the  $w$ -plane.

- (b) (i) Show that  $C$  is a circle with centre the origin.  
 (ii) Write down a cartesian equation of  $C$  in terms of  $u$  and  $v$ . (6)

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7. (a) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta \quad (5)$$

(b) Hence find the five distinct solutions of the equation

$$16x^5 - 20x^3 + 5x + \frac{1}{2} = 0$$

giving your answers to 3 decimal places where necessary. (5)

(c) Use the identity given in (a) to find

$$\int_0^{\frac{\pi}{4}} (4 \sin^5 \theta - 5 \sin^3 \theta) d\theta$$

expressing your answer in the form  $a\sqrt{2} + b$ , where  $a$  and  $b$  are rational numbers. (4)

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8. (a) Show that the substitution  $x = e^z$  transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 2y = 3 \ln x, \quad x > 0 \tag{I}$$

into the equation

$$\frac{d^2y}{dz^2} + \frac{dy}{dz} - 2y = 3z \tag{II}$$

**(7)**

(b) Find the general solution of the differential equation (II). **(6)**

(c) Hence obtain the general solution of the differential equation (I) giving your answer in the form  $y = f(x)$ . **(1)**

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