

Mark Scheme (Results)

Summer 2016

Pearson Edexcel IAL in Further Pure Mathematics 1 (WFM01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.
- 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- _ or d... The second mark is dependent on gaining the first mark

- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.
- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

 $(x^2+bx+c) = (x+p)(x+q)$, where |pq| = |c|, leading to $x = \dots$

 $(ax^2+bx+c) = (mx+p)(nx+q)$, where |pq| = |c| and |mn| = |a|, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme		Notes	Marks			
1.	$\sum_{r=1}^n r(r^2 -$	$(-3) = \sum_{r=1}^{n} r^3 - 3\sum_{r=1}^{n} r^3$					
		$=\frac{1}{4}n^2(n+1)^2 - 3\left(\frac{1}{2}n(n+1)\right)$	Attempts to expand $r(r^2 - 3)$ and attempts to substitute at least one correct standard formula into their resulting expression.	M1			
			Correct expression (or equivalent)	A1			
		$=\frac{1}{4}n(n+1)\left[n(n+1)-6\right]$ a	dependent on the previous M mark Attempt to factorise at least $n(n+1)$ having ttempted to substitute both the standard formulae	dM1			
		$=\frac{1}{4}n(n+1)\left[n^2+n-6\right]$	{this step does not have to be written]				
		$= \frac{1}{4}n(n+1)(n+3)(n-2)$	Correct completion with no errors	A1 cso			
				(4)			
			Ouestion 1 Notes	4			
1.	Note	Applying eg. $n = 1, n = 2, n = 3$ to	the printed equation without applying the standard	d formulae			
		to give $a = 1, b = 3, c = -2$ or another combination of these numbers is M0A0M0A0.					
	Alt	Alternative Method: Obtains $\sum_{r=1}^{n} r(r^2 - 3) \equiv \frac{1}{4}n(n+1)\left[n(n+1) - 6\right] \equiv \frac{1}{4}n(n+a)(n+b)(n+c)$					
		$30 \ u = 1, n = 1 \implies -2 = -\frac{1}{4}(1)(2)$	$(1+b)(1+c)$ and $n-2 \implies 0 - \frac{1}{4}(2)(3)(2+b)(2+b)(2+b)(2+b)(2+b)(2+b)(2+b)(2+b$	- ()			
	11/1	leading to either $b = -2, c = 3$ or	b = 3, c = -2				
	aMI	dependent on the previous M mark. Substitutes in values of n and solves to find h and n					
	A1	Finds $a = 1, b = 3, c = -2$ or another combination of these numbers.					
	Note	Using only a method of "proof by induction" scores 0 marks unless there is use of the standard					
		formulae when the first M1 may be scored.					
	Note	Allow final dM1A1 for $\frac{1}{4}n^4 + \frac{1}{2}n^3 - \frac{5}{4}n^2 - \frac{3}{2}n$ or $\frac{1}{4}n(n^3 + 2n^2 - 5n - 6)$					
		or $\frac{1}{4}(n^4 + 2n^3 - 5n^2 - 6n) \to \frac{1}{4}$	n(n+1)(n+3)(n-2), from no incorrect working.				
	Note	Give final A0 for eg. $\frac{1}{4}n(n+1)[n]$	$n^{2} + n - 6$] $\rightarrow = \frac{1}{4}n(n+1)(x+3)(x-2)$ unless rec	covered.			

Question Number		Scheme	Notes	Marks
2.	$P: y^2 = 2$	$8x \text{ or } P(7t^2, 14t)$		
(a)	$(y^2 = 4ax)$	$a \Rightarrow a = 7) \Rightarrow S(7,0)$	Accept (7,0) or $x = 7$, $y = 0$ or 7 marked on the <i>x</i> -axis in a sketch	B1
(b)	$\{A \text{ and } B\}$	have x coordinate} $\frac{7}{2}$	Divides their <i>x</i> coordinate from (a) by 2 and substitutes this into the parabola equation and takes the square root to	(1)
	So $y^2 = 2$	$8\left(\frac{7}{2}\right) \Rightarrow y^2 = 98 \Rightarrow y = \dots$	find $y = \dots$ or applies	M1
	$y = \sqrt{2(2)}$	$(7) - 3.5)^2 - (3.5)^2 \left\{ = \sqrt{(10.5)^2 - (3.5)^2} \right\}$	$y = \sqrt{\left(2("7") - \left(\frac{"7"}{2}\right)\right)^2 - \left(\frac{"7"}{2}\right)^2}$	
	$7t^2 = 3.5$	$\Rightarrow t = \sqrt{0.5} \Rightarrow y = 2(7)\sqrt{0.5}$	or solves $7t^2 = 3.5$ and finds $y = 2(7)$ "their t"	
	$y = (\pm)7$	$\sqrt{2}$	<i>At least one</i> correct exact value of <i>y</i> . Can be un-simplified or simplified.	A1
	A, B have	coordinates $\left(\frac{7}{2}, 7\sqrt{2}\right)$ and $\left(\frac{7}{2}, -7\sqrt{2}\right)$		
	Area trian $ \frac{1}{2} $	gle $ABS = \left(2(7\sqrt{2})\right)\left(\frac{7}{2}\right)$	dependent on the previous M mark A full method for finding	dM1
	• $\frac{1}{2}$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	he area of triangle ABS.	uwn
		$=\frac{49}{2}\sqrt{2}$	Correct exact answer.	A1
				(4)
		Questio	n 2 Notes	5
2. (a) (b)	Note 1 st M1	You can give B1 for part (a) for correct r Allow a slip when candidates find the x c 0 < their midpoint $<$ their a	elevant work seen in either part (a) or part coordinate of their midpoint as long as	(b)
	Note	Give 1 th M0 if a candidate finds and uses	y = 98	
	I st AI	Allow any exact value of either $7\sqrt{2}$, –	$7\sqrt{2}, \sqrt{98}, -\sqrt{98}, 14\sqrt{0.5}, \text{ awrt } 9.9 \text{ or av}$	wrt – 9.9
	2 nd dM1	Either $\frac{1}{2} (2 \times \text{their } "7\sqrt{2}") (\text{their } x_{\text{midpoint}})$) or $\frac{1}{2} (2 \times \text{their} "7\sqrt{2}") (\text{their} "7" - x_{\text{midpt}})$	pint)
	Note	Condone area triangle $ABS = \left(7\sqrt{2}\right)\left(\frac{7}{2}\right)$, i.e. (their " $7\sqrt{2}$ ")($\frac{\text{their "7"}}{2}$)	
	2 nd A1	Allow exact answers such as $\frac{49}{2}\sqrt{2}$, $\frac{49}{\sqrt{2}}$	$\frac{1}{2}$, 24.5 $\sqrt{2}$, $\frac{\sqrt{4802}}{2}$, $\sqrt{\frac{4802}{4}}$, 3.5 $\sqrt{2}$, 49 $\sqrt{\frac{1}{2}}$	
		or $\frac{7}{2}\sqrt{98}$ but do not allow $\frac{1}{2}(3.5)(2\sqrt{9})$	(8) seen by itself	
	Note	Give final A0 for finding 34.64823228	. without reference to a correct exact value.	

Question Number	Scheme			Notes	Marks
3.	$f(x) = x^2 + \frac{3}{x} - 1, x < 0$				
(a)	$f'(x) = 2x - 3x^{-2}$		At one of ei	ther $x^2 \rightarrow \pm Ax$ or $\frac{3}{x} \rightarrow \pm Bx^{-2}$	M1
			where	Correct differentiation	A1
	$f(-1.5) = -0.75$, $f'(-1.5) = -\frac{13}{3}$	Eit awr	ther f(-1.5) t -4.33 or : Ca	= -0.75 or $f'(-1.5) = -\frac{13}{3}$ or a correct numerical expression for either $f(-1.5)$ or $f'(-1.5)$ an be implied by later working	B1
		75	depe	ndent on the previous M mark	
	$\left\{\alpha \simeq -1.5 - \frac{f(-1.5)}{f'(-1.5)}\right\} \Rightarrow \alpha \simeq -1.5 - \frac{-0.5}{-4.332}$.75 3333	Valid at their	ttempt at Newton-Raphson using values of $f(-1.5)$ and $f'(-1.5)$	dM1
	$\left\{ \alpha = -1.67307692 \text{ or } -\frac{87}{52} \right\} \Rightarrow \alpha = -1.67307692$	7	depo (I	endent on all 4 previous marks -1.67 on their first iteration gnore any subsequent iterations)	A1 cso cao
	Correct differentiation followed by	a corr	ect answer s	scores full marks in (a)	
	Correct answer with <u>no</u> y	workin	g scores no	marks in (a)	
(b) Way 1	f(-1.675) = 0.01458022 f(-1.665) = -0.0295768		Chooses within ± 0 .	a suitable interval for x, which is 005 of their answer to (a) and at ast one attempt to evaluate $f(x)$	(5) M1
	Sign change (positive, negative) (and $f(x)$ is continuous) therefore (a root) $\alpha = -1.67$ (2 dg	p)	Both v	values correct awrt (or truncated) 1 sf, sign change and conclusion.	A1 cso
					(2)
(b)	Alt 1: Applying Newton-Raphson again E	Eg. Usir	ng $\alpha = -1.6^{\circ}$	7, -1.673 or $-\frac{87}{52}$	
Way 2	• $\alpha \simeq -1.67 - \frac{-0.007507185629}{-4.415692926} \left\{ = -\frac{-0.007507185629}{-4.415692926} \right\}$ • $\alpha \simeq -1.673 - \frac{0.005743106396}{-4.41783855} \left\{ = -\frac{-1}{-4.417893838} \right\}$	-1.6717 -1.6717 .67170	700115} 700019} 036}	Evidence of applying Newton- Raphson for a second time on their answer to part (a)	M1
	So $\alpha = -1.67 (2 \text{ dp})$			$\alpha = -1.67$	A1
		<u> </u>			(2)
					7

		Question 3 Notes								
3. (a)	Note	Incorrect differentiation followed by their estimate of α with no evidence of applying the								
		NR formula is final dM0A0.								
	B1	B1 can be given for a correct numerical expression for either $f(-1.5)$ or $f'(-1.5)$								
		Eg. either $(-1.5)^2 + \frac{3}{(-1.5)} - 1$ or $2(-1.5) - \frac{3}{(-1.5)^2}$ are fine for B1.								
	Final	This mark can be implied by applying at least one correct value of either $f(-1.5)$ or $f'(-1.5)$								
	dM1	in $-1.5 - \frac{f(-1.5)}{f'(-1.5)}$. So just $-1.5 - \frac{f(-1.5)}{f'(-1.5)}$ with an incorrect answer and no other evidence								
		scores final dM0A0.								
	Note	Give final dM0 for applying $1.5 - \frac{f(-1.5)}{f'(-1.5)}$ without first quoting the correct N-R formula.								
3. (b)	A1	Way 1: correct solution only								
		Candidate needs to state both of their values for $f(x)$ to awrt (or truncated) 1 sf along with								
		a reason and conclusion. Reference to change of sign or eg. $f(-1.6/5) \times f(-1.665) < 0$								
		or a diagram or < 0 and > 0 or one positive, one negative are sufficient reasons. There must								
		be a (minimal, not incorrect) conclusion, eg. $\alpha = -1.07$, root (or α or part (a)) is correct, QED								
		A minimal exceptable. Ignore the presence of absence of any reference to continuity.								
		A minimal acceptable reason and conclusion is change of sign, hence root.								
	Note	Stating "root is in between -1.675 and -1.665 " without some reference to $\alpha = -1.67$ is not								
		sufficient for A1								
	Note	Accept 0.015 as a correct evaluation of $f(-1.675)$								
	A1	Way 2: correct solution only								
		Their conclusion in Way 2 needs to convey that they understand that $\alpha = -1.67$ to 2 decimal								
		places. Eg. "therefore my answer to part (a) [which must be $-1.6/$] is correct" is fine for A1.								
	Note	$\int -1.67 - \frac{f(-1.67)}{f'(1.67)} = -1.67(2 \text{ dp}) \text{ is sufficient for M1A1 in part (b).}$								
	Note	The root of $f(x) = 0$ is -1.67169988 , so candidates can also choose x_1 which is less than								
		-1.67169988 and choose x_2 which is greater than -1.67169988 with both x_1 and x_2 lying								
		in the interval $\begin{bmatrix} -1.675, -1.665 \end{bmatrix}$ and evaluate $f(x_1)$ and $f(x_2)$.								
3. (b)	Note	Helpful Table								
		x f(x)								
		-1.675 0.014580224								
		-1.674 0.010161305								
		-1.673 0.005743106								
		-1.672 0.001325627								
		-1.671 -0.003091136								
		-1.009 $-0.011922323-1.668$ -0.016337151								
		-1.667 $-0.010537151-1.667$ -0.020751072								
		-1.666 -0.025164288								
		-1.665 -0.029576802								
		11000 0102/010002								

Past Paper (Mark Scheme)

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WF	M01
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Question Number		Scheme		Notes	Mar	'ks
4.	$\mathbf{A} = \begin{pmatrix} k \\ -1 \end{pmatrix}$	$\begin{pmatrix} 3 \\ k+2 \end{pmatrix}$, where <i>k</i> is a constant and let <i>g</i>	$g(k) = k^2 + 2k + k$	3		
(a)	$\left\{ \det(\mathbf{A}) = \right.$	= $\frac{k(k+2)+3}{k(k+2)+3}$ or k^2+2k+3	Correct det(A), un-simplified or simplified	B1	
Way 1		$(k+1)^2 - 1 + 3$	Att	tempts to complete the square [usual rules apply]	M1	
	=	$(k+1)^2 + 2 > 0$		$(k+1)^2 + 2$ and > 0	A1 c	so
	()	~ 1 / 1			(3)
(a)	$det(\mathbf{A}) =$	= $\frac{k(k+2)+3 \text{ or } k^2+2k+3}{k^2+2k+3}$	Correct det(A), un-simplified or simplified	B1	
Way 2	$\left\{b^2-4ac\right\}$	$r = \begin{cases} 2^2 - 4(1)(3) \end{cases}$	Applie	es " $b^2 - 4ac$ " to their det(A)	M1	
	All of	2				
	• b	z - 4ac = -8 < 0				
	• sc • sc	to det(A) > 0	e the x-axis	Complete solution	A1	cso
		× /				(3)
(a)	$\left\{ \mathbf{g}(k) = \mathbf{d} \right\}$	$\det(\mathbf{A}) = \begin{cases} k(k+2) + 3 \text{ or } k^2 + 2k + 3 \end{cases}$	Correct det(A), un-simplified or simplified	B1	
Way 3	$\mathbf{g}'(k) = 2k$	$k+2 = 0 \Longrightarrow k = -1$	Finds the v	alue of <i>k</i> for which $g'(k) = 0$	M1	
	$g_{\min} = (-$	$1)^2 + 2(-1) + 3$	and substit	tutes this value of k into $g(k)$	1111	
	$g_{\min} = 2,$	so $det(\mathbf{A}) > 0$	Ę	$g_{\min} = 2$ and states det(A) > 0	A1	cso
(b)	$\mathbf{A}^{-1} = \frac{1}{k^2}$	$\frac{1}{\binom{2}{2}+2k+3}\binom{k+2}{1} \frac{-3}{k}$		$\frac{1}{\text{their det}(\mathbf{A})} \begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$	M1	(3)
				Correct answer in terms of k	A1	
						(2)
		Ques	stion 4 Notes			
4. (a)	B1	Also allow $k(k+2) - 3$				
	Note	Way 2: Proving $b^2 - 4ac = -8 < 0$	by itself could n	the near that $det(\mathbf{A}) > 0$ or $det(\mathbf{A})$	A)<0	
	Note	To gain the final A1 mark for Way 2,	candidates need	l to show $b^2 - 4ac = -8 < 0$ a	nd m	ake
		some reference to $k^2 + 2k + 3$ being a positive or evaluates det(A) for any	bove the x-axis value of k to give	(eg. states that coefficient of A e a positive result or sketches A	k ² is a	
	Noto	Δ 4 transfer to solve dat(Δ) = 0 by or	is) before the gued	taking that $\det(\mathbf{A}) > 0$.	<u>[</u>];	
	Note	is enough to score the M1 mark for W	pprynig the quad Vav 2. To gain A	A 1 these candidates need to ma	v 21 ke	
		some reference to $k^2 + 2k + 3$ being a positive or evaluates det(A) for any	above the x-axis value of k to give	(eg. states that coefficient of <i>a</i>	k^2 is	
		quadratic curve that is above the x-ax	is) before then s	tating that $det(\mathbf{A}) > 0$.		
(b)	A1	Allow either $\frac{1}{(k+1)^2 + 2} \begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$) or $\left(\frac{\frac{k+2}{k^2+2k}}{\frac{1}{k^2+2k}}\right)$	$\frac{-3}{k^3} = \frac{-3}{k^2 + 2k + 3}$ or equivalen $\frac{k}{k^3} = \frac{k}{k^2 + 2k + 3}$	t.	

Past Paper (Mark Scheme)

Question Number		Scheme	Notes	Marks		
5.	$2z + z^* =$	$\frac{3+4i}{7+i}$				
Way 1	$\Big\{2z+z^*=$	$= \left\{ 2(a+\mathrm{i}b) + (a-\mathrm{i}b) \right\}$	Left hand side = $2(a+ib) + (a-ib)$ Can be implied by eg. $3a + ib$ Note: This can be seen anywhere in their solution	B1		
	= ($\frac{(3+4i)}{(7+i)}\frac{(7-i)}{(7-i)}$	Multiplies numerator and denominator of the right hand side by $7 - i$ or $-7 + i$	M1		
	=	$\frac{25+25i}{50}$	Applies $i^2 = -1$ to and collects like terms to give right hand side = $\frac{25 + 25i}{50}$ or equivalent	A1		
	So, 3 <i>a</i> + i	$\mathbf{i}b = \frac{1}{2} + \frac{1}{2}\mathbf{i}$	dependent on the previous B and M marks Equates either real parts or imaginary parts to give at least one of $a = \dots$ or $b = \dots$	ddM1		
	$\Rightarrow a = \frac{1}{6},$	$b = \frac{1}{2}$ or $z = \frac{1}{6} + \frac{1}{2}i$	Either $a = \frac{1}{6}$ and $b = \frac{1}{2}$ or $z = \frac{1}{6} + \frac{1}{2}i$	A1		
				(5)		
Way 2	$\Big\{2z+z^*=$	$= \left\{ 2(a+\mathrm{i}b) + (a-\mathrm{i}b) \right\}$	Left hand side = $2(a+ib) + (a-ib)$ Can be implied by eg. $3a + ib$	B1		
	(3a + ib)($(7 + i) = \dots$	Multiplies their $(3a + ib)$ by $(7 + i)$	M1		
	21a + 3ai	$+7bi - b = \dots$	Applies $i^2 = -1$ to give left hand side = $21a + 3ai + 7bi - b$	A1		
	So, (21 <i>a</i> - gives 21 <i>a</i>	(-b) + (3a+7b) = 3 + 4i a - b = 3, 3a + 7b = 4	dependent on the previous B and M marks Equates both real parts and imaginary parts to give at least one of $a =$ or $b =$	ddM1		
	$\Rightarrow a = \frac{1}{6},$	$b = \frac{1}{2}$ or $z = \frac{1}{6} + \frac{1}{2}i$	Either $a = \frac{1}{6}$ and $b = \frac{1}{2}$ or $z = \frac{1}{6} + \frac{1}{2}i$	A1		
				(5)		
				5		
		1	Question 5 Notes			
5.	Note	Some candidates may let $z = x$	$z + iy$ and $z^* = x - iy$.			
		So apply the mark scheme with $x \equiv a$ and $y \equiv b$.				
	Note For the final A1 mark, you can accept exact equivalents for <i>a</i> , <i>b</i> .					

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Past Paper (Mark Scheme)

Question Number	Scheme		Notes	Marks	
6.	$H: xy = 25, P\left(5t, \frac{5}{t}\right)$ is a general point or	n H			
(a)	Either $5t\left(\frac{5}{t}\right) = 25$ or $y = \frac{25}{x} = \frac{25}{5t} = \frac{25}{5t}$	$=\frac{5}{t}$ or	$x = \frac{25}{y} = \frac{25}{\frac{5}{t}} = 5t$ or states $c = 5$	B1	
					(1)
(b)	$y = \frac{25}{x} = 25x^{-1} \Rightarrow \frac{dy}{dx} = -25x^{-2} = -\frac{25}{x^2}$		$\frac{dy}{dx} = \pm k x^{-2}$ where k is a numerical value		
	$xy = 25 \Longrightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$		Correct use of product rule. The sum of two terms, one of which is correct.	M1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \cdot \frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{5}{t^2} \left(\frac{1}{5}\right)$		$\frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{1}{\mathrm{their}} \frac{\mathrm{d}x}{\mathrm{d}t}$		
	$\left\{ \text{At } A, \ t = \frac{1}{2}, \ x = \frac{5}{2}, \ y = 10 \right\} \Longrightarrow \frac{dy}{dx} = -4$		Correct numerical gradient at <i>A</i> , which is found using calculus. Can be implied by later working	A1	
	So, $m_N = \frac{1}{4}$	Appli	ies $m_N = \frac{-1}{m_T}$, to find a numerical m_N , where m_T is found from using calculus.	M1	
	• $y-10 = \frac{1}{4} \left(x - \frac{3}{2} \right)$		Correct line method for a normal where a numerical $m_N (\neq m_T)$ is found	M1	
	• $10 = \frac{1}{4} \left(\frac{5}{2}\right) + c \Rightarrow c = \frac{75}{8} \Rightarrow y = \frac{1}{4}$	$x + \frac{75}{8}$	from using calculus. Can be implied by later working		
	leading to $8y-2x-75=0$ (*)		Correct solution only	A1	
					(5)
(c)	$y = \frac{25}{x} \implies 8\left(\frac{25}{x}\right) - 2x - 75 = 0$	or $x =$	$\frac{25}{y} \Rightarrow 8y - 2\left(\frac{25}{y}\right) - 75 = 0$		
	or $x = 5t, y = \frac{5}{t} = \frac{5}{t}$	$\Rightarrow 8(5t) -$	$-2\left(\frac{5}{t}\right) - 75 = 0$	M1	
	Substitutes $y = \frac{25}{x}$ or $x = \frac{25}{y}$ or x	x = 5t and	d $y = \frac{5}{t}$ into the printed equation		
	or their normal equation to obtain an	n equation	in either x only, y only or t only		
	$2x^2 + 75x - 200 = 0 \text{or} 8y^2 - 75y - 3y^2 = 0$	50 = 0 or	$2t^2 + 15t - 8 = 0 \text{ or } 10t^2 + 75t - 40 = 0$		
	$(2x-5)(x+40) = 0 \Rightarrow x = \dots \text{ or } (y-10)(8y+5) = 0 \Rightarrow y = \dots \text{ or } (2t-1)(t+8) = 0 \Rightarrow t = \dots$ dependent on the previous M mark Correct attempt of solving a 3TQ to find either $x = \dots$, $y = \dots$ or $t = \dots$				
	Finds at least one of e	either $x =$	-40 or $y = -\frac{5}{8}$	A1	
	$B\left(-40, -\frac{5}{8}\right)$ st	Both coated they	orrect coordinates (If coordinates are not can be paired together as $x =, y =$)	A1	
					(4)
					10

		Question 6 Notes						
6. (a)	Note	A conclusion is not required on this occasion in part (a).						
	B 1	Condone reference to $c = 5$ (as $xy = c^2$ and $\left(ct, \frac{c}{t}\right)$ are referred in the Formula book.)						
(b)	Note	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \cdot \frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{5}{t^2} \left(\frac{1}{5}\right) = -\frac{1}{t^2} \Rightarrow m_N = t^2 \Rightarrow y - 10 = t^2 \left(x - \frac{5}{2}\right)$ scores only the first M1.						
		When $t = \frac{1}{2}$ is substituted giving $y - 10 = \frac{1}{4} \left(x - \frac{5}{2} \right)$						
(a)	Note	the response then automatically gets A1(implied) M1(implied) M1						
(C)	note	• $8\left(\frac{25}{x}\right) - 2x - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$						
		• $8y - 2\left(\frac{25}{y}\right) - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$						
		• $8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$						
		with no intermediate working.						
		You can also imply the middle dM1A1 marks for either						
		• $8\left(\frac{25}{x}\right) - 2x - 75 = 0 \rightarrow x = -40$						
		• $8y - 2\left(\frac{25}{y}\right) - 75 = 0 \rightarrow y = -\frac{5}{8}$						
		• $8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0 \rightarrow x = -40 \text{ or } y = -\frac{5}{8}$						
	with no intermediate working.							
	Note	Writing $x = -40$, $y = -\frac{5}{8}$ followed by $B\left(40, \frac{5}{8}\right)$ or $B\left(-\frac{5}{8}, -40\right)$ is final A0.						
	Note	Ignore stating $B\left(\frac{5}{2}, 10\right)$ in addition to $B\left(-40, -\frac{5}{8}\right)$						

Number (a)RotationB17. (a)RotationEither $\operatorname{artan}(\frac{12}{5}), \operatorname{tan}^{-1}(\frac{12}{5}), \operatorname{cos}^{-1}(\frac{13}{5}), $	Question Number	Scheme		Notes	Marks
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	7 (a)	Rotation		Rotation	B1
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	7• (a)	Kotation	Eithe	remain	DI
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		67 dagraas (anticlockwise)	Eithe	$\frac{1}{5}, \tan\left(\frac{1}{5}\right), \tan\left(\frac{1}{5}\right), \sin\left(\frac{1}{13}\right), \cos\left(\frac{1}{13}\right),$	B1 o e
$\begin{array}{ c c c c c c } \hline \begin{array}{ c c c c } \hline \begin{array}{ c c } \hline \end{array} \hline \begin{array}{ c } \hline \begin{array}{ c c } \hline \end{array} \hline \begin{array}{ c c } \hline \end{array} \hline \begin{array}{ c c } \hline \begin{array}{ c c } \hline \end{array} \hline \begin{array}{ c } \hline \end{array} \hline \end{array} \hline \end{array} \hline \begin{array}{ c } \hline \end{array} \hline $		or degrees (anticiockwise)	awrt 67 de	grees, awrt 1.2, truncated 1.1 (anticlockwise),	D1 0.e.
$\begin{array}{c c c c c c c c c c c c c c c c c c c $			awrt T	293 degrees clockwise or awrt 5.1 clockwise	
About (0,0)About (0,0) or about O or about the originabout (0,0)Note: Give 2^{nd} B0 for 67 degrees clockwise o.e.(3)(b) $\left[\mathbf{Q}=\right]\begin{pmatrix} 0 & 1\\ 1 & 0 \end{pmatrix}$ Correct matrix(c) $\left[\mathbf{R}=\mathbf{P}\mathbf{Q}=\right]\begin{pmatrix} \frac{1}{13} & -\frac{12}{13}\\ \frac{1}{13} & \frac{1}{13} \end{pmatrix} \begin{pmatrix} 0 & 1\\ 1 & 0 \end{pmatrix} := \begin{pmatrix} -\frac{12}{13} & \frac{1}{13}\\ \frac{1}{3} & \frac{1}{13} \end{pmatrix}$ Multiplies P by their Q in the correct order and finds at least one element(d)(d)(-\frac{12}{13} & \frac{1}{13}) \begin{pmatrix} x\\ xx \end{pmatrix} = \begin{pmatrix} x\\ xx \end{pmatrix}(1)(e) $\left(-\frac{12}{13} & \frac{1}{13}\right) \begin{pmatrix} x\\ xx \end{pmatrix} = \begin{pmatrix} x\\ xx \end{pmatrix}$ (1)(f)(1)(1)(1)(1)(h)(h)(1)(1)(h)(h)(h)(h)(about (0,0)	1	previous B marks being awarded.	dB1
Note: Give 2^{ud} B0 for 67 degrees clockwise o.e.(3)(b) $\left[\mathbf{Q}=\right]\begin{pmatrix} 0 & 1\\ 1 & 0 \end{pmatrix}$ Correct matrixB1(c) $\left[\mathbf{R}=\mathbf{P}\mathbf{Q}=\right]\begin{pmatrix} \frac{5}{13} & \frac{12}{13}\\ \frac{1}{33} & \frac{13}{13} \end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & 0 \end{pmatrix}; = \begin{pmatrix} -\frac{12}{13} & \frac{5}{13}\\ \frac{5}{13} & \frac{13}{13} \end{pmatrix}$ Multiplies \mathbf{P} by their \mathbf{Q} in the correct order and finds at least one element order and finds at least one element order and finds at least one element of the correct atrixM1(d) $\left(-\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}\begin{pmatrix} x\\ kx \end{pmatrix} = \begin{pmatrix} x\\ kx \end{pmatrix}$ (1)(d) $\left(-\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \\ \frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \\ \frac{12}{13} & \frac{12}{13} \\ \frac{5}{13} & \frac{12}{13} \\ \frac{12}{13} & $		uoout (0, 0)		About $(0, 0)$ or about <i>O</i> or about the origin	dD1
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		Note: Give 2 nd B0 for 67 degrees clockwise o.e.			(3)
(b) $\begin{cases} \mathbf{Q} = \left\{ \begin{array}{c} 0 & 1 \\ 1 & 0 \end{array} \right\} & \text{Correct matrix} & \text{B1} \\ \hline \\ \mathbf{Q} = \left\{ \begin{array}{c} 0 & 1 \\ 1 & 0 \end{array} \right\} & \frac{1}{12} & \frac{1}{13} & \frac{1}{13} \\ \hline \\ \frac{1}{13} \frac{1}{13} \\ \frac{1}{13} \\ \hline \\ \frac{1}{13} \\ \frac$		$\begin{pmatrix} 0 & 1 \end{pmatrix}$			
$(c) \qquad \{\mathbf{R} = \mathbf{PQ} = \} \begin{pmatrix} \frac{x}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{x}{13} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} : = \begin{pmatrix} -\frac{12}{13} & \frac{x}{13} \\ \frac{x}{13} & \frac{12}{13} \end{pmatrix} \qquad $	(b)	$\left\{\mathbf{Q}=\right\}\left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right]$		Correct matrix	B1
(c) $ \begin{cases} \left\{ \mathbf{R} = \mathbf{PQ} = \right\} \begin{pmatrix} \frac{x}{13} & -\frac{12}{13} \\ \frac{x}{13} & \frac{12}{13} \\ \frac{x}{13} & \frac{12}{13} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ \frac{x}{13} & \frac{x}{13} \\ \frac{x}{13} & \frac{x}{13} \end{pmatrix} \begin{pmatrix} x \\ kx \end{pmatrix} = \begin{pmatrix} x \\ kx \end{pmatrix} Multiplies \mathbf{P} by their \mathbf{Q} in the correct order and finds at least one element of the correct order and finds at least one element of the correct order and finds at least one element of the correct order and finds at least one element of the correct order and finds at least one element of the correct order and finds at least one element of the correct order and finds at least one element of the correct order and finds at least one element of the correct order and finds at least one element of the correct order and finds at least one element of the correct order and finds at least one element or the equation in the and program and equation in the and program and equation in the and program and the element order and finds at lea$					(1)
(c) $ \left\{ \mathbf{R} = \mathbf{PQ} = \right\} \left[\frac{13}{12} - \frac{13}{13} \\ \frac{13}{12} - \frac{13}{13} \\ \frac{1}{10} $		$\begin{pmatrix} 5 & 12 \end{pmatrix}$	(12 5)	Multiplies P by their O in the correct	(1)
(d) (d) (d) (d) (e) (e) (d) (e) (f) (f) (f) (f) (f) (f) (f) (f	(c)	$\{\mathbf{R} = \mathbf{PO} =\} \begin{bmatrix} \frac{1}{13} & -\frac{12}{13} \\ \frac{1}{13} & \frac{1}{13} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} :=$	$\begin{vmatrix} -\frac{12}{13} & \frac{5}{13} \end{vmatrix}$	order and finds at least one element	M1
(d) Way 1 $\begin{pmatrix} -\frac{12}{13} & \frac{1}{13} \\ \frac{5}{13} & \frac{1}{12} \\ \frac{5}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{1}{12} \\ \frac{5}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{1}{12} \\ \frac{5}{13} & \frac{1}{12} \\ \frac{5}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{1}{12} \\ \frac{5}{13} & \frac{5}{13} \\ $	(0)	$\begin{pmatrix} 12 & -4 & 0 \end{pmatrix} \begin{pmatrix} 12 & 5 \\ 13 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix}'$	$\left(\begin{array}{cc} \frac{5}{13} & \frac{12}{13} \end{array}\right)$	Correct matrix	A1
(d) (d) (d) (e) (e) (f) (f) (f) (f) (f) (f) (f) (f			× ,		(2)
(d) (d) (e) (d) (e) (f) (f) (f) (f) (f) (f) (f) (f					(=)
(d) Way 1 $ \begin{bmatrix} \begin{bmatrix} -\frac{12}{13} & \frac{13}{13} \\ \frac{1}{51} & \frac{12}{13} \end{bmatrix} \begin{bmatrix} x \\ kx \end{bmatrix} = \begin{bmatrix} x \\ kx \end{bmatrix} $ Forming the equation "their matrix \mathbf{K} $\begin{bmatrix} kx \end{bmatrix} = \begin{bmatrix} kx \end{bmatrix} $ M1 Allow x being replaced by any non-zero number eg. 1. Can be implied by at least one correct ft equations below. $ \begin{bmatrix} -\frac{12}{13}x + \frac{5kx}{13} = x \text{ or } \frac{5}{13}x + \frac{12kx}{13} = kx \Rightarrow k = \dots $ Uses their matrix equation to form an equation in k and progresses to give K = numerical value So $k = 5$ Dependent on all previous marks being scored in this part. Either • Solves both $-\frac{12}{13}x + \frac{5kx}{13} = x$ and $\frac{5}{13}x + \frac{12kx}{13} = kx$ to give $k = 5$ • Finds $k = 5$ and checks that it is true for the other component • Confirms that $\begin{pmatrix} -\frac{12}{13} & \frac{5}{13}\\ \frac{5}{13} & \frac{12}{13} \end{pmatrix} \begin{pmatrix} x\\ 5x \end{pmatrix} = \begin{pmatrix} x\\ 5x \end{pmatrix}$ (d) Way 2 (d) $\begin{bmatrix} (d)\\ Kx = 1 \tan\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right) $ Full method of finding 2θ , then θ and applying tan θ M1 $\tan\left(\frac{1}{2} \operatorname{arccos}\left(-\frac{12}{13}\right)\right) $ Full method of finding 2θ , then θ and applying tan θ M1 $\tan\left(\frac{1}{2} \operatorname{arccos}\left(-\frac{12}{13}\right)\right) $ All conduction to the conduction of the equation only All $\tan\left(\frac{1}{2} \operatorname{arccos}\left(-\frac{12}{13}\right)\right) $ All conduction the conduction only All $\tan\left(\frac{1}{2} \operatorname{arccos}\left(-\frac{12}{13}\right)\right) $ All conduction the conduction only the previous form an equation in the matrix \mathbf{R} M1 $\tan\left(\frac{1}{2} \operatorname{arccos}\left(-\frac{12}{13}\right)\right) $ All conduction the conduction only All $\tan\left(\frac{1}{2} \operatorname{arccos}\left(-\frac{12}{13}\right)\right) $ All conduction only All All (4) (4) (5) $k = 5$ (2) $k = 6$ (3) $k = 5$ (3) $k = 5$ (4) $k = 5$ (4) $k = 1$ (4) (4) (5) $k = 5$ (4) $k = 5$ (4) $k = 5$ (4) $k = 1$ (4) (4) (5) $k = 5$ (4) $k = 5$ (4) $k = 5$ (4) $k = 1$ (4) (4) (4) (5) $k = 5$ (4) $k = 5$ (4) $k = 1$ (4) (4) (5) $k = 5$ (4) $k = 5$ (4) $k = 1$ (4) (4) (5) $k = 5$ (4) $k = 5$ (4) $k = 1$ (4) (5) $k = 5$ (4) k		$(\mathbf{x} \mathbf{x})(\mathbf{x})(\mathbf{x})$	г ·		
Way 1 $\begin{pmatrix} \frac{5}{13} & \frac{12}{13} \\ kx \end{pmatrix} = \begin{pmatrix} kx \end{pmatrix}$ Allow x being replaced by any non-zero number eg. 1. Can be implied by at least one correct fi equations below.Mil $-\frac{12}{13}x + \frac{5kx}{13} = x$ or $\frac{5}{13}x + \frac{12kx}{13} = kx \Rightarrow k =$ Uses their matrix equations to form an equation in k and progresses to give $k = numerical value$ M1So $k = 5$ dependent on only the previous M mark $k = 5$ A1 caoDependent on all previous marks being scored in this part. Either • Solves both $-\frac{12}{13}x + \frac{5kx}{13} = x$ and $\frac{5}{13}x + \frac{12kx}{13} = kx$ to give $k = 5$ A1 cao• Confirms that $\begin{pmatrix} -\frac{12}{13}x + \frac{5kx}{13} = x \text{ and } \frac{5}{13}x + \frac{12kx}{13} = kx$ to give $k = 5$ A1 cso• Confirms that $\begin{pmatrix} -\frac{12}{13}x + \frac{5kx}{13} = x \text{ and } \frac{5}{5x} \end{pmatrix} = \begin{pmatrix} x \\ 5x \end{pmatrix}$ A1 cso(d)Either $\cos 2\theta = -\frac{12}{13}$, $\sin 2\theta = \frac{5}{13}$ or $\tan 2\theta = -\frac{5}{12}$ Correct follow through equation in 2θ based on their matrix \mathbf{R} Way 2 $\{k = \} \tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ Full method of finding 2\theta, then θ and applying $\tan \theta$ M1 $\tan(awrt 1.37)$. Can be implied.A1So $k = 5$ $k = 5$ by a correct solution onlyA1 (4) (4) (4)	(d)	$\left[\begin{array}{ccc} -\frac{12}{13} & \frac{5}{13} \end{array}\right] \left[\begin{array}{ccc} x \\ - \end{array}\right] \left[\begin{array}{ccc} x \\ - \end{array}\right]$	Formi	ng the equation "their matrix \mathbf{R} " =	241
$(d) \qquad (d) $	Way 1	$\left \frac{5}{12} \frac{12}{12} \right $		$\begin{pmatrix} kx \end{pmatrix} \begin{pmatrix} kx \end{pmatrix}$	IVI I
$\begin{array}{ c c c c c } \hline Can be implied by at least one correct ft equations below. \\ \hline \hline -\frac{12}{13}x + \frac{5kx}{13} = x \text{ or } \frac{5}{13}x + \frac{12kx}{13} = kx \Rightarrow k = \dots \\ \hline \hline Can be implied by at least one correct ft equations below. \\ \hline \hline -\frac{12}{13}x + \frac{5kx}{13} = x \text{ or } \frac{5}{13}x + \frac{12kx}{13} = kx \Rightarrow k = \dots \\ \hline \hline Can be implied by at least one correct ft equations below. \\ \hline \hline Can be implied by at least one correct ft equations below. \\ \hline \hline Can be implied by at least one correct ft equations below. \\ \hline \hline Can be implied by at least one correct ft equations below. \\ \hline \hline Can be implied by at least one correct ft equations below. \\ \hline \hline Can be implied by at least one correct ft equations below. \\ \hline \hline Can be implied by at least one correct ft equation to form an equation in k and progresses to give k = 10 \text{ meth} only the previous M mark k = 5 k = 5 k = 5 and checks that it is true for the other component \\ \hline \hline Confirms that \begin{pmatrix} -\frac{12}{13} & \frac{x}{13} \\ \frac{x}{5x} & \frac{12}{13} \\ \frac{x}{5x} & \frac{12}{5x} \end{pmatrix} = \begin{pmatrix} x \\ 5x \end{pmatrix} A1 coolumn (4) \\ \hline \hline Confirms that \begin{pmatrix} -\frac{12}{13} & \frac{x}{13} \\ \frac{x}{5x} & \frac{12}{13} \\ \frac{x}{5x} & \frac{12}{5x} \end{pmatrix} = \begin{pmatrix} Can be implied by at least one correct follow through equation in 2\theta based on their matrix R M1 A1 A1 A1 A1 A1 A1 A1 A$		$\left(\begin{array}{c} 13 \\ 13 \end{array}\right) \left(\begin{array}{c} \kappa \lambda \end{array}\right) \left(\begin{array}{c} \kappa \lambda \end{array}\right)$	Allow x bei	ng replaced by any non-zero number eg. 1.	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			Can be impl	ied by at least one correct ft equations below.	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		12 5kx 5 12kx		Uses their matrix equation to form an	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$-\frac{1}{13}x + \frac{1}{13} = x \text{ or } \frac{1}{13}x + \frac{1}{13}x$	$= kx \implies k = \dots$	equation in k and progresses to give $k =$ numerical value	MI
So $k = 5$ Comparison of the previous of the previous of the tark is a collection of the other of the other of the other is a collection of the tark is a collection of the other componentA1 caoDependent on all previous marks being scored in this part. EitherA1 cao• Solves both $-\frac{12}{13}x + \frac{5kx}{13} = x$ and $\frac{5}{13}x + \frac{12kx}{13} = kx$ to give $k = 5$ A1 cao• Finds $k = 5$ and checks that it is true for the other componentA1 cso• Confirms that $\begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{12} \end{pmatrix} \begin{pmatrix} x \\ 5x \end{pmatrix} = \begin{pmatrix} x \\ 5x \end{pmatrix}$ (4)(d)Either $\cos 2\theta = -\frac{12}{13}$, $\sin 2\theta = \frac{5}{13}$ or $\tan 2\theta = -\frac{5}{12}$ Correct follow through equation in 2θ based on their matrix R (d)M1(d)Full method of finding 2θ , then θ and applying $\tan \theta$ M1(d)Full method of finding 2θ , then θ and applying $\tan \theta$ M1(d) $k = 3$ tan $\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right)$ A1(d)So $k = 5$ $k = 5$ by a correct solution only of tan (awrt 78.7°) or $A1$ (d)So $k = 5$ $k = 5$ by a correct solution only A1		~		dependent on only the previous M mark	
Dependent on all previous marks being scored in this part. Either• Solves both $-\frac{12}{13}x + \frac{5kx}{13} = x$ and $\frac{5}{13}x + \frac{12kx}{13} = kx$ to give $k = 5$ A1 cso• Finds $k = 5$ and checks that it is true for the other component• Confirms that $\begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix} \begin{pmatrix} x \\ 5x \end{pmatrix} = \begin{pmatrix} x \\ 5x \end{pmatrix}$ A1 cso(d)Either $\cos 2\theta = -\frac{12}{13}$, $\sin 2\theta = \frac{5}{13}$ or $\tan 2\theta = -\frac{5}{12}$ Correct follow through equation in 2θ based on their matrix R M1Way 2Full method of finding 2θ , then θ and applying $\tan \theta$ M1 $k = \frac{1}{2} \tan\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right)$ Full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\tan\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right)$ $\tan\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right)$ or $\tan\left(\operatorname{awrt} 78.7^{\circ}\right)$ or $\operatorname{atn}\left(\operatorname{awrt} 1.37\right)$. Can be implied.So $k = 5$ $k = 5$ by a correct solution only A110		So $k = 5$		k = 5	A1 cao
$(d) Way 2 (d) \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13} \exp\left(-\frac{12}{13}\right)}_{\{k=\}} \tan\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right) Full method of finding 2\theta, then \theta and applying tan \theta \\ (d) \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13} \text{ or } \tan 2\theta = -\frac{5}{12}}_{\{k=\}} \underbrace{\text{Correct follow through equation in } 2\theta \text{ based on their matrix } \mathbf{R} \\ (d) \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13} \text{ or } \tan 2\theta = -\frac{5}{12}}_{\{k=\}} \underbrace{\text{Correct follow through equation in } 2\theta \text{ based on their matrix } \mathbf{R} \\ (d) \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13} \text{ or } \tan 2\theta = -\frac{5}{12}}_{\{k=\}} \underbrace{\text{Correct follow through equation in } 2\theta \text{ based on their matrix } \mathbf{R} \\ (d) \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13} \text{ or } \tan 2\theta = -\frac{5}{12}}_{\{k=\}} \underbrace{\text{Correct follow through equation in } 2\theta \text{ based on their matrix } \mathbf{R} \\ (d) \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13} \text{ or } \tan 2\theta = -\frac{5}{12}}_{\{k=\}} \underbrace{\text{Correct follow through equation in } 2\theta \text{ based on their matrix } \mathbf{R} \\ (d) \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13} \text{ or } \tan 2\theta = -\frac{5}{12}}_{\{k=\}} \underbrace{\text{Correct follow through equation in } 2\theta \text{ based on their matrix } \mathbf{R} \\ (d) \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13} \text{ or } \tan 2\theta = -\frac{5}{12}}_{\{k=\}} \underbrace{\text{Correct follow through equation in } 2\theta \text{ based on their matrix } \mathbf{R} \\ (d) \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13} \text{ or } \tan 2\theta = -\frac{5}{12}}_{\{k=\}} \underbrace{\text{Correct follow through equation in } 2\theta \text{ based on their matrix } \mathbf{R} \\ (d) \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13} \text{ or } \tan 2\theta = -\frac{5}{12}}_{\{k=\}} \underbrace{\text{Either } \cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13} \text{ or } \tan 2\theta = -\frac{5}{12}_{\{k=\}} \text{ or } (12)_{\{k=\}} \text{ or } (12)_{\{k=\}$		Dependent on all previous marks	s being scored	l in this part. Either	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		• Solves both $-\frac{12}{x} + \frac{5kx}{x}$	$-r$ and $\frac{5}{r}$	$+\frac{12kx}{k} - kx$ to give $k = 5$	
• Finds $k = 5$ and checks that it is true for the other componentA1 cso• Confirms that $\begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix} \begin{pmatrix} x \\ 5x \end{pmatrix} = \begin{pmatrix} x \\ 5x \end{pmatrix}$ (4)(d)Either $\cos 2\theta = -\frac{12}{13}$, $\sin 2\theta = \frac{5}{13}$ or $\tan 2\theta = -\frac{5}{12}$ Correct follow through equation in 2θ based on their matrix R M1(d)Either $\cos 2\theta = -\frac{12}{13}$, $\sin 2\theta = \frac{5}{13}$ or $\tan 2\theta = -\frac{5}{12}$ Full method of finding 2θ , then θ and applying $\tan \theta$ M1(d) $k = \frac{1}{2} \tan\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right)$ Full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\left\{k = \frac{1}{2} \tan\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right)\right)$ $\tan\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right)$ or $\tan\left(awrt 78.7^{\circ}\right)$ or $\tan\left(awrt 78.7^{\circ}\right)$ or $\tan\left(awrt 1.37\right)$. Can be implied.A1So $k = 5$ $k = 5$ by a correct solution onlyA1Image: transmitted structure in the stru		13^{-13}	$-x$ and 13^{x}	$+$ $\frac{-\kappa}{13}$ $\frac{13}{13}$	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		• Finds $k = 5$ and checks the	at it is true for	the other component	A1 cso
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		$\begin{pmatrix} 12 & 5 \end{pmatrix}$	x (x)		
$(\frac{5}{13} \frac{12}{13})(5x) (5x) $ (4) (d) $($		• Confirms that $\begin{bmatrix} -\frac{1}{13} & \frac{1}{13} \end{bmatrix}$			
(c) $y + (c) y$ (4)(d)Either $\cos 2\theta = -\frac{12}{13}$, $\sin 2\theta = \frac{5}{13}$ or $\tan 2\theta = -\frac{5}{12}$ Correct follow through equation in 2θ based on their matrix R M1Way 2Full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\{k = \} \tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ Full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\sin\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\sin\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ full method of finding 2θ , then θ and applying 10 M1 $\sin\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ full method of finding 2θ full method 10 $\sin\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ full method of finding 2θ full method 10 $\sin\left(\frac{1}{2}\operatorname{arccos}\left(-\frac{12}{13}\right)\right)$ full method 10 full method 10		$\left(\begin{array}{c} \frac{5}{13} & \frac{12}{13} \end{array}\right)$	$5x \mid 5x \mid$		
(d) Way 2 $ \begin{array}{c} (d) \\ (d$, , ,		(4)
(d) Either $\cos 2\theta = -\frac{12}{13}$, $\sin 2\theta = \frac{3}{13}$ or $\tan 2\theta = -\frac{3}{12}$ Contect follow intoduct equation in 2θ based on their matrix R M1 Way 2 Full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\left\{k = \right\} \tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ Full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ or $\tan\left(\operatorname{awrt} 78.7^\circ\right)$ or $\tan\left(\operatorname{awrt} 78.7^\circ\right)$ or $\tan\left(\operatorname{awrt} 78.7^\circ\right)$ or $\tan\left(\operatorname{awrt} 1.37\right)$. Can be implied. So $k = 5$ $k = 5$ by a correct solution only A1 (4)		12 5		5 Correct follow through equation in	(4)
Way 2Full method of finding 2θ , then θ and applying $\tan \theta$ M1 $\{k =\} \tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ $\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ or $\tan\left(\operatorname{awrt} 78.7^\circ\right)$ or $\operatorname{aut}\left(\operatorname{awrt} 1.37\right)$. Can be implied.A1(4)10	(d)	Either $\cos 2\theta = -\frac{12}{12}$, $\sin 2\theta = \frac{3}{12}$	or $\tan 2\theta = -\frac{1}{1}$	$\frac{5}{2}$ Correct follow through equation in $\frac{2}{2}$ $\frac{2\theta}{\theta}$ based on their matrix R	M1
$\frac{\left\{k=\right\} \tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)}{\operatorname{So} k=5} \qquad \qquad$	Way 2	15 15	Eull met	20° oused on their matrix A	M1
$\begin{cases} k = \frac{1}{2} \tan\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right) & \tan\left(\frac{1}{2} \arccos\left(-\frac{12}{13}\right)\right) \text{ or } \tan\left(\operatorname{awrt} 78.7^{\circ}\right) \text{ or } \left(\operatorname{awrt} 78.7^{\circ}\right) \text{ or } \left(\operatorname{awrt} 78.7^{\circ}\right) \text{ or } \tan\left(\operatorname{awrt} 78.7^{\circ}\right) \text{ or } \left(\operatorname{awrt} 78.7^{\circ}$,,uj 2		i un men	$\begin{pmatrix} 1 & (12) \end{pmatrix}$	1411
$\begin{array}{c} (2 & (-13)) & (2 & (-13)) & (-13) & (-1$		$\{k=\}$ tan $\left(\frac{1}{-}\arccos\left(-\frac{12}{-}\right)\right)$		$\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{12}\right)\right)$ or $\tan\left(\operatorname{awrt} 78.7^{\circ}\right)$ or	
tan(awrt 1.37). Can be implied.So $k = 5$ $k = 5$ by a correct solution onlyA1(4)10		(2 (13))		(2 (13)) (1)	A1
So $k = 5$ $k = 5$ by a correct solution onlyA1(4)10				tan(awrt 1.37). Can be implied.	
(4) 10		So $k = 5$		k = 5 by a correct solution only	A1
10				•	(4)
					10

		Question 7 Notes					
7. (a)	Note	Condone "Turn" for the 1 st B1 mark.					
	Note	Penalise the first B1 mark for candidates giving a combination of transformations.					
(c)	Note	Allow 1 st M1 for eg. "their matrix $\mathbf{R}'' \begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$ or "their matrix $\mathbf{R}'' \begin{pmatrix} k \\ k^2 \end{pmatrix} = \begin{pmatrix} k \\ k^2 \end{pmatrix}$					
		or "their matrix $\mathbf{R} = \begin{pmatrix} \frac{1}{k} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{k} \\ 1 \end{pmatrix}$ or equivalent					
	Note	$y = (\tan\theta)x : \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} = \begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}$					

Question Number	Scheme		Notes	Marks	
8.	$f(z) = z^4 + 6z^3 + 76z^2 + az + b$, a, b a	$a^4 + 6z^3 + 76z^2 + az + b$, a, b are real constants. $z_1 = -3 + 8i$ is given.			
(a)	-3-8i		-3-8i	B1	
				(1)	
		Att	tempt to expand $(z - (-3 + 81))(z - (-3 - 81))$		
			$-3+8i \rightarrow z+3-+8i \rightarrow z^2+6z+9=-64$	N/1	
(b)	$z^2 + 6z + 73$	υg. ₄ –	or sum of roots -6 , product of roots 73	MI	
			to give $z^2 + (sum)z + product$		
			$\frac{1}{2} \frac{1}{2} \frac{1}$	A1	
			Attempts to find the other quadratic factor.		
	2	e	g. using long division to get as far as $z^2 +$	M1	
	$f(z) = (z^2 + 6z + 73)(z^2 + 3)$		or eg. $f(z) = (z^2 + 6z + 73)(z^2 +)$		
		-	$z^{2}+3$	A1	
		G	dependent on only the previous M mark	dM1	
	$\left\{z^2 + 3 = 0 \Longrightarrow z = \right\} \pm \sqrt{3}i$	Corre	$\frac{1}{2}$ ct method of solving the 2 nd quadratic factor	A 1	
			v31 and -v31	AI (6)	
(c)			Criteria	(0)	
	Im		• $-3\pm 8i$ plotted correctly in		
			quadrants 2 and 3 with some		
	N 8		 Their other two complex roots 		
			(which are found from solving their		
	$ $ $\langle \rangle$		2 nd quadratic in (b)) are plotted		
	$\sqrt{1}\sqrt{3}$		symmetry about the x-axis		
	V v		Satisfies at least one of the two aritaria	D1 ft	
	-3 Re		Satisfies at least one of the two citteria	DIII	
	$/ -\sqrt{3}$				
			Satisfies both criteria with some		
			indication of scale or coordinates stated.	B1 ft	
			All points (arrows) must be in the correct	DIR	
			positions relative to each other.		
		Oue	stion 8 Notes	9	
8. (b)	Note Give 3^{rd} M1 for $z^2 + k = 0$	$k > 0 \implies s$	at least one of either $z = \sqrt{k}$ i or $z = -\sqrt{k}$	i	
	Note Give 3^{rd} M0 for $r^2 + k = 0$, $k > 0 \Rightarrow r = \pm h$:				
	Note Give $2^{rd} M0$ for $-2^2 + k = 0$,	$k > 0 \rightarrow -$	$x = \pm k \text{ or } x = \pm \sqrt{k}$		
	Note Give 3 th MU for $z^{-} + k = 0$, $k > 0 \Rightarrow z = \pm k$ or $z = \pm \sqrt{k}$				
		ing <i>u</i> = 10,			

Question Number	Scheme		N	otes	Marks	
9.	$2x^2 + 4x - 3 = 0$ has roots α, β					
(a)	$\alpha + \beta = -\frac{4}{2}$ or -2 , $\alpha\beta = -\frac{3}{2}$		Both $\alpha + \beta = -\frac{4}{2}$ and seen or implied a	d $\alpha\beta = -\frac{3}{2}$. This may be anywhere in this question.	B1	
(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \dots$		Use of a con (May	rrect identity for $\alpha^2 + \beta^2$ be implied by their work)	M1	
	$= (-2)^2 - 2\left(-\frac{3}{2}\right) = 7$			7 from correct working	A1 cso	
(ii)	$\alpha^{3} + \beta^{3} = (\alpha + \beta)^{3} - 3\alpha\beta(\alpha + \beta) = \dots$ or $= (\alpha + \beta)(\alpha^{2} + \beta^{2} - \alpha\beta) = \dots$		Use of an (May	appropriate and correct identity for $\alpha^3 + \beta^3$ be implied by their work)	M1	
	$= (-2)^{3} - 3(-\frac{3}{2})(-2) = -17$ or $= (-2)(7\frac{3}{2}) = -17$		-1	7 from correct working	A1 cso	
					(5)	
(b)	Sum = $\alpha^2 + \beta + \beta^2 + \alpha$ = $\alpha^2 + \beta^2 + \alpha + \beta$ = 7 + (-2) = 5	Sum = $\alpha^2 + \beta + \beta^2 + \alpha$ = $\alpha^2 + \beta^2 + \alpha + \beta$ = 7 + (-2) = 5 Uses at least one of their $\alpha^2 + \beta^2$ or $\alpha + \beta$ in an attempt to find a numerical value for the sum of $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$		ir $\alpha^2 + \beta^2$ or $\alpha + \beta$ in an rical value for the sum of $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$	M1	
	Product = $(\alpha^2 + \beta)(\beta^2 + \alpha)$ = $(\alpha\beta)^2 + \alpha^3 + \beta^3 + \alpha\beta$	Expands $(\alpha^2 + \beta)(\beta^2 + \alpha)$ and uses at least one of their $\alpha\beta$ or $\alpha^3 + \beta^3$ in an attempt to find a numerical		M1		
	$=\left(-\frac{3}{2}\right) -17 - \frac{3}{2} = -\frac{65}{4}$	value for the product of $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$				
	x ² - 5x - $\frac{65}{4} = 0$ Applies x ² - (sum)x + product (Can ("= 0" n			product (Can be implied) (" = 0" not required)	M1	
	$4x^2 - 20x - 65 = 0$		Any integer multip	ple of $4x^2 - 20x - 65 = 0$, including the "= 0"	A1	
					(4)	
	<u>Alternative</u> : Finding $\alpha^2 + \beta$ and $\beta^2 + \alpha$ explicitly					
(b)	Eg. Let $\alpha = \frac{-4 + \sqrt{40}}{4}$, $\beta = \frac{-4 + \sqrt{40}}{4}$ and so $\alpha^2 + \beta = \frac{5 - 3\sqrt{10}}{2}$, $\beta^2 + \alpha = \frac{5 + 3\sqrt{10}}{2}$					
	$\left(x - \left(\frac{5 - 3\sqrt{10}}{2}\right)\right) \left(x - \left(\frac{5 + 3\sqrt{10}}{2}\right)\right) \qquad \text{Uses } \left(x - \left(\alpha^2 + \beta\right)\right) \left(x - \left(\beta^2 + \alpha\right)\right) \\ \text{with exact numerical values. (May expand first)}\right)$			M1		
	$= x^{2} - \left(\frac{5+3\sqrt{10}}{2}\right)x - \left(\frac{5-3\sqrt{10}}{2}\right)x + \left(\frac{5-3\sqrt{10}}{2}\right)\left(\frac{5+3\sqrt{10}}{2}\right) \qquad \qquad$					
	$\Rightarrow x^2 - 5x - \frac{65}{4} = 0$		Со	llect terms to give a 3TQ. (" = 0" not required)	M1	
	$4x^2 - 20x - 65 = 0$		Any integer multip	ple of $4x^2 - 20x - 65 = 0$, including the "= 0"	A1	
					(4)	
					9	

	Question 9 Notes					
9. (a)	1 st A1	$\alpha + \beta = 2, \ \alpha\beta = -\frac{3}{2} \Rightarrow \alpha^2 + \beta^2 = 4 - 2\left(-\frac{3}{2}\right) = 7$ is M1A0 cso				
(a)	Note	Finding $\alpha + \beta = -2$, $\alpha\beta = -\frac{3}{2}$ by writing down or applying $\frac{-4 + \sqrt{40}}{4}$, $\frac{-4 + \sqrt{40}}{4}$ but then				
		writing $\alpha^{2} + \beta^{2} = (\alpha + \beta)^{2} - 2\alpha\beta = 4 + 3 = 7$ and $\alpha^{3} + \beta^{3} = (\alpha + \beta)^{3} - 3\alpha\beta(\alpha + \beta) = -8 - 9 = -17$				
		scores B0M1A0M1A0 in part (a).				
	Note	Applying $\frac{-4 + \sqrt{40}}{4}$, $\frac{-4 + \sqrt{40}}{4}$ explicitly in part (a) will score B0M0A0M0A0				
		Eg: Give no credit for $\left(\frac{-4+\sqrt{40}}{4}\right)^2 + \left(\frac{-4+\sqrt{40}}{4}\right)^2 = 7$				
		or for $\left(\frac{-4+\sqrt{40}}{4}\right)^3 + \left(\frac{-4+\sqrt{40}}{4}\right)^3 = -17$				
(b)	Note	Candidates are allowed to apply $\frac{-4+\sqrt{40}}{4}$, $\frac{-4+\sqrt{40}}{4}$ explicitly in part (b).				
	Note	A correct method leading to a candidate stating $a = 4$, $b = -20$, $c = -65$ without writing a final answer of $4x^2 - 20x - 65 = 0$ is final M1A0				

Question Number	Scheme	Notes	Marks			
10.	$u_1 = 5, u_{n+1} = 3u_n + 2, n \ge 1$. Required to prove the result, $u_n = 2 \times (3)^n - 1, n \in \mathbb{D}^+$					
(i)	$n=1: u_1 = 2(3) - 1 = 5$	$u_1 = 2(3) - 1 = 5$ or $u_1 = 6 - 1 = 5$	B1			
	(Assume the result is true for $n = k$)					
	$u_{k+1} = 3(2(3)^k - 1) + 2$	Substitutes $u_k = 2(3)^k - 1$ into $u_{k+1} = 3u_k + 2$	M1			
		dependent on the previous M mark	dM1			
	$= 2(3)^{k+1} - 1$	Expresses u_{k+1} in term of 3^{k+1}				
		$u_{k+1} = 2(3)^{k+1} - 1$ by correct solution only	A1			
	If the result is true for $n = k$, then it is true for $n = k + 1$. As the result has been shown to be					
	true for $n = 1$, then the result is true for all n					
	Required to prove the result $\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3+2n)}{3^n}$, $n \in \mathbb{D}^+$					
(ii)	$n = 1:$ LHS $= \frac{4}{2}$, RHS $= 3 - \frac{5}{2} = \frac{4}{2}$	Shows or states both LHS = $\frac{4}{3}$ and RHS = $\frac{4}{3}$	B1			
	5 5 5	or states LHS = RHS = $\frac{4}{3}$				
	(Assume the result is true for $n = k$)					
	$\sum_{r=1}^{k+1} \frac{4r}{3^r} = 3 - \frac{(3+2k)}{3^k} + \frac{4(k+1)}{3^{k+1}}$	Adds the $(k+1)^{\text{th}}$ term to the sum of k terms	M1			
		dependent on the previous M mark				
	3(3+2k) = A(k+1)	Makes 3^{k+1} or $(3)3^k$	dM1			
	$= 3 - \frac{3(3+2k)}{2^{k+1}} + \frac{4(k+1)}{2^{k+1}}$	a common denominator for their fractions.				
		Correct expression with common	A1			
		denominator 3 th or (3)3 ^t for their fractions.				
	$= 3 - \left(\frac{3(3+2k) - 4(k+1)}{3^{k+1}}\right) = 3 - \left(\frac{5+2k}{3^{k+1}}\right)$					
	$= 3 - \frac{(3+2(k+1))}{3^{k+1}}$	$3 - \frac{(3+2(k+1))}{3^{k+1}}$ by correct solution only	A1			
	If the result is true for $n = k$, then it is true	for $n = k + 1$. As the result has been shown to be				
	true for $n = 1$, then the result is true for all n					
			6			
		 Question 10 Notes				
(i) & (ii)	Note Final A1 for narts (i) and (ii) is	dependent on all previous marks being scored in th	at part			
(.) & (ii)	It is gained by candidates convey	ing the ideas of all four underlined points	P V			
	either at the end of their solution or as a narrative in their solution.					
(i)	Note $u_1 = 5$ by itself is not sufficient for the 1 st B1 mark in part (i).					
	Note $u_1 = 3 + 2$ without stating $u_2 = 2(3) - 1 = 5$ or $u_2 = 6 - 1 = 5$ is B0					
(ii)	Note LHS = RHS by itself is not sufficient for the 1 st B1 mark in part (ii).					
(11)	The Law rate by ison is not sufficient for the T D1 mark in part (ii).					