Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Core Mathematics C12

Advanced Subsidiary

Tuesday 10 January 2017 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Pearson

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1. Given $y = \frac{x^3}{3} - 2x^2 + 3x + 5$

(a) find $\frac{dy}{dx}$, simplifying each term.

(3)

(b) Hence find the set of values of x for which $\frac{dy}{dx} > 0$

(4)

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Question 1 continued

Q1

(Total 7 marks)

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2. A circle, with centre C and radius r, has equation

$$x^2 + y^2 - 8x + 4y - 12 = 0$$

Find

(a) the coordinates of C,

(2)

(b) the exact value of r.

(2)

The circle cuts the y-axis at the points A and B.

(c) Find the coordinates of the points A and B.

(3)

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Question 2 continued

Q2

(Total 7 marks)

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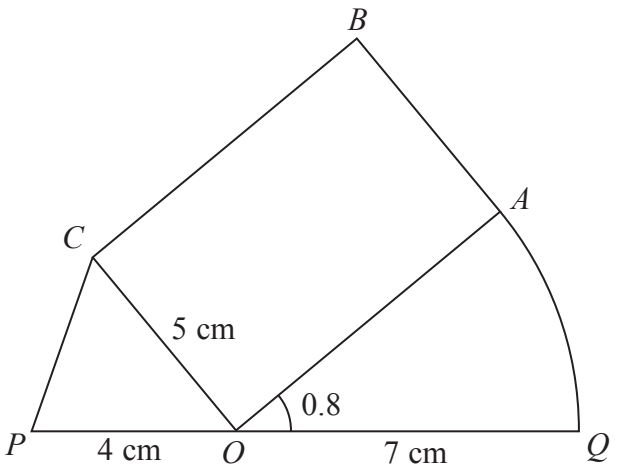
3.



Not to scale

Figure 1

The shape $POQABCP$, as shown in Figure 1, consists of a triangle POC, a sector OQA of a circle with radius 7 cm and centre O, joined to a rectangle $OABC$.

The points P, O and Q lie on a straight line.

$PO = 4$ cm, $CO = 5$ cm and angle $AOQ = 0.8$ radians.

(a) Find the length of arc AQ.

(2)

(b) Find the size of angle POC in radians, giving your answer to 3 decimal places.

(2)

(c) Find the perimeter of the shape $POQABCP$, in cm, giving your answer to 2 decimal places.

(4)

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Question 3 continued

Q3

(Total 8 marks)

Leave blank

4. An arithmetic series has first term a and common difference d.

Given that the sum of the first 9 terms is 54

(a) show that

$$a + 4d = 6$$

(2)

Given also that the 8th term is half the 7th term,

(b) find the values of a and d.

(4)

Leave blank

Question 4 continued

Q4

(Total 6 marks)

Leave blank

5. (a) Given that

$$y = \log_3 x$$

find expressions in terms of y for

(i) $\log_3\left(\frac{x}{9}\right)$

(ii) $\log_3 \sqrt{x}$

Write each answer in its simplest form.

(3)

(b) Hence or otherwise solve

$$2\log_3\left(\frac{x}{9}\right) - \log_3 \sqrt{x} = 2$$

(4)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 7 marks)

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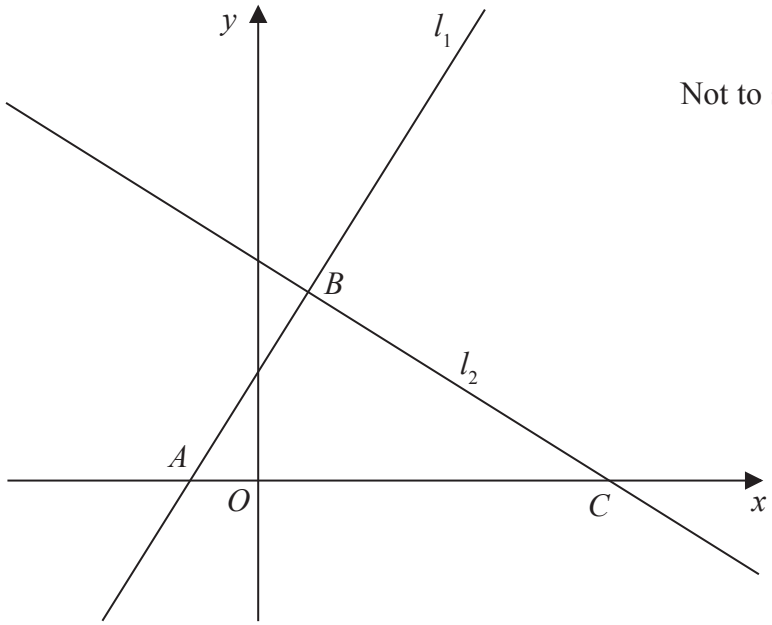

Figure 2

The straight line l_1 has equation $2y = 3x + 5$

The line l_1 cuts the x-axis at the point A, as shown in Figure 2.

(a) (i) State the gradient of l_1

(ii) Write down the x coordinate of point A.

(3)

Another straight line l_2 intersects l_1 at the point B with x coordinate 1 and crosses the x-axis at the point C, as shown in Figure 2.

Given that l_2 is perpendicular to l_1

(b) find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers,

(5)

(c) find the exact area of triangle ABC.

(3)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 11 marks)

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7. (i) Find

$$\int \frac{2 + 4x^3}{x^2} \mathrm{d}x$$

giving each term in its simplest form.

(4)

(ii) Given that k is a constant and

$$\int_2^4 \left(\frac{4}{\sqrt{x}} + k \right) \mathrm{d}x = 30$$

find the exact value of k.

(5)

Leave blank

Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 9 marks)

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8. $f(x) = 2x^3 - 5x^2 - 23x - 10$

(a) Find the remainder when $f(x)$ is divided by $(x - 3)$.

(2)

(b) Show that $(x + 2)$ is a factor of $f(x)$.

(2)

(c) Hence fully factorise $f(x)$.

(4)

(d) Hence solve

$$2(3^{3t}) - 5(3^{2t}) - 23(3^t) = 10$$

giving your answer to 3 decimal places.

(2)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

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9.

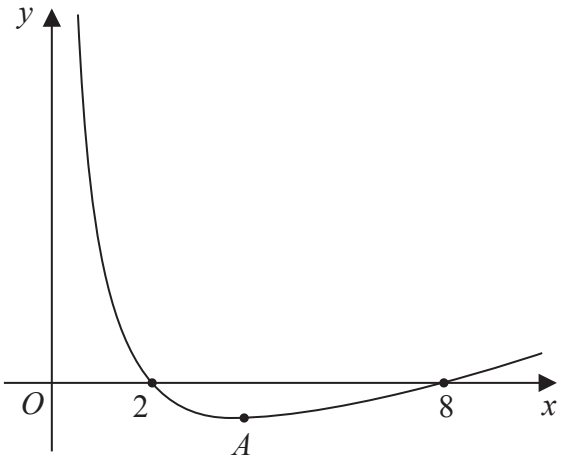

Figure 3

Figure 3 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \frac{8}{x} + \frac{1}{2}x - 5, \qquad 0 < x \leqslant 12$$

The curve crosses the x-axis at (2, 0) and (8, 0) and has a minimum point at A.

(a) Use calculus to find the coordinates of point A.

(5)

(b) State

(i) the roots of the equation $2f(x) = 0$

(ii) the coordinates of the turning point on the curve $y = f(x) + 2$

(iii) the roots of the equation $f(4x) = 0$

(3)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 8 marks)

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10. The first 3 terms, in ascending powers of x, in the binomial expansion of $(1 + ax)^{20}$ are given by

$$1 + 4x + px^2$$

where a and p are constants.

(a) Find the value of a.

(2)

(b) Find the value of p.

(2)

One of the terms in the binomial expansion of $(1 + ax)^{20}$ is qx^4, where q is a constant.

(c) Find the value of q.

(2)

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Question 10 continued

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Question 10 continued

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Question 10 continued

Q10

(Total 6 marks)

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11. In this question solutions based entirely on graphical or numerical methods are not acceptable.

(i) Solve, for $0 \leqslant x < 2\pi$,

$$3\cos^2 x + 1 = 4\sin^2 x$$

giving your answers in radians to 2 decimal places.

(5)

(ii) Solve, for $0 \leqslant \theta < 360°$,

$$5\sin(\theta + 10°) = \cos(\theta + 10°)$$

giving your answers in degrees to one decimal place.

(5)

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Question 11 continued

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Question 11 continued

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Question 11 continued

Q11

(Total 10 marks)

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12.

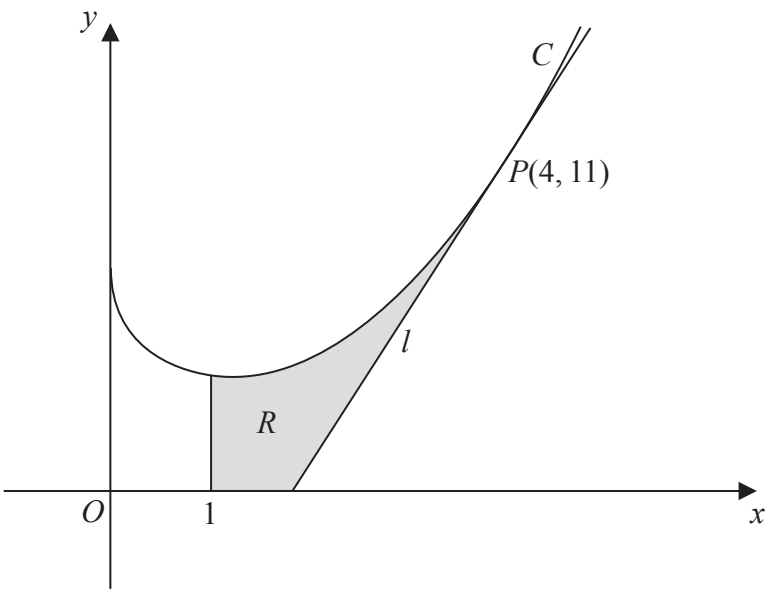

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{3}{4}x^2 - 4\sqrt{x} + 7, \quad x > 0$$

The point P lies on C and has coordinates (4, 11).

Line l is the tangent to C at the point P.

(a) Use calculus to show that l has equation $y = 5x - 9$

(5)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the line $x = 1$, the x-axis and the line l.

(b) Find, by using calculus, the area of R, giving your answer to 2 decimal places.

(6)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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Question 12 continued

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Question 12 continued

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Question 12 continued

Q12

(Total 11 marks)

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13. (a) On separate axes sketch the graphs of

(i) $y = c^2 - x^2$

(ii) $y = x^2(x - 3c)$

where c is a positive constant.

Show clearly the coordinates of the points where each graph crosses or meets the x-axis and the y-axis.

(5)

(b) Prove that the x coordinate of any point of intersection of

$$y = c^2 - x^2 \text{ and } y = x^2(x - 3c)$$

where c is a positive constant, is given by a solution of the equation

$$x^3 + (1 - 3c)x^2 - c^2 = 0$$

(2)

Given that the graphs meet when $x = 2$

(c) find the exact value of c, writing your answer as a fully simplified surd.

(4)

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Question 13 continued

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Question 13 continued

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Question 13 continued

Q13

(Total 11 marks)

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14. A geometric series has a first term a and a common ratio r.

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(4)

A liquid is to be stored in a barrel.

Due to evaporation, the volume of the liquid in a barrel at the end of a year is 7% less than the volume at the start of the year.

At the start of the first year, a barrel is filled with 180 litres of the liquid.

(b) Show that the amount of the liquid in this barrel at the end of 5 years is approximately 125.2 litres.

(2)

At the start of each year a new identical barrel is filled with 180 litres of the liquid so that, at the end of 20 years, there are 20 barrels containing varying amounts of the liquid.

(c) Calculate the total amount of the liquid, to the nearest litre, in the 20 barrels at the end of 20 years.

(3)

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Question 14 continued

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Question 14 continued

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Question 14 continued

Q14

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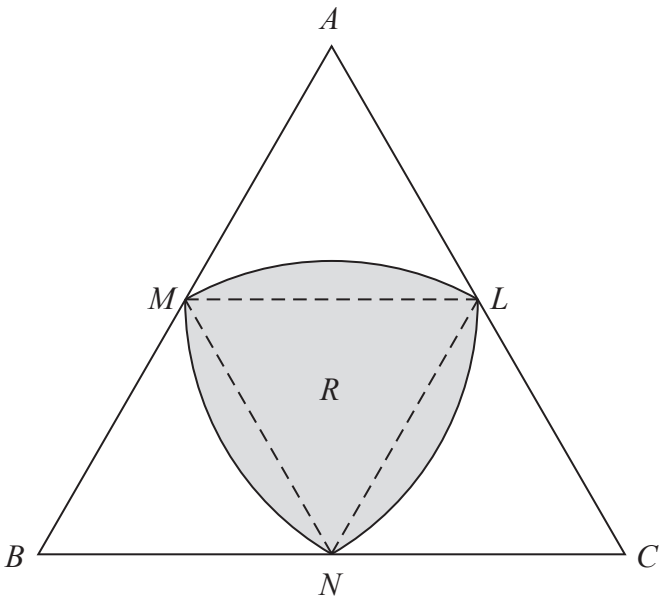

Figure 5

Figure 5 shows the design for a logo.

The logo is in the shape of an equilateral triangle ABC of side length $2r$ cm, where r is a constant.

The points L, M and N are the midpoints of sides AC, AB and BC respectively.

The shaded section R, of the logo, is bounded by three curves MN, NL and LM.

The curve MN is the arc of a circle centre L, radius r cm.

The curve NL is the arc of a circle centre M, radius r cm.

The curve LM is the arc of a circle centre N, radius r cm.

Find, in cm^2, the area of R. Give your answer in the form kr^2, where k is an exact constant to be determined.

(5)

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Question 15 continued

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Question 15 continued

Q15

(Total 5 marks)

TOTAL FOR PAPER: 125 MARKS

END