Mark Scheme (Results)

Summer 2018

Pearson Edexcel International Advanced Level
Core Mathematics C12 (WMA01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- ⎿ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x = \ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

June 2018
International A Level WMA01/01 Core Mathematics C12
Mark Scheme

Question Number	Scheme	Marks
1.(a)	<table><tr><td>x</td><td>0</td><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td></tr><tr><td>y</td><td>1</td><td>0.5</td><td>0.378</td><td>0.316</td><td>0.277</td><td>**0.25**</td></tr></table>	B1 **[1]**
(b)	State h = 3, or use of $\frac{1}{2}\times 3$;	B1
	For sight of the expression $1+0.25+2(\ 0.5+0.378+0.316+0.277\ )$	M1
	Area= $\frac{1}{2}\times 3\times\{1+0.25+2(\ 0.5+0.378+0.316+0.277\ )\}$, = awrt 6.29	A1ft, A1 **[4]**
		5 marks

Notes

(a)

B1: 0.25 or exact equivalent. Allow 0.250.
It may not be in the table. Award even if it is within the trapezium rule.

(b)

B1: for using $\frac{1}{2}\times 3$ or h = 3 or sight of $1.5\{........(...)\}$ Award for the expression $\frac{15-0}{5}$ or similar.

M1: For an attempt at the correct $\{......\}$ bracket structure.
Just look for the correct sequence of terms within an expression. $1+0.25+2(\ 0.5+0.378+0.316+0.277\ )$
Condone this bracketing error $1.5\times(1+0.25)+2(0.5+0.378+0.316+0.277)$ for the M mark
If there is a copying error or one value is omitted from the 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if **all** values used in brackets are x values instead of y values
A1ft: for a correct un simplified expression with correct bracketing following through on their h and their 0.25.
Note that candidates may recover following a bracketing error if the final answer is correct
The bracketing error (with h =3) usually results in an answer of 4.817. This scores B1M1A0A0 in (b)
A1: for answer which rounds to 6.29

NB: Separate trapezia may be used: B1 for 3, M1 for 1/2 $h(a + b)$ used 4 or 5 times (and A1 if it is all correct) Then A1 as before.

Question Number	Scheme	Marks
2.	$\mathrm{f}(x) = ax^3 + 2x^2 + bx - 3$	
(a)	Attempts $\mathrm{f}(\pm\frac{1}{2})$ and puts expression equal to 1 **or** Use long division and sets remainder =1	M1
	$\mathrm{f}(\frac{1}{2}) = a\left(\frac{1}{8}\right) + 2\left(\frac{1}{4}\right) + b\left(\frac{1}{2}\right) - 3 = 1$ so $a + 4b = 28$* **or** $-3 + b\left(\frac{1}{2}\right) + \frac{1}{2} + \frac{a}{8} = 1$ so $a + 4b = 28$*	A1 **[2]**
(b)	Attempts $\mathrm{f}(\pm 1)$ and puts expression equal to -17	M1
	Or Use long division and sets remainder equal to -17	
	$\Rightarrow -3 - b - a + 2 = -17$ { so $a + b = 16$ }	A1
	Solve simultaneous equations to give values for a **and** b	dM1
	$a = 12$ **and** $b = 4$	A1 **[4]**
		6 marks
	Notes	

(a)

M1: Puts $\mathrm{f}(\pm\frac{1}{2}) = 1$ or $\mathrm{f}(\pm\frac{1}{2}) - 1 = 0$

Alternatively uses long division and produces a remainder in a and b that is set equal to 1

A1: cao Note that answer is printed so some working, which needs to be correct, (see scheme) needs to be seen.

Accept $1a + 4b = 28$ as well as $28 = 1a + 4b$

(b)

M1: Attempts $\mathrm{f}(\pm 1) = \pm 17$ or may attempt $\mathrm{f}(\pm 1) + 17 = 0$

Alternatively uses long division and produces a remainder in a and b that is set equal to -17

A1: A correct un-simplified equation but the powers of -1 must have been processed correctly– does not need to be simplified to $a + b$=16

dM1: Solves their equation with a+4b =28 to arrive at values for a and b. Do not be worried by the processing of this Allow the answer(s) to appear from two equations. It is dependent upon the previous M.

A1: Correct values

Question Number	Scheme	Marks
3. (a)	Gradient = $\frac{4-(-8)}{-1-5}=-2$	M1 A1 **[2]**
(b)	Perpendicular line has gradient $\frac{-1}{m}\left(=\frac{1}{2}\right)$	M1
	Line has equation $y-(-8)=\frac{1}{2}(x-5)$ or $y=\frac{1}{2}x+c,\ with\ c=-8-\frac{1}{2}\times 5\ so\ y=\frac{1}{2}x-10\frac{1}{2}$	M1 A1
	So $x-2y-21=0$	A1 **[4]**
		6 marks
	Notes	

(a)

M1: Attempts to use gradient formula $\frac{\Delta y}{\Delta x}$

You may condone only one sign slip even after a correct formula is quoted.

It may be implied by $\frac{-12}{4},\frac{12}{-4},\frac{-4}{-6},\frac{4}{6},\frac{12}{6},2$

The correct answer of -2 implies both marks.

Alternatively solves simultaneous equations $-8=5m+c$ and $4=-1m+c$ and proceeds to find m. Allow one sign slip here.

A1: cao. Do not allow fractions

(b)

M1: Uses or states negative reciprocal of their gradient

M1: Uses line equation with point $(5,-8)$ and a **changed** gradient. Condone one sign slip Eg $y-8=\frac{1}{2}(x-5)$

If the form $y=mx+c$ is used they must proceed as far as c = ...

A1: Any un-simplified form of correct line equation

A1: cao – accept $k(x-2y-21)=0$ where k is an integer ≠ 0 and accept any order of the terms = 0.

Allow $1x-2y-21=0$

Allow the candidate to state $a=1,b=-2,c=-21$ but do not penalise after a correct answer.

Question Number	Scheme	Marks
4. (a)	$y^{-\frac{1}{2}} = \left(\frac{64x^6}{25}\right)^{-\frac{1}{2}} = \frac{5}{8}x^{-3}$	M1 A1 A1 **[3]**
(b)	$(25y)^{\frac{2}{3}} = 16x^4$	B1, B1 **[2]**
		5 marks
	Notes	

(a)

M1: Sight of 5 or 0.2, 8 or 0.125, x^3 or x^{-3}

Do not award if the 5 is 5^2 or the x^3 is $(x^3)^{-2}$

A1: For achieving the correct coefficient $\frac{5}{8}x^p, \frac{5}{8x^p}, \frac{1}{1.6}x^p, 0.625\,x^p$ in their final answer

or the correct index qx^{-3} in their final answer.

A1: $\frac{5}{8}x^{-3}$ cao final answer . Accept $0.625\,x^{-3}$

Note that $\frac{0.625}{x^3}$ is not in the correct form. See the demand of the question

Do not withhold the mark if $\frac{5}{8}x^{-3}$ is followed by $\frac{5}{8x^3}$ in the candidate's response.

(b)

B1: 16 or x^4 correct, in the final answer

B1: 16 x^4 cao final answer. Allow $16 \times x^4$

Question Number	Scheme	Marks
5.	(a) $\left(1+\frac{x}{3}\right)^{18} = 1+\binom{18}{1}.\left(\frac{x}{3}\right)+\binom{18}{2}.\left(\frac{x}{3}\right)^2+\binom{18}{3}.\left(\frac{x}{3}\right)^3...$	M1
	$=1+6x,+17x^2,+\frac{272}{9}x^3...$	B1, A1, A1
		[4]
	(b) Use $x = 0.1$	B1
	$=1+6\times0.1+17\times(0.1)^2+\frac{272}{9}\times(0.1)^3...$ or equivalent	M1
	$= 1.8002$	A1cao **[3]**
		7 marks
	Notes	

(a)

M1: The **method** mark is awarded for an attempt at Binomial to get the second third or fourth term – need to see $\frac{x}{3}$ used with a **correct** power of x and **a correct** binomial coefficient. Eg $\binom{18}{3}.\left(\frac{x^3}{3}\right)$ is fine for M1

Accept any notation for ${}^{18}C_1$, ${}^{18}C_2$ and ${}^{18}C_3$, e.g. $\binom{18}{1}$, $\binom{18}{2}$ and $\binom{18}{3}$ (un-simplified) or 18, 153 and 816 from Pascal's triangle. This mark may be given if no working is shown, but either or both of the terms including x correct.

B1: For the first two terms. The coefficient of x may be un simplified, but the 1^{18} must become 1.

Accept $1+18\left(\frac{x}{3}\right)$ or listed as $1, 6x$

A1: For either $+17x^2,$ or $+\frac{272}{9}x^3...$ which must be in the simplified form

A1: is cao and is for all of the terms correct and simplified –(ignore extra terms). Accept $30\frac{2}{9}x^3$ or $30.\dot{2}x^3$

It is OK to write as a list $1, 6x, 17x^2, \frac{272}{9}x^3$

Remember to isw after the correct answer. (Some students will go on to multiply by 9)

(b)

B1: States or uses $x = 0.1$ or equivalent such as $\frac{1}{10}$ or $\frac{3}{30}$ This must be seen, it is a demand of the question

M1: This is for fully substituting their value into their series expansion with at least 4 terms.

Accept sight of a value of x substituted into their expression as evidence of the M1 but do not allow $x=\frac{31}{30}$ or $\frac{31}{30}^{18}$

A1: This is for 1.8002 and is cao.

Note: The calculator answer to $\left(\frac{31}{30}\right)^{18}$ is 1.8044 This scores B0 M0 A0 ....unless $x = 0.1$ is stated and then scores B1 M0 A0

Note: A candidate just writing $\left(\frac{31}{30}\right)^{18} = 1.8002$, the correct answer, scores SC B0 M1 A1

Question Number	Scheme	Marks
6.	Use or state $2\log_5(x+5)=\log_5(x+5)^2$	M1
	Use or states $\log_5(x+5)^2-\log_5(2x+2)=\log_5\frac{(x+5)^2}{(2x+2)}$ or $\log_5(2x+2)+\log_5 5^2=\log_5 5^2(2x+2)$ etc	M1
	Use or state $\log_5 25=2$	M1
	$(x+5)^2=25(2x+2)$ or equivalent	A1
	$x^2-40x-25=0$	A1
	Solves their quadratic to give $x=$ (use formula, calculator or completing the square)	M1
	$x=20\pm5\sqrt{17}$	A1 **[7]**
		7 marks

Notes

M1: Uses or states $2\log_5(x+5)=\log_5(x+5)^2$ Can be scored without sight of the base 5 of the log

M1: Uses addition (or subtraction) law correctly at least once. Can be scored without sight of the base 5 on the log

This may follow an incorrect line. Eg. $\log_5 2(x+5)-\log_5(2x+2)=\log_5\frac{2(x+5)}{(2x+2)}$ would be fine for this mark as would

$\log_5 10+\log_5(2x+2)=\log_5 10(2x+2)$ but $2\log_5(x+5)-\log_5(2x+2)=2\log_5\frac{(x+5)}{(2x+2)}$ would not score this mark as it is incorrect subtraction law. If the lhs is going to score this mark, the coefficient of "2" must have been dealt with.

M1: Connects 2 with 25 OR 5^2 correctly

A1: Correct equation, not involving logs, in any form (un-simplified). **Dependent upon all 3 M's being awarded**.

A1: Obtains correct 3TQ **Dependent upon all 3 M's being awarded.**

M1: Solves a 3TQ by formula, calculator or completing the square to give a surd answer.

A1: CSO $x=20\pm5\sqrt{17}$

If they reject one of the solutions, usually $x=20-5\sqrt{17}$ then withhold the final mark.

....

There are students who make two or more errors and fortuitously manage to form the correct equation.

Eg $2\log_5(x+5)-\log_5(2x+2)=2\Rightarrow\frac{2\log_5(x+5)}{\log_5(2x+2)}=2\Rightarrow\frac{\log_5(x+5)^2}{\log_5(2x+2)}=2\Rightarrow\frac{(x+5)^2}{(2x+2)}=5^2$

This student scores M1 (shown) M0 (incorrect subtraction law), M1 (shown).

As they have not scored the 3 M marks they only have access to the final M for a total 3 out of 7

Students who start $2\log_5(x+5)=2\log_5 2+2\log_5 5$ will only have access to M3

Question Number	Scheme	Marks
7.		
(a)	$u_2 = -2,\ u_3 = -7$ and $u_4 = -12$	M1, A1 **[2]**
(b)	$d = -5$ and arithmetic	B1
	Uses *a* + (*n* – 1) *d* with *a* = 3 and *n* = 100, to give -492	M1, A1 **[3]**
(c)	$S_{100} = \frac{n}{2}(2a+(n-1)d)$ *or* $\frac{n}{2}(a+l)$.	M1
	$S_{100} = \frac{100}{2}(6+99\times -5)$ *or* $\frac{100}{2}(3+-492)$	dM1
	$= -24\,450$	A1 **[3]**
		8 marks
	Notes	

(a)

M1: Attempt to use formula correctly at least twice. ("Subtract 5") Follow through on an incorrect u_2 or u_3
A1: three correct answers

(b)

B1: Assumes AP and uses or states that $d = -5.$ Hence B0 if you see for example $d = -5,$ followed by $3\times(-5)^{99}$
You may assume an AP if you see any AP formula.
M1: Correct formula used and processed correctly. Look for $3+99\times"d"$ or $-2+98\times"d"$ with their *d*.
The (*n* -1) must be multiplied by *d*.
So, students that write $S_{100} = a+(n-1)d = 3+(100-1)-5 = 97$ score B1 M0 A0 for incorrect processing
A1: -492 (cao)

(c)

M1: States or uses a correct sum formula for an AP with *n* = 100 with any values for *a*, *d* and *l*
dM1: Uses and processes a correct sum formula for an AP with $a = 3$ *or* $-2, d = \pm 5$ and ft on their *l*

Note that students who write $S_{100} = \frac{n}{2}(2a+(n-1)d) = \frac{100}{2}(6+(100-1)-5) = 5000$ score M1 dM0 A0

A1: Obtains $-24\,450$

Question Number	Scheme	Marks
8. **(a)**	$(k-4)x^2-4x+k-2=0$	
	Uses b^2-4ac with $a=k-4,\ b=-4$ and $c=k-2$	M1
	Uses $b^2-4ac<0$ or $b^2<4ac \Rightarrow$ *Eg.* $16-4(k-4)(k-2)<0$, $16<4k^2-24k+32$ oe	dM1
	proceeds correctly to $k^2-6k+4>0$ *	A1*
		[3]
(b)	Attempts to solve $k^2-6k+4=0$ to give $k=$	M1
	$\Rightarrow$ Critical values, $k=3\pm\sqrt{5}$	A1
	$k^2-6k+4>0$ gives $k>3+\sqrt{5}$ (or) $k<3-\sqrt{5}$	M1 A1cao
		[4]
		7 marks
	Notes	

You may mark (a) and (b) as one whole question

(a)

M1: Attempts b^2-4ac with $a=k-4,\ b=-4$ and $c=k-2$ condoning one slip. Eg $a=k+4$
or uses the quadratic formula to solve equation
or uses the discriminant on two sides of an equation or inequation e.g. $b^2<4ac$ or $b^2=4ac$ or $b^2>4ac$
This cannot be awarded with $a=(k-4)x^2,\ b=-4x$ and $c=k-2$

dM1: Uses $b^2-4ac<0$ or $b^2<4ac$ with correct *a*, *b* and *c* and forms a correct inequality, in any form, seen at least once before the given answer.

A1*: CSO. Uses discriminant condition and proceeds to given answer with no errors. The inequality cannot just appear on the last line. Condone missing bracket on $-4^2=16$

(b)

M1: Uses formula, or completion of square method to find two answers to the given quadratic.
They may use their calculator to find the answers, implied by awrt 5.24 and 0.76

A1: Obtains the correct critical values $3\pm\sqrt{5}$ (which may be un simplified $\frac{6\pm\sqrt{20}}{2}$ and may be within an inequality)

M1: Chooses outside region **(** $k<$ Their Lower Limit $k>$ Their Upper Limit) for their critical values.
Do not award simply for diagram or table. Condone (for this mark) the inclusion of the boundary, or the outside region expressed as *x* not *k*.

A1: $k>3+\sqrt{5}$ (or) $k<3-\sqrt{5}$ must be exact. Allow $k>3+\sqrt{5} \cup k<3-\sqrt{5}$
Withhold if this is given in terms of *x*, has "**and" or "&"** between the two inequalities or is just one inequality.
$3+\sqrt{5}<k<3-\sqrt{5}$ scores M1 A0

Question Number	Scheme	Marks
9. (a)	Assumes GP and uses or states r = 1.06	B1
	$u_8 = ar^7 = 12\times(1.06)^7 =$	M1
	$= 12\times(1.06)^7$ = 18.04 so approximately 18* .	A1* **[3]**
(b)	$\frac{12(1.06^N - 1)}{1.06-1} = 1200 \Rightarrow r^N = k, \quad k>0$	M1
	$(1.06)^N = 7$	A1
	$N = \frac{\log(7)}{\log 1.06} = 33.395 \Rightarrow N = 34$	M1 A1 **[4]**
(c)	Distance on day N is $1200 - \frac{12((1.06)^{N-1} - 1)}{1.06-1} =$	M1
	= awrt 32km	A1 **[2]**
		9 marks
	Notes	

(a)
B1: Assumes GP (implied by any GP formula or term by term increase ×1.06) and uses or states r = 1.06 or 106% or 1+6%
M1: Uses **correct** formula with **correct** a, r and n
A1: Obtains awrt 18.0 or awrt 18.04 before writing = / ≈ 18(km)
You may see just terms. 12, 12.72, 13.48, 14.29, 15.15, 16.06, 17.02, 18.04
Three values correct to 1 dp scores B1. Eight values correct to 1dp scores M1, with conclusion scores all three marks
(b) Now marked M1 A1 M1 A1
M1: Uses correct sum formula with their r and 1200 and proceeds to $r^N = k, \quad k>0$.
$\frac{12(1-1.06^N)}{1-1.06} = 1200$ is also a correct starting point
A1: $(1.06)^N = 7$
M1: Uses a correct method using logs to solve power equation.
May be scored from $\frac{12(1.06^{N-1} - 1)}{1.06-1} = 1200$ or even a term equation
A1: N=34 cao It cannot be scored via incorrect inequality work.

It is possible you may see a trial and improvement solution. It is possible to use a tighter interval.
Score M1: Uses GP sum formula with a =13, r =1.06 and a value of n, $33 \le n \le 34$
A1: Finds an accurate value for the sum. Note: S (33)= 1168 and S (34) =1250 respectively
M1: Substitutes both 33 and 34 (or a tighter interval spanning 33.395) into the formula and achieves awrt 1168 and awrt 1250 respectively
A1**:** N=34 cao

(c)
M1: For a correct expression or for using 1200 – (sum of their first (N-1) days) or 1200 – '1168' via trial and improvement.
A1: 31.88 km or awrt 32km (they do not need to state km)

Please note that there are alternative ways of finding this:Eg.Finds $12\times 1.06^{33} - \left(\frac{12(1.06^{34} - 1)}{1.06-1} - 1200\right)$

Question Number	Scheme	Marks
10.(a)	$XZ^2 = 3^2 + 3^2 - 2\times3\times3\cos1.3$, or $\sin0.65 = \frac{x}{3}$ *so* $XZ = 2\times x$	M1
	$XZ = 3.63$	A1 **[2]**
(b)	Arc length $ZY = 3\times\theta$,= 3 × (π – 1.3) (= 5.52 / 5.53)	M1, A1
	Perimeter = 3 + 3 + arc ZY + chord XZ = 15.2 (cm)	dM1 A1 **[4]**
(c)	Area of triangle $OXZ = \frac{1}{2}\times3\times3\times\sin1.3$ (=4.34)	M1
	Area of sector is $\frac{1}{2}r^2\theta = \frac{1}{2}\times3^2\times(\pi-1.3)$ (= 8.28 / 8.29)	M1
	Total area is $\frac{1}{2}\times3^2\times(\pi-1.3)+\frac{1}{2}\times3\times3\times\sin1.3$	dM1
	= 12.6 (cm^2)	A1 **[4]**
		10 marks

Notes

(a)

M1: Uses cosine rule – must be correct. Allow $XZ^2 = 3^2 + 3^2 - 2\times3\times3\cos1.3$, for the M1
Or splits into right angled triangles correctly, uses sin 0.65 and then doubles the result
Uses angles in a triangle rule with the sine rule to find the required side. Eg $\frac{x}{\sin1.3} = \frac{3}{\sin0.92}$

A1: awrt 3.63

(b)

M1: Arc length formula $r\,\theta$ with r = 3 and $\theta = 1.3, (\pi - 1.3)$ or $(2\pi - 1.3)$ If decimals are seen accept 1.8 or 5.0
If the degree formula is being used look for $\frac{\theta}{360}\times2\pi r$ with $\theta = 74° - 75°$ or $\theta = 105° - 106°$

A1: Uses arc length formula with a correct angle. It does not need to be processed
Allow $3(\pi-1.3), 3\times1.84,$ awrt $5.52/5.53$ In degrees look for the minimum accuracy of $\frac{105.5}{360}\times2\pi\times3$

dM1: Complete method for perimeter. It is dependent upon the previous M. Look for $6+(a)+\text{arc length}$

A1: awrt15.2 (cm) – you do not need to see units

(c)

M1: Uses area formula for triangle correctly. If $\frac{1}{2}bh$ is used it must be the correct combinations found using a correct method.

M1: Uses the formula $\frac{1}{2}r^2\theta$ to find the area of the correct sector. There must be some valid attempt to use the correct angle. Allow as a minimum awrt 1.8 radians $(3.1-1.3)$

dM1: Adds two correct area formulae together. Both M's must have been awarded

A1: Accept awrt 12.6 (do not need units)

Alt (c)

M1: Attempts to find the area of the segment $\frac{1}{2}\times3^2(1.3-\sin1.3)$

M1: Attempts area of semi circle **along with** the area of segment

dM1: Finds area of the semi circle - segment $\frac{\pi\times3^2}{2}-\frac{1}{2}\times3^2(1.3-\sin1.3)$

A1: awrt 12.6

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Question Number	Scheme	Marks
11. (a)	$\dfrac{5x^2+4}{2\sqrt{x}} = \dfrac{5}{2}x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$	B1
	$\mathrm{f}(x) = \dfrac{\frac{5}{2}x^{\frac{5}{2}}}{\frac{5}{2}} + \dfrac{2x^{\frac{1}{2}}}{\frac{1}{2}} - 5x \quad (+c)$	M1 A1A1
	Uses f(4) = 14 to find c =	
	$c = -6$ and so $\mathrm{f}(x) = x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - 5x - 6$ o.e. e.g $x^{\frac{1}{2}}\left(x^2+4\right) - 5x - 6$	dM1A1
		[6]
(b)	Gradient of curve at (4, 14) is $\mathrm{f}'(4) = \dfrac{84}{4} - 5 = 16$	M1 A1
	So $(y - 14) = $ '16' $(x - 4)$ and $y = 16x - 50$	dM1 A1
		[4]
		10 marks

(a)

B1: $\dfrac{5x^2+4}{2\sqrt{x}} = \dfrac{5}{2}x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$ which may have un simplified coefficients. Allow decimal indices. This B mark may be implied by later work.

M1: Attempt to integrate - one power, even if incorrect, increased by one. Usually scored for $-5 \to -5x$

Allow for $x^{\frac{3}{2}} \to x^{\frac{3}{2}+1}$

Do not award if the candidate integrates the numerator and denominator without first attempting division.

A1: **Two** of the three terms in x correct un-simplified or simplified– (ignore no constant here).

The indices must now be simplified / calculated.

A1: All **three terms** correct **un-simplified. There is no need to have + *c***

dM1: Uses x = 4 when f(x) = 14 to find numerical value for c (may make slips). They must have attempted to integrate.

A1cao: All four terms **correct simplified** with **-6 included.** You may condone the omission of f (x) =

(b)

M1: For an attempt to substitute x = 4 into $\mathrm{f}'(x) = \dfrac{5x^2+4}{2\sqrt{x}} - 5$ or their 'simplified' function from (a). Also allow a candidate to differentiate their answer to part (a) and substitute x = 4 in the result.

Look for evidence but allow f ' (4) =... Condone slips (eg. forgetting to subtract 5) BUT do not allow this if an incorrect value just appears from nowhere.

A1: Get f '(4) = 16

dM1: Linear equation with their gradient through (4,14). It must be their $\mathrm{f}'(4)$ and not a "normal"

If they use the form $y = mx + c$ they must proceed as far as $c = \ldots$

A1: cao: $y = 16x - 50$

Question Number	Scheme	Marks
12 (i)	$\sin(...) = -\frac{2}{5}$	M1
	.... = $-$ 23.6° (or 203.6° or 336.4°)	A1
	So x = 138.6° or 271.4° (allow awrt)	dM1 A1
		[4]
(ii)	$12(1-\cos^2\theta)+\cos\theta = 6$	M1
	Solves their three term quadratic "$12\cos^2\theta - \cos\theta - 6 = 0$" to give roots	dM1
	So $(\cos\theta =)\ -\frac{2}{3}$ or $\frac{3}{4}$	A1
	$\theta = 2.30,\ 3.98,\ 0.723$ or 5.56	M1 A1 A1
		[6]
		10 marks
	Notes	

(i)

M1: As in scheme. Allow for $\text{invsin}\left(\frac{2}{5}\right) = 23.6°$ or one of the given angles.

(In radians allow for awrt 0.41, 2.73)

A1: Requires one of the answers given in the scheme. This is implied by a correct final answer

dM1: For subtracting 65 from any of **their** answers. Dependent upon the first M. Allow for -88.6
This may be implied by one of the final answers

A1: cao (Work in radians gets first M mark only unless all converted inc 65). Withhold this mark if there are any extra solutions within the range 0 to 360 degrees.

(ii)

M1: Attempts to use $\sin^2\theta = (1-\cos^2\theta)$

dM1: Solves three term quadratic and proceeds to find roots
This is implied by the sight of ± the correct values.

A1: For $-\frac{2}{3}$ or $\frac{3}{4}$

M1: Uses inverse cosine to obtain a correct value of θ for their $\cos\theta$ Allow in degrees (nearest degree) or radians (to 1 dp) Do not allow this on trivial values such as $\cos\theta = 0$ *or* ± 1

A1: Two angles correct **in degrees or radians.** Degree answers are awrt 41.4, 319, 132, 228

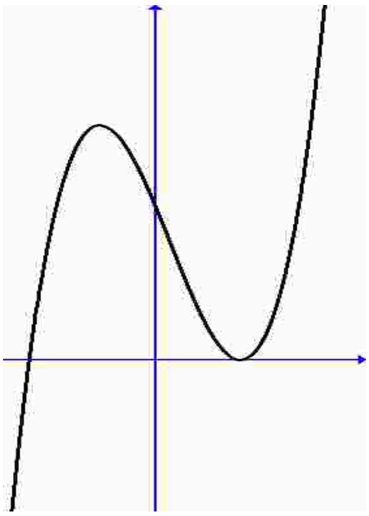
A1: All four correct (awrt) and no extra's. Condone 2.3 for 2.30
Allow multiples of π So allow awrt $0.732\pi, 1.267\pi, 0.230\pi, 1.770\pi$

Question number	Scheme	Marks
13 (a)	See $(9)^2 + (\pm 13)^2 = r^2$	M1
	$r = \sqrt{250} = 5\sqrt{10}$	A1 **[2]**
(b)	$x^2 + y^2 = 250$	B1 **[1]**
(c)	Substitute $x = 9$ when $y = -13$ to give $k = 1$	B1 **[1]**
(d)	Attempts to combine $2y + 3x = k$ and their $x^2 + y^2 = r^2$	M1
	i.e. $13x^2 - 6x - 999 = 0$ **or** when $13y^2 - 4y - 2249 = 0$	A1
	Solve to give $x =$ (**or** $y =$)	M1
	Substitute to give $y =$ (**or** $x =$)	M1
	$\left(-\frac{111}{13}, \frac{173}{13}\right)$	A1 A1 **[6]**
		(10 marks)

Notes

(a)
M1 : Allow for a correct expression for r^2 or r Implied by awrt 15.8
The method is scored for the distance from $(9,-13)$ to the origin. Some candidates are finding the formula for a circle centre $(9,-13)$ passing through the origin.
A1: For $\sqrt{250}$ *or* $5\sqrt{10}$ Either value implies the previous M. (See above).
(b)
B1: Accept any multiple of $x^2 + y^2 = 250$
Even accept $(x\pm 0)^2 + (y\pm 0)^2 = 250$, $x^2 + y^2 = \left(\sqrt{250}\right)^2$, $x^2 + y^2 = \left(5\sqrt{10}\right)^2$ and $x^2 + y^2 = \sqrt{250}^2$
(c)
B1: $k = 1$ stated or implied by $2y + 3x = 1$
(d)
M1: Eliminates x or y from their two equations to get an equation in just one variable. Allow with a numerical or algebraic k
The two equations must be of the form $2y + 3x = k$ and $(x\pm a)^2 + (y\pm b)^2 = r^2$
Do not allow the circle equation to be incorrectly simplified to $y = 5\sqrt{10} - x$
A1: Correct 3TQ equation in x or in y. The three terms need not be on the same side of the equation, just look for the correct 3 terms. You may see " $\frac{13}{4}y^2 - y = \frac{2249}{4}$ " for instance
M1: Solve a 3TQ, using a correct method, to give at least one value of x or y.
If a calculator is used 2sf is OK. You will have to use a calculator to check.
M1: Substitute x or y (in either equation) to give a value for y or x that is not -13 or 9.
It is dependent upon having started with allowable equations and having solved a 2 or 3 term quadratic equation by a correct method.
A1: One correct coordinate $x = -\frac{111}{13}$ *or* $-8\frac{7}{13}$, $y = \frac{173}{13}$ *or* $13\frac{4}{13}$
A1: Both correct answers. See above. Allow separately (not in coordinate form)

NOTE: It is possible to solve this question by geometry where *M*, the mid- point of the chord *AB* is found by solving $2y + 3x = 1$ and $y = \frac{2}{3}x$ simultaneously.

Question Number	Scheme	Marks
14. (a)	$\mathrm{f}(x)=(x-2)^2(2x+1)=2x^3-7x^2+4x+4$	M1
	So $\mathrm{f}'(x)=6x^2-14x\ +4$	M1 A1
	Puts $\mathrm{f}'(x)$ =0 and solves three term quadratic to obtain for example 2(3*x* – 1)(*x* – 2) = 0 so *x* =	M1
	$x=\frac{1}{3}$ (with *x* = 2)	A1
	Calculates f(their *x*) and find *y* $\Rightarrow\left(\frac{1}{3},\frac{125}{27}\right)$ Allow $x=\frac{1}{3}, y=4\frac{17}{27}$	dM1 A1 **[7]**
(b)	$y=(x-1)^2(2x+3)$	B1 **[1]**
(c)	When *x* = 0, *y* = 3	M1 A1 **[2]**
(d)	(1, 0) and $\left(-\frac{2}{3}, "\frac{125}{27}"\right)$	M1 A1ft **[2]**
(e)	 M1: Shape same as before, +ve cubic, but moved. Don't be overly concerned about the position of the maximum point.	M1
	A1: Shape same as before but moved to the **left** (maximum must be in second quadrant and minimum on +ve *x* - axis) and graph lies in three quadrants	A1
	A1: (1,0) and (-1.5,0) or marked on the x axis as 1 and -1.5	A1 **[3]**
		15 marks
	Notes	

(a)

M1: Expand brackets, must have a four term cubic with or without collected terms.
M1: Differentiates to a quadratic– reduction of a power by one seen at least once
A1: Completely correct $f'(x) = 6x^2 - 14x + 4$
M1: Puts their derivative = 0 and solves to find the other root to '2'. The derivative must be a 3TQ expression.
A1: Allow exact equivalences including recurring decimals. May include $x = 2$
dM1: Substitutes their 1/3 into f(x) to find the y coordinates. Implied by y = awrt 4.63 Dependent upon previous M
A1: $x = \frac{1}{3}, y = \frac{125}{27}$ must be exact. Allow mixed numbers, allow recurring decimals

....

The first 3 marks could be done by the product rule
M1: For $f'(x) = A(x-2)^2 + B(2x+1)(x-2)$
M1 A1: For $f'(x) = 2(x-2)^2 + 2(2x+1)(x-2)$

....

(b)
B1: cao. Must be in the form y =....or f (x) = or f (x+1) =
Allow $y = 2(x+1)^3 - 7(x+1)^2 + 4(x+1) + 4$ You may isw after seeing this.
Do not allow the mark if the function is left in the form $y = (x+1-2)^2(2(x+1)+1)$

(c)
M1: Puts x = 0 into their new function. Allow embedded values or correct ft.
A1: y = 3 The function must have been correct, but not necessarily simplified, to score this mark,
Condone lack of y = if the candidates work implies that y is being found at x = 0

(d)
M1: Either coordinate pair correct. Follow through their point P.
So (1,0) or (a – 1, b) where P had coordinates (a, b)
A1ft: Both pairs correct, follow through **only** on the ***y* coordinate** of P
You may condone a decimal approximation such as 0.33
So if $P = \left(\frac{1}{3}, 2\right)$ the answer of $(1,0)$ *and* $\left(-\frac{2}{3}, 2\right)$ would score M1 A1ft

Note: If they do differentiate again they only score the marks as above. They cannot be awarded from the sketch in (e)

(e)
M1: Curve moved in any way. Evidence could be, for example, the maximum to the left of the y axis or the minimum not on the x axis or a point adapted. Be tolerant on slips in shape.
A1: Shape same as before but translated to the **left** (maximum must be in second quadrant and minimum on +ve x - axis) and graph lies in three quadrants. If the maximum looks on the y - axis, do not allow.
A1: For the new curve having a minimum point on the x axis at (1,0) and passing through the x axis at -1.5. Allow this mark if it just stops at the x axis at -1.5. (It would lose the earlier A1 for not appearing in quadrant 3)

Watch for the curve been superimposed on Figure 2. If it appears twice, on blank page and on Figure 2, the blank page takes precedence. Be tolerant of slips on shape especially for the M1. Also do not penalise changes in height as we need to mark this attempt in exactly the same way as an attempt on its own.

Question Number	Scheme	Marks
15.	Line $y = 8x + 38$ and curve $y = 4x^2 + 6, \ -2 < x < 5$	
	Way 1: Integrates separately	
(a)	Attempts integration $\int (4x^2+6)\mathrm{d}x = \frac{4x^3}{3} + 6x$	M1 A1
	Uses limits and finds area under curve $\left[\frac{4x^3}{3}+6x\right]_{-2}^{4} = \left((109\tfrac{1}{3}) - (-22\tfrac{2}{3}) = 132\right)$	dM1 A1
	Full method: Area under trapezium $\frac{1}{2}\times(4+2)(22+70) - "132"$ or $\left[4x^2+38x\right]_{-2}^{4} - "132"$	M1
	So area = $276 - 132 = 144$	A1
		[6]
(b)	Attempts to find area of R_2 *i.e.* $\left[\frac{4x^3}{3}+6x\right]_{4}^{5} - \frac{(5-4)(70+78)}{2}$	M1
	So area of R_2 = $196\tfrac{2}{3} - (109\tfrac{1}{3}) - 74 = 13\tfrac{1}{3}$	A1
	Total Area shaded = $144 + 13\tfrac{1}{3} = 157\tfrac{1}{3}$	A1
		[3]
(c)	(k)= 10.8 oe	B1 **[1]**
	Way 2: Integrates line - curve	
(a)	Subtracts, and integrates: $\pm\int\left\{(8x+38)-(4x^2+6)\right\}\mathrm{d}x = \pm\left\{8\frac{x^2}{2}+38x-\frac{4x^3}{3}-6x\right\}$	M1 A1
	Uses correct limits $\pm\left[+4x^2+32x-\frac{4x^3}{3}\right]_{-2}^{4} = \left(106\frac{2}{3}\right)-\left(-37\frac{1}{3}\right)$	dM1 A1
	Full method (awarded on line 1) So area R_1 = 144	M1 A1
		[6]
(b)	Attempts to find the area of R_2 using correct limits $\pm\left[-4x^2-32x+\frac{4x^3}{3}\right]_{4}^{5} = \left(106\frac{2}{3}\right)-\left(93\frac{1}{3}\right)$	M1
	So area of R_2 = $13\tfrac{1}{3}$	A1
	Total Area shaded = $144 + 13\tfrac{1}{3} = 157\tfrac{1}{3}$	A1 **[3]**
(c)	(k) = 10.8 oe	B1 **[1]**
	Notes	

Way 1: Integrates separately. Note that this is now scored M1 A1 M1 A1 M1 A1

(a)

M1: Correct integration method for $\int (4x^2 + 6)\mathrm{d}x$ – increase power by one

A1: $\frac{4x^3}{3} + 6x$

dM1: Uses limits 4 and -2 within an integrated function– see some embedded values unless implied by correct answer

A1: For achieving 132

M1: For a full attempt at the area of R_1

Look for the area of the trapezium or the area under line (by integration) and subtract their 132

Eg. $\frac{1}{2}\times 6\times(22+70) - "132"$ or $\left[4x^2 + 38x\right]_{-2}^{4} - "132"$ either way around.

A1: 144

(b)

M1: Uses limits 5 and 4 either way round in their $\frac{4x^3}{3} + 6x$ and subtracts (or subtracts from) the area of a trapezium $\frac{1}{2}\times 1\times("78"+70)$ The 78 must have been attempted using a correct method. (Not using the quadratic function)

Alternatively Uses limits 5 and 4 either way round in their $\frac{4x^3}{3} + 6x$ and their $4x^2 + 38x$ and subtracts either way

A1: For $\pm\ 13\frac{1}{3}$ **(may be implied by final answer)**

Allow alternative/international forms $13.\dot{3}$ *and* $13.\bar{3}$ for recurring

A1: For $157\frac{1}{3}$

(c)

B1: For 10.8 oe

....

Way 2 : Integrates a combined function (Eg. line -curve) Note that this is now scored M1 A1 M1 A1 M1 A1

M1: Attempts to combine (hopefully subtract) and integrate . Correct integration method – increase power by one seen at least once. Condone bracketing error. It can be scored if they add. (Penultimate M mark is not scored)

A1: For $\pm\left(4x^2 + 32x - \frac{4}{3}x^3\right)$ This may be left un-simplified

dM1: Uses the limits 4 and -2 within an integrated function– see some working either embedded value or (...) - (...)

A1: Correct values seen, either embedded or as in scheme (...) - (...) for $\pm\left(4x^2 + 32x - \frac{4}{3}x^3\right)$

M1: For a full method. This is implied by line 1 with the functions subtracted. Condone bracketing issues

A1: For 144 following correct work.

(b)

M1: Uses limits 5 and 4 either way round in their $\pm\left(4x^2 + 32x - \frac{4}{3}x^3\right)$ or the result of their subtracted functions

A1: For $\pm\ 13\frac{1}{3}$ **(may be implied by final answer)** You may see the alternative forms for recurring.

A1: For $157\frac{1}{3}$

(c)

B1: For 10.8 oe

....

Note the demand of the question is "use integration"

If candidate writes Area = $\int_{-2}^{4}\left\{(8x+38) - (4x^2+6)\right\}\mathrm{d}x = 144$ they can score SC M0 A0 M1 A0 M1 A0

If candidate writes Area = $\int_{-2}^{4}\left\{(8x+38) - (4x^2+6)\right\}\mathrm{d}x = \left[4x^2 + 38x - \frac{4}{3}x^3 - 6x\right]_{-2}^{4} = 144$ they can score M1 A1 M1 A1 M1 A1

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