



Mark Scheme (Results)

Summer 2017

Pearson Edexcel International A-Level
In Core Mathematics C12 (WMA01)

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PEARSON EDEXCEL IAL MATHEMATICS**General Instructions for Marking**

1. The total number of marks for the paper is 125
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - \square or d... The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \dots$$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0, \quad \text{leading to } x = \dots$$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1 (a)	$a + 14d = 6 + 14 \times 10 = 146$	M1 A1 (2)
(b)	$\frac{20}{2}(2a + 19d) = 10(12 + 190) = 2020$	M1 A1 (2) (4 marks)

(a)

M1 For attempting $a + 14d$ with $a = 6$ and $d = 10$.
Alternatively lists 6, 16, 26, 36, with 15 terms and picks out the 15th term
A1 146

(b)

M1 For attempting $\frac{n}{2}(2a + (n-1)d)$ with $n = 20$, $a = 6$ and $d = 10$
For attempting $\frac{n}{2}(a + l)$ with $n = 20$, $a = 6$ and $l = a + 19d$
Alternatively lists 6+ 16+ 26+ 36+... (with or without the +signs) with 20 terms and adds
A1 2020

Do not allow misreads in either case. For example, finding the 20th term in part (a) or the sum to 15 terms in part (b) is M0

Question Number	Scheme	Marks
2 (a)	$\frac{1}{3}x^2$	B1 (1)
(b)	$\left(\frac{x}{\sqrt{2}}\right)^{-2} = \frac{2}{x^2}$	B1 (1)
(c)	$\sqrt{3}(x) \div \sqrt{\frac{48}{x^4}} = \frac{\sqrt{3}}{\sqrt{48}} \times x\sqrt{x^4} = \frac{1}{4}x^3$	M1A1 (2) (4 marks)

(a)

B1 $\frac{1}{3}x^2$ Accept exact alternatives like $\frac{x^2}{3}$ and $0.\dot{3}x^2$ but not expressions such as $0.33x^2$

(b)

B1 $\frac{2}{x^2}$ Accept exact alternatives such as $2 \times x^{-2}$ or $2 \times \frac{1}{x^2}$ (All forms must have a '2')

(c)

M1 Either the correct coefficient (accept $\frac{1}{4}$ or 0.25) or the correct power of x (accept x^3 or $\frac{1}{x^{-3}}$)

A1 Only accept $\frac{1}{4}x^3$ or simplified equivalents such as $0.25 \times x^3$ Do NOT accept $\frac{1}{4x^{-3}}$ for this mark.

Question Number	Scheme	Marks
3	$2x + 3y = 6 \Rightarrow y = -\frac{2}{3}x + \dots$ Equation of l_2 is $y = -\frac{2}{3}x + c$ Substitutes $3, -5$ into $y = -\frac{2}{3}x + c \Rightarrow -5 = -\frac{2}{3} \times 3 + c$ $y = -\frac{2}{3}x - 3$	B1 M1 M1 A1 (4 marks)
Alt	Equation of l_2 is $2x + 3y = c$ Substitutes $3, -5$ into $2x + 3y = c \Rightarrow 6 - 15 = c$ $2x + 3y = -9$ $y = -\frac{2}{3}x - 3$	B1 M1 M1 A1 (4 marks)

B1 States or implies the gradient of l_1 is $-\frac{2}{3}$ Alternatively accept l_1 written in the form $y = -\frac{2}{3}x + \dots$

M1 States or implies the gradient of l_2 is **the same** as l_1 Eg. $y = -\frac{2}{3}x + c$.

If the gradient of l_2 is incorrect then you must see

- Evidence that the gradient used for l_2 has been linked with the gradient of l_1
For example $2x + 3y = 6$ Gradient of l_1 is 2 so equation of l_2 is $y = 2x + c$
- Or a statement that the gradients are the same. $2x + 3y = 6$ The gradient of l_1 is 6 so gradient of l_2 is 6.

You must see some evidence of the candidate using equal gradients. They cannot just write down a gradient for this mark. So for example, given $2x + 3y = 6$ gradient of l_2 is 2 or equation of l_2 is $y = 2x + c$ scores M0, as there is insufficient evidence of the candidate using equal gradients.

In the alternative scheme the first two marks can be implied by stating that the new equation is of the form $2x + 3y = c$

M1 Substitutes $3, -5$ into their $y = -\frac{2}{3}x + c \Rightarrow c = \dots$ Also score for $-\frac{2}{3} = \frac{y - -5}{x - 3}$ oe

It is for knowing how to find an equation of a line knowing the gradient with the point $3, -5$

Hence follow through on an incorrect gradient, even a perpendicular one.

A1 $y = -\frac{2}{3}x - 3$ Accept forms like this $y = \left(-\frac{2}{3}\right)x + (-3)$

$$y = -\frac{2}{3}x + \dots \Rightarrow y = \frac{3}{2}x + c \Rightarrow -5 = \frac{3}{2} \times 3 + c \Rightarrow c = ..$$

Question Number	Scheme	Marks
4(a)(i)	$\frac{dy}{dx} = 6x^{0.5} - 24x^{-1.5}$	M1A1A1
(ii)	$\frac{d^2y}{dx^2} = 3x^{-0.5} + 36x^{-2.5}$	M1A1
		(5)
(b)	$\frac{dy}{dx} = 0 \Rightarrow 6x^{0.5} - 24x^{-1.5} = 0$	M1
	$x^2 = 4 \Rightarrow x = 2$	dM1, A1
	Substitutes their $x = 2$ into $y = 4x\sqrt{x} + \frac{48}{\sqrt{x}} - \sqrt{8} \Rightarrow y = 30\sqrt{2}$	M1,A1
		(5)
(c)	Substitutes their $x = 2$ into their $\frac{d^2y}{dx^2} = 3x^{-0.5} + 36x^{-2.5}$	M1
	Statement +reason. ie $\frac{d^2y}{dx^2} > 0 \Rightarrow$ minimum	A1 cso
		(2)
		(12 marks)

(a)(i)

M1 For a correct power on any of the 'three' terms including the $\sqrt{8} \rightarrow 0$.

A1 Two of the three terms correctly differentiated (can be unsimplified)

You may accept $6x^{0.5}$ as $4 \times 1.5x^{1.5-1}$ and $-24x^{-1.5}$ as $+48 \times -\frac{1}{2}x^{-0.5-1}$

A1 Cao but remember to isw. Accept alternatives for the terms in x such as $x^{0.5} = \sqrt{x} = x^{\frac{1}{2}}$

Allow expressions given in the form of the question $\left(\frac{dy}{dx} = \right) 6\sqrt{x} - \frac{24}{x\sqrt{x}}$

(a)(ii)

M1 Differentiating again. Scored for reducing any **fractional** power by one (seen once allowing follow through)

A1 Cao. See part (i) notes for acceptable alternatives. Eg accept $\left(\frac{d^2y}{dx^2} = \right) \frac{3}{\sqrt{x}} - \frac{36}{x^2\sqrt{x}}$

(b)

M1 Sets (or implies that) their $\frac{dy}{dx} = 0$

dM1 Dependent upon the previous M. For forming an equation of the type $x^n = A$, **following correct index work.**

A1 $x = 2$ (Ignore any reference to $x = -2$). Part (a) must be correct and both M's must have been scored.

M1 For substituting their solution (of $\frac{dy}{dx} = 0$) into $y = 4x\sqrt{x} + \frac{48}{\sqrt{x}} - \sqrt{8} \Rightarrow y = \dots$

A1 $(y) = 30\sqrt{2}$ Part (a) must be correct and all three M's must have been scored.

(c)

M1 For substituting their $x = 2$ into their $\left(\frac{d^2y}{dx^2} = \right) 3x^{-0.5} + 36x^{-2.5}$ and finding (or implying to find)

a numerical result. Alternatively, for substituting their $x = 2$ into their $\left(\frac{d^2y}{dx^2} = \right) 3x^{-0.5} + 36x^{-2.5}$

and considering the sign. Eg When $x = 2 \Rightarrow 3 \times 2^{-0.5} + 36 \times 2^{-2.5} > 0$

A1 CSO Requires a correct $x = 2$ and a correct $\left(\frac{d^2y}{dx^2} = \right) 3x^{-0.5} + 36x^{-2.5}$

A statement and a conclusion is required to score this mark.

Allow the candidate to state that when $x = 2$ $\frac{d^2y}{dx^2} = 3 \times 2^{-0.5} + 36 \times 2^{-2.5} > 0 \Rightarrow \text{minimum}$

If the candidate gives the numerical value to $\frac{d^2y}{dx^2}$, it must be correct. Accept $6\sqrt{2}$ oe or awrt 8.5

Alternatives in part (c)

M1 Finding the value of 'y' at $x = 2$, left of 2 and right of 2.

Alternatively finding the $\frac{dy}{dx}$ at $x = 2$, left of 2 and right of 2

A1 A statement and a conclusion is required to score this mark. A sketch graph can be used instead of a statement. Numerical values must be correct.

Question Number	Scheme	Marks
5(a)	Attempts $f(\pm 1)$ Remainder = 2	M1 A1 (2)
(b)	Attempts $f(\pm 3) = -4 \times (\pm 3)^3 + 16 \times (\pm 3)^2 - 13 \times (\pm 3) + 3$ Remainder = 0 $\Rightarrow (x - 3)$ is a factor	M1 A1* (2)
(c)	Divides their $f(x)$ by $(x - 3)$ to get the quadratic factor $(-4x^2 + 4x - 1)$ $f(x) = (x - 3) \times -(2x - 1)(2x - 1) = (3 - x)(2x - 1)^2$	M1 A1 dM1A1 (4)
(d)	$f(x) \leq 0 \Rightarrow (3 - x)(2x - 1)^2 \leq 0$ $x = \frac{1}{2}, x \geq 3$	B1, B1 (2) (10 marks)

(a)

M1 Attempts to calculate $f(\pm 1)$ condoning slips. Algebraic division does not score this mark.A1 (Remainder is) 2 Accept sight of $f(1) = 2$ for both marks

(b)

M1 Attempts to find $f(\pm 3)$ condoning slips. We must see some substitution or calculation

$$-4 \times (\pm 3)^3 + 16 \times (\pm 3)^2 - 13 \times (\pm 3) + 3 \quad \text{or} \quad -108 + 144 - 39 + 3 \quad \text{is OK.}$$

We cannot accept just sight of ' $f(3) = 0$ ' Algebraic division does not score this mark.

A1* It is a "show that" so requires both a statement and a conclusion. For example

$$f(3) = -4 \times (3)^3 + 16 \times (3)^2 - 13 \times (3) + 3 = 0 \Rightarrow (x-3) \text{ is a factor.}$$

The conclusion could be QED or a tick following the working to $f(3) = 0$

It could also as a preamble before the working.

(c)

M1 Divides $f(x)$ by $(x-3)$ to get a quadratic. If part (b) is done by division it can be awarded in (c)from work in (b). If inspection is used look for first and last terms. Eg $f(x) = (x-3)(\pm 4x^2 + \dots x \pm 1)$

$$\begin{array}{r} \text{If division is used look for first two terms} \quad x-3 \overline{) \begin{array}{l} \pm 4x^2 \pm 4x \dots \dots \dots \\ -4x^3 + 16x^2 - 13x + 3 \\ \hline -4x^2 + 12x^2 \\ \hline 4x^2 \end{array}} \end{array}$$

$$A1 \quad (-4x^2 + 4x - 1)$$

dM1 Dependent upon the previous M. For further factorisation of their $(-4x^2 + 4x - 1)$

Method and accuracy marks may be awarded if seen in part (d)

$$A1 \quad f(x) = (3-x)(2x-1)^2, f(x) = -(x-3)(2x-1)^2 \quad f(x) = (x-3)(2x-1)(-2x+1)$$

$$f(x) = 4(3-x) \left(x - \frac{1}{2}\right) \left(x - \frac{1}{2}\right) \text{ oe} \quad \text{You do not need to see the } f(x)$$

If there is little or no working in (c) with no quadratic factor seen then.....

award all marks if the answer is correct (See above for possible options)

award M1 A0 M1 A0 by implication for $(x-3)(2x-1)^2$ or equivalentaward no marks for answers such as $(x-3)\left(x - \frac{1}{2}\right)^2$

(d) It is quite difficult to ascertain whether candidates are finding roots or solving the question so mark as follows

B1 Sight of either $\frac{1}{2}$ and/or 3 seen in part (d).B1 Both of $x = \frac{1}{2}, x \geq 3$ Do not accept this if candidates write down roots first and just $x \geq 3$ laterAccept set language Eg $\left\{x : x = \frac{1}{2} \cup x \geq 3\right\}$

Question Number	Scheme	Marks
6(a)	$\cos \angle BAC = \frac{12^2 + 10^2 - 6^2}{2 \times 12 \times 10} \Rightarrow \angle BAC = 0.5223$	M1A1 (2)
(b)	Arc $BD = r\theta = 10 \times 0.5223$ Perimeter = $6 + 2 + 10 \times 0.5223 = 13.22$ (m)	M1 dM1, A1 (3)
(c)	Area of sector $BAD = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 10^2 \times 0.5223$ (= 26.116) Area of triangle ABC $\frac{1}{2} ab \sin C = \frac{1}{2} \times 12 \times 10 \times \sin 0.5223$ (= 29.932) Area of flowerbed $BCD = \frac{1}{2} \times 12 \times 10 \times \sin 0.5223 - \frac{1}{2} \times 10^2 \times 0.5223$ = 3.81 / 3.82 (m ²)	M1 M1 dM1 A1 (4) (9 marks)

(a)

M1 Attempts use of the formula $6^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \cos A$ or $\cos \angle BAC = \frac{12^2 + 10^2 - 6^2}{2 \times 12 \times 10}$
The sides must be in the correct "position" within the formula. Condone different notation Eg. θ

A1 $\angle BAC =$ awrt 0.5223 The angle in degrees (awrt 29.9°) is A0

(b)

M1 Attempts arc formula: In radians uses Arc $BD = r\theta = 10 \times "0.5223"$

In degrees uses Arc $BD = \frac{\theta}{360} \times 2\pi r = \frac{"29.9"}{360} \times 2\pi \times 10$

dM1 Dependent upon the arc formula having been used. It is for calculating the perimeter as 8 + arc length.

A1 Perimeter = awrt 13.22 (m)

(c)

M1 Attempts area of sector formula: Area of sector $BAD = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 10^2 \times "0.5223"$

In degrees uses Area of sector $BAD = \frac{\theta}{360} \times \pi r^2 = \frac{"29.9"}{360} \times \pi \times 10^2$

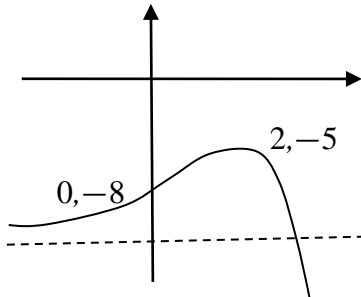
M1 Attempts area of triangle formula: Area of triangle $ABC = \frac{1}{2} ab \sin C = \frac{1}{2} \times 12 \times 10 \times \sin "0.5223"$

You may see Herons formula used with $S = \frac{10+6+12}{2} = (14)$ and $A = \sqrt{S(S-10)(S-6)(S-12)}$

Watch for other methods including the calculation of a perpendicular.

dM1 Dependent upon both correct formulae. It is scored for finding area of triangle - area of sector

A1 Allow awrt 3.81 or 3.82 (m²)

Question Number	Scheme	Marks
7 (a)	(8, 5)	B1 (1)
(b)	$y = 7$	B1 (1)
(c)	$5 < k < 10$	M1A1 (2)
(d)	 <p>Shape (0, -8) and (2, -5)</p> <p>Asymptote $y = -10$</p>	B1 B1 B1 (3) (7 marks)

(a)

B1 Accept (8, 5) or $x = 8, y = 5$ or a sketch of $y = f\left(\frac{1}{4}x\right)$ with a minimum point marked at (8, 5)

(b)

B1 $y = 7$. It must be an equation and not just '7'

(c)

M1 Accept one "side" of the inequality condoning a misunderstanding of whether the boundary is included or not. Allow for $k > 5$, $k \geq 5$, $k < 10$, $k \leq 10$ Condone a different variable for the M1

A1 cao $5 < k < 10$. Allow $k > 5$ **and** $k < 10$ $k > 5, k < 10$ (5,10) $\{k \in \mathbb{R} : 5 < k < 10\}$
Do not allow $k > 5$ **or** $k < 10$

(d)

B1 For a reflection of the original curve in the x – axis.

Look for the shape shown in the scheme but be tolerant of slips at either end.

B1 For the graph to have an intercept of (0, -8) and a (single) **maximum** point of (2, -5)

Accept -8 being marked on the y - axis and the graph passing through this.

Condone (-8,0) as long as it is marked on the correct axis

B1 For giving the equation of the asymptote as $y = -10$

The graph must clearly be asymptotic but be tolerant of slips. See practice items for clarification.

Question Number	Scheme	Marks
8 (a)	$\int (3x^2 + 4x - 15)dx = x^3 + 2x^2 - 15x + c$	M1A1A1 (3)
(b)	$\int_b^4 (3x^2 + 4x - 15)dx = \left[x^3 + 2x^2 - 15x + (c) \right]_b^4 = 36$ $(64 + 32 - 60) - (b^3 + 2b^2 - 15b) = 36$ $b^3 + 2b^2 - 15b = 0$	M1 A1* (2)
(c)	$b = 0$ $b^2 + 2b - 15 = 0 \Rightarrow (b + 5)(b - 3) = 0$ $b = -5, 3$	B1 M1 A1 (3)
		(8 marks)

(a)

M1 Raises the index of any term in x by one

A1 Two of the three algebraic terms correct (unsimplified). For example accept $2x^2 = \frac{4}{2}x^{1+1}$

A1 cao including the +c

(b)

M1 Substitutes 4 and b into their integrated expression, subtracts either way around and sets equal to 36

A1* Simplifies to the given solution. This is a given answer and therefore the intermediate line(s) must be correct. Minimum expectation for an intermediate line is $36 - (b^3 + 2b^2 - 15b) = 36$ or equivalent with the bracket removed.

(c)

B1 $b = 0$

M1 Factorises/ attempts to solve the quadratic

A1 $b = -5, 3$

Question Number	Scheme	Marks
9(i)	$2\log_{10}(x-2) - \log_{10}(x+5) = 0 \Rightarrow \log_{10}(x-2)^2 = \log_{10}(x+5)$ $\Rightarrow (x-2)^2 = (x+5)$ $\Rightarrow x^2 - 5x - 1 = 0$ $x = \frac{5 \pm \sqrt{29}}{2} \Rightarrow x = \frac{5 + \sqrt{29}}{2} \text{ only}$	M1 M1 A1 M1,A1 (5)
9(ii)	$\log_p(4y+1) - \log_p(2y-2) = 1 \Rightarrow \log_p\left(\frac{4y+1}{2y-2}\right) = \log_p p$ $\Rightarrow \left(\frac{4y+1}{2y-2}\right) = p$ $\Rightarrow 4y+1 = 2py - 2p \Rightarrow y = \frac{1+2p}{2p-4}$	M1, M1 A1 M1A1 (5) (10 marks)

(i)

M1 Use of the power law of logs

M1 For 'undoing' the logs by either setting $\log_{10} \dots = \log_{10} \dots$ or using the subtraction law and $0 = \log_{10} 1$ A1 A correct simplified quadratic $x^2 - 5x - 1 = 0$

M1 A correct attempt to find a solution to a 3TQ of equivalent difficulty (ie no factors). Allow formula, completing the square and use of a calculator giving exact or decimal answers

A1 cso $\frac{5 + \sqrt{29}}{2}$ or exact simplified equivalent without extra answers.

(ii)

M1 Use of subtraction (or addition) law of logs

M1 For using $1 = \log_p p$ or equivalent in an attempt to get an equation not involving logs.
$$\log_p(4y+1) - \log_p(2y-2) = 1 \Rightarrow (4y+1) - (2y-2) = p \text{ implies this and scores M0 M1.}$$
A1 A correct equation in p and y not involving logs. Accept $\left(\frac{4y+1}{2y-2}\right) = p^1$ M1 Score for an attempt to change the subject. This must include cross multiplication, collection of terms in y , followed by factorisation of the y term.A1 cso $y = \frac{1+2p}{2p-4}$ or equivalent such as $y = \frac{-1-2p}{4-2p}$

Special cases in (i): Case 1 Allow the subtraction law either way around as the rhs of the equation will be 1

$$\text{Case 2 } \log_{10} \frac{(x-2)^2}{(x+5)} = 0 \Rightarrow \frac{(x-2)^2}{(x+5)} = 1 \Rightarrow (x-2)^2 = (x+5) \Rightarrow x^2 - 5x - 1 = 0$$

$$\Rightarrow x = \frac{5 + \sqrt{29}}{2} \text{ only will be awarded M1 M0 A1 M1 A0}$$

$$\text{Special cases in (ii): } \log_p(4y+1) - \log_p(2y-2) = 1 \Rightarrow \frac{\log_p(4y+1)}{\log_p(2y-2)} = \log_p p \Rightarrow \left(\frac{4y+1}{2y-2}\right) = p^1$$

$$\Rightarrow 4y+1 = 2py - 2p \Rightarrow y = \frac{1+2p}{2p-4} \text{ will be awarded M0 M1A1 M1 A0}$$

Question Number	Scheme	Marks
10 (a)	2^{10} OR 1024 seen as the constant term $\left(2 - \frac{x}{8}\right)^{10} = 2^{10} + {}^{10}C_1 2^9 \left(-\frac{x}{8}\right)^1 + {}^{10}C_2 2^8 \left(-\frac{x}{8}\right)^2 + \dots$ $= 1024 - 640x + 180x^2$	B1 M1A1 A1 (4)
(b)	$\left(2 - \frac{x}{8}\right)^{10} (a + bx) = (1024 - 640x + 180x^2)(a + bx)$ $1024a = 256 \Rightarrow a = \frac{1}{4}$ oe	M1A1 (2)
(c)	$1024b - 640a = 352 \Rightarrow b = \frac{1}{2}$	M1A1 (2) (8 marks)

(a)

B1 2^{10} OR 1024 seen as the constant term

M1 For a correct attempt at the binomial expansion for $(a + b)^n$ with $a = 2$, $b = \pm \frac{x}{8}$ and $n = 10$

Condone missing brackets. Accept any unsimplified term in x as evidence

Accept a power series expansion on $(1 \pm kx)^{10} = 1 + 10(\pm kx) + \frac{10 \times 9}{2}(\pm kx)^2$ condoning missing brackets. Again accept any unsimplified term in x as evidence

A1 A completely correct unsimplified solution.

Accept $= 2^{10} + {}^{10}C_1 2^9 \left(-\frac{x}{8}\right)^1 + {}^{10}C_2 2^8 \left(-\frac{x}{8}\right)^2 + \dots$

Accept $= 2^{10} \left(1 + 10 \times \left(-\frac{x}{16}\right) + \frac{10 \times 9 \times \left(-\frac{x}{16}\right)^2}{2!} + \dots \right)$

A1 $1024 - 640x + 180x^2$ Accept $1024 + -640x + 180x^2$
 Can be listed with commas or appear on separate lines. Accept in reverse order.

(b)

M1 Sets their '1024' $\times a = 256$

A1 $a = \frac{1}{4}$. Accept equivalents such as 0.25.

Accept this for both marks (it can be done by substituting $x = 0$ into both sides of the expression) as long as it is not found from an incorrect method

(c)

M1 Sets their '1024' $\times b \pm$ their '640' $a = 352$

A1 $b = \frac{1}{2}$ or 0.5

Question Number	Scheme	Marks
11 (a)	Attempts $U_4 = 6000 \times (1.015)^3 = 6274$ (tonnes)	M1A1* (2)
(b)	Attempts $U_N = 6000 \times 1.015^{N-1} > 8000$ $1.015^{N-1} > \frac{8000}{6000}$ oe $\log(1.015^{N-1}) > \log\left(\frac{4}{3}\right) \Rightarrow N > \frac{\log\left(\frac{4}{3}\right)}{\log(1.015)} + 1 = (20.3)$ $(N) = 21$	M1 A1 M1A1 A1 (5)
(c)	Attempts $S_n = \frac{a(1-r^n)}{(1-r)}$ with $n=10$ $a=6000/30000$ and $r=1.015$ $S = 5 \times \frac{6000(1.015^{10}-1)}{(1.015-1)}$ OR $S = \frac{30000(1.015^{10}-1)}{(1.015-1)}$ Awrt £321 000	M1 A1 A1 (3)
		(10 marks)

(a)

- M1 Attempts to use ar^3 with $a=6000$, $r=1.015$. Accept $r=1+1.5\%$
Condone for this mark $r=1.15$ or 1.0015 Accept a list of 4 terms with the same conditions
- A1* cso $6000 \times (1.015)^3 = 6274$ (tonnes).

If candidate states $U_4 = 6000 \times (1.015)^3 = 6274.07$ (tonnes) or 6274.0 (or anything that rounds to 6274) they don't need to round to the given answer.

(b)

- M1 Attempts to use $ar^{n-1} \dots 8000$ or $ar^n \dots 8000$ with $a=6000$, $r=1.015$ or $1+1.5\%$ condoning values of r being 1.15 or 1.0015
- A1 For reaching the intermediate result $1.015^{n-1} \dots \frac{4}{3}$ or $1.015^n \dots \frac{4}{3}$.
Allow $\frac{4}{3}$ to be rounded or truncated to 1.33 (to 2dp or better)
- M1 Uses logs correctly to get n or $n-1$ This mark may be awarded from a sum formula
- A1 This is scored for a 'correct' (unrounded) answer. It may be left in log form. If the candidate has used n instead of $n-1$, they will not score this unless they subsequently reach a final answer of 21. Allow for N or n .

Accept versions of $n \dots \frac{\log\left(\frac{4}{3}\right)}{\log(1.015)} + 1 = (20.3)$ or $n \dots \log_{1.015}\left(\frac{4}{3}\right) + 1 = (20.3)$ or

$$n \dots \frac{\log(1.33)}{\log(1.015)} + 1 = (20.15)$$

- A1 $(N) = 21$ Do not accept $N > 21$ etc
The two final A marks may be implied by finding 'n' and adding 1 to reach 21

So trial and improvement above and below

M1 Attempts $(U_{20}) = 6000 \times (1.015)^{19}$ or $(U_{21}) = 6000 \times (1.015)^{20}$

A1 Achieves either $(U_{20}) = \text{awrt } 7960$ or $(U_{21}) = \text{awrt } 8080$

M1 Attempts $(U_{20}) = 6000 \times (1.015)^{19}$ and $(U_{21}) = 6000 \times (1.015)^{20}$

A1 Achieves both $(U_{20}) = \text{awrt } 7960$ and $(U_{21}) = \text{awrt } 8080$

A1 States $(N) = 21$

Candidate uses a "solve" option on a calculator.

M1 $6000 \times 1.015^{N-1} = 8000$ M1 A1 A1 $(N) = 20.3$ (to 1 dp at least) A1 states 21

Candidates can attempt to write down 20 terms which would follow the same scheme as trial and improvement

(c)

M1 Attempts $S_n = \frac{a(1-r^n)}{(1-r)}$ with $n = 10$ $a = 6000 / 30000$ and $r = 1.015 / 1 + 1.5\% / 1.15 / 1.0015$

Alternatively accept a list of ten terms starting with £30 000, £30450, £30906.75, ... added
Or £6000, £6090, £6181.35,added

A1 A correct unsimplified answer to the **cost** for 10 years

Accept either $S = 5 \times \frac{6000(1.015^{10} - 1)}{(1.015 - 1)}$ OR $S = \frac{30000(1.015^{10} - 1)}{(1.015 - 1)}$

A1 cso awrt (£) 321 000

Question Number	Scheme	Marks
12(a)	$y = x^3 - 9x^2 + 26x - 18 \Rightarrow \frac{dy}{dx} = 3x^2 - 18x + 26$ $\text{At } x = 4 \Rightarrow \frac{dy}{dx} = 3 \times 4^2 - 18 \times 4 + 26 (= 2)$ $\text{Equation of normal is } y - 6 = -\frac{1}{2}(x - 4) \Rightarrow 2y + x = 16$	M1A1 M1 dM1A1* (5)
(b)	Sub $x = 1$ in $(y) = 1 - 9 + 26 - 18 = 0$	B1* (1)
(c)	$\int x^3 - 9x^2 + 26x - 18 dx = \left[\frac{1}{4}x^4 - 3x^3 + 13x^2 - 18x \right]$ $\text{Normal meets } x \text{ axis at } x = 16$ $\text{Area of triangle} = \frac{1}{2} \times (16 - 4) \times 6 = (36)$ $\text{Correct method for area} = \left[\frac{1}{4}x^4 - 3x^3 + 13x^2 - 18x \right]_1^{16} + \frac{1}{2} \times (16 - 4) \times 6$ $= 51.75$	M1A1 B1 M1 dM1 A1 (6) (12 marks)

(a)

M1 Two of the three terms correct (may be unsimplified).

A1 $\left(\frac{dy}{dx} = \right) 3x^2 - 18x + 26$, need not be simplified. You may not see the $\frac{dy}{dx}$

M1 Substitutes $x = 4$ into their $\frac{dy}{dx}$

dM1 The candidate must have scored both M's. It is for the correct method of finding the equation of a normal. Look for $y - 6 = -\frac{dx}{dy} \Big|_{x=4} \times (x - 4)$

If the form $y = mx + c$ is used it is for proceeding as far as $c = ..$

A1* cso $2y + x = 16$ Note that this is a given answer. $x + 2y = 16$ is ok

(b)

B1* Either substitute $x = 1$ in $(y) = 1^3 - 9 \times 1^2 + 26 \times 1 - 18 = 0$ or $(y) = 1 - 9 + 26 - 18 = 0$

Or substitute $y = 0$ reach $(x - 1)(x^2 - 8x + 18)$ by inspection or division and state $x = 1$

To mark consistently decide on the method first

(c) Way One: Finding area under curve C

- M1 Integrates $x^3 - 9x^2 + 26x - 18$ with at least two terms correct (unsimplified)
- A1 $\int x^3 - 9x^2 + 26x - 18 dx = \left[\frac{1}{4}x^4 - 3x^3 + 13x^2 - 18x \right]$ or equivalent. May be unsimplified
- B1 For sight of the normal meeting the x axis at 16. May be embedded within a formula or on the diagram
- M1 For correct method for the area of the triangle. Either $\frac{1}{2} \times ('16' - 4) \times 6$ or correct integration with limits of 4 to their 16 $\left[8x - \frac{1}{4}x^2 \right]_{x=4}^{x='16'}$
- dM1 Dependent upon the candidate having scored both M's. It is a fully correct method of finding the area of R . The limits of the integral(s) must be correct
- Correct method for area = $\left[\frac{1}{4}x^4 - 3x^3 + 13x^2 - 18x \right]_1^4 + \frac{1}{2} \times ('16' - 4) \times 6$
- A1 cso 51.75 or such as $\frac{207}{4}$

Alternatives in part (c). In some cases candidates may try to find the area between the line and the curve. If you see an attempt where this occurs score as follows.

Way Two: Finding area between line and curve

- M1 Integrates their $\left(8 - \frac{1}{2}x \right) - (x^3 - 9x^2 + 26x - 18)$ either way around with at least two terms correct (unsimplified). Condone slips on the straight line (eg $8 - x$, $16 - x$ or on the subtraction)
- A1 Completely correct integration of their expression either way around $\pm \left\{ \left(8x - \frac{1}{4}x^2 \right) - \left(\frac{1}{4}x^4 - 3x^3 + 13x^2 - 18x \right) \right\}$
- You may follow through on their equation of the line or on their combined expression of line and curve. This A1 is for correct integration following the award of the M mark.
- B1 For sight of the normal meeting the x axis at 16. May be implied within a triangle formula or from the diagram
- M1 For correct method for the area of the large triangle. The base length of the triangle is (their 16 - 1). The height of the triangle must be an attempt to find the y value on the normal at $x=1$
- Either by $\frac{1}{2} \times ('16' - 1) \times y_{\text{NORM at } x=1} = \left(\frac{1}{2} \times ('16' - 1) \times '7.5' = (56.25) \right)$ or by correct integration with limits of 1 to their 16 $\left[8x - \frac{1}{4}x^2 \right]_{x=1}^{x='16'}$
- dM1 Dependent upon the candidate having scored both M's. It is a fully correct method of finding the area of R . The limits of the integral(s) must be correct
- Correct method for area = $\frac{1}{2} \times ('16' - 1) \times '7.5' - \left[\left(8x - \frac{1}{4}x^2 \right) - \left(\frac{1}{4}x^4 - 3x^3 + 13x^2 - 18x \right) \right]_1^4$
- A1 cso 51.75

Question Number	Scheme	Marks
13(a)	$5 \cos x + 1 = \sin x \tan x \Rightarrow 5 \cos x + 1 = \sin x \times \frac{\sin x}{\cos x}$ $5 \cos^2 x + \cos x = \sin^2 x$ $5 \cos^2 x + \cos x = 1 - \cos^2 x$ $6 \cos^2 x + \cos x - 1 = 0$	M1 A1 M1 A1* (4)
(b)	$6 \cos^2 k + \cos k - 1 = 0 \Rightarrow (3 \cos k - 1)(2 \cos k + 1) = 0$ $\Rightarrow \cos k = \frac{1}{3}, -\frac{1}{2}$ <p>Either $\cos 2\theta = \frac{1}{3} \Rightarrow 2\theta = 70.53, 289.47$ Or $\cos 2\theta = -\frac{1}{2} \Rightarrow 2\theta = 120, 240$</p> $\Rightarrow \theta = 35.3^\circ, 144.7^\circ, 60^\circ, 120^\circ$	M1 A1 M1A1M1A1 (6) (10 marks)

(a)

- M1 Uses $\tan x = \frac{\sin x}{\cos x}$ in the given equation. You may see it used in the form $\tan x \cos x = \sin x$
- A1 A correct equation (not involving fractions) in both sin and cos. Eg $5 \cos^2 x + \cos x = \sin^2 x$ oe
- M1 Replaces $\sin^2 x$ by $1 - \cos^2 x$ to produce a quadratic equation/expression in just $\cos x$
- A1* Proceeds correctly to the given answer $6 \cos^2 x + \cos x - 1 = 0$
- All notation should be consistent and correct including $\cos x$ instead of \cos and $\sin^2 x$ instead of $\sin x^2$

(b)

M1 Attempts to factorise and solve expression of the form $6\cos^2 k + \cos k - 1 = 0$

Accept use of formula or GC They may solve $6y^2 + y - 1 = 0$ which is fine

A1 Correct answers for quadratic Accept $\cos k = \frac{1}{3}, -\frac{1}{2}$ with $k = \theta$ or 2θ or x or even $y = \frac{1}{3}, -\frac{1}{2}$

M1 Correct order of operations to produce at least one value for θ in the range $0 \rightarrow 360^\circ$

Look for $\cos 2\theta = \frac{1}{3} \Rightarrow \theta = \frac{\arccos\left(\frac{1}{3}\right)}{2}$ or $\cos 2\theta = -\frac{1}{2} \Rightarrow \theta = \frac{180 - \arccos\left(\frac{1}{2}\right)}{2}$

A1 Two of $\theta = \text{awrt } 35.3^\circ, 144.7^\circ, 60^\circ, 120^\circ$ Any of these values would imply M1

M1 Correct order of operations to produce at least one other value for θ in the range $0 \rightarrow 180^\circ$ from their principal value

Look for $\cos 2\theta = \frac{1}{3} \Rightarrow \theta = \frac{360 - \arccos\left(\frac{1}{3}\right)}{2}$ or $\cos 2\theta = -\frac{1}{2} \Rightarrow \theta = \frac{180 + \arccos\left(\frac{1}{2}\right)}{2}$

A1 All four of $\theta = \text{awrt } 35.3^\circ, 144.7^\circ, 60^\circ, 120^\circ$ with no additional solutions in the range.

If the answers are given in radians $0.615, 2.526, \frac{\pi}{3} (1.047), \frac{2\pi}{3} (2.094)$ just withhold the final A1

Special Case 1: If candidates solve part (b) giving values for 2θ (perhaps mistakenly thinking that the question was $6\cos^2 \theta + \cos \theta - 1 = 0$) then you may award special case 1 1 1 0 0 0 for sight of both solutions $(\theta) = \text{awrt } 70.5^\circ, 120^\circ$ in the range $0 \rightarrow 180^\circ$

Question Number	Scheme	Marks
14 (a)	$\left(\frac{1+7}{2}, \frac{4+8}{2}\right) = (4, 6)$	M1A1 (2)
(b)	$\frac{\sqrt{(7-1)^2 + (8-4)^2}}{2}$ Or $\sqrt{('4'-1)^2 + ('6'-4)^2}$ Or $\sqrt{(7-'4')^2 + (8-'6')^2}$ (Radius of circle) = $\sqrt{13}$	M1 A1 (2)
(c)	Equation of C ₂ is $x^2 + y^2 = r^2$ Attempts either value of r as $\left(\sqrt{'4'^2 + '6'^2} \pm \text{their } r\right)$ When $r = \sqrt{52} - \sqrt{13} = \sqrt{13} \Rightarrow x^2 + y^2 = 13$ When $r = \sqrt{52} + \sqrt{13} = 3\sqrt{13} \Rightarrow x^2 + y^2 = 117$	M1 M1 A1 A1 (4) (8marks)

(a)

M1 For an attempt at $\left(\frac{1+7}{2}, \frac{4+8}{2}\right)$ May be implied by either correct coordinate

A1 (4, 6). No working is required, Correct answer scores both marks. Condone lack of brackets

(b)

M1 Scored for using Pythagoras' theorem to find the distance between their centre and a point. Look for an attempt at $\sqrt{('4'-1)^2 + ('6'-4)^2}$ or similar. If the original coordinates are used then there must be some attempt to halve.

A1 $= \sqrt{13}$ Correct answer scores both marks

(c)

M1 For stating the equation of C₂ is $x^2 + y^2 = r^2$ or $(x-0)^2 + (y-0)^2 = r^2$ for any 'r' including an algebraic 'r' Accept $x^2 + y^2 = k$ If a value of k is given then k must be positive

M1 Attempts either value of r Look for $\left(\sqrt{'4'^2 + '6'^2} \pm \text{their } r\right)$ Accept $r = \frac{\sqrt{4^2 + 6^2}}{2}$

A1 Either of $x^2 + y^2 = 13$ or $x^2 + y^2 = 117$

Allow for this mark variations like $(x-0)^2 + (y-0)^2 = \sqrt{13}^2$

A1 Both of $x^2 + y^2 = 13$ and $x^2 + y^2 = 117$. Equations must be simplified as seen here
Any one correct equation will imply the first two M's.

Alt method to find equations using the intersections:

M1: As above

M1: Solves 'their' $y = \frac{3}{2}x$ with their $(x-'4')^2 + (y-'6')^2 = '13'$ \Rightarrow Intersections (2,3) and (6,9)

So this time the method is scored for either $\sqrt{'2'^2 + '3'^2}$ or $\sqrt{'6'^2 + '9'^2}$

A1 A1 as before

Question Number	Scheme	Marks
15 (a)	$t = 1 \Rightarrow H = 4 + 1.5 \sin\left(\frac{\pi}{6}\right) = 4.75(\text{m})$	B1* (1)
(b)	$t = 14 \Rightarrow H = 4 + 1.5 \sin\left(\frac{14\pi}{6}\right) = \text{awrt } 5.3(\text{m})$	M1A1 (2)
(c)	$H = 3 \Rightarrow 3 = 4 + 1.5 \sin\left(\frac{\pi t}{6}\right)$ $\Rightarrow \sin\left(\frac{\pi t}{6}\right) = -\frac{2}{3}$ $\Rightarrow \sin\left(\frac{\pi t}{6}\right) = -\frac{2}{3} \Rightarrow t = \frac{6 \times \left(\pi + \arcsin\left(-\frac{2}{3}\right)\right)}{\pi} = \text{awrt } 7.4$ $\text{And } \Rightarrow t = \frac{6 \times \left(2\pi - \arcsin\left(-\frac{2}{3}\right)\right)}{\pi} = \text{awrt } 10.6$ <p>Times are 7:23/7:24 am and 10:36/10:37 am</p>	M1 A1 M1A1 M1 A1 (6) (9 marks)

(a)

B1* This is a given answer. Score for sight of $(H =) 4 + 1.5 \sin\left(\frac{\pi}{6}\right) = 4.75(\text{m})$

Alternatively they can work backwards with $H = 4.75 \Rightarrow \frac{\pi t}{6} = \frac{\pi}{6} \Rightarrow t = 1 \Rightarrow$ Hence (time) = 1a.m

(b)

M1 For substituting $t = 14$ into H and an attempt to calculate.

Score for $4 + 1.5 \sin\left(\frac{14\pi}{6}\right) = ..$

A1 awrt 5.3 (m) or $\frac{16 + 3\sqrt{3}}{4}$ m following correct work.

$4 + 1.5 \sin\left(\frac{2\pi}{6}\right) = 5.30(\text{m})$ is M0A0 unless there is an explanation that states that the period is 12 hours so

$$4 + 1.5 \sin\left(\frac{2\pi}{6}\right) = 4 + 1.5 \sin\left(\frac{14\pi}{6}\right)$$

(c)

M1 For substituting $H=3$ into $H = 4 + 1.5 \sin\left(\frac{\pi t}{6}\right)$ WITH some attempt to make $\sin\left(\frac{\pi t}{6}\right)$ the subject.

You may see the $\left(\frac{\pi t}{6}\right)$ being replaced by another variable which is fine for the first two marks

A1 $\sin\left(\frac{\pi t}{6}\right) = -\frac{2}{3}$ oe Condone awrt $\sin\left(\frac{\pi t}{6}\right) = -0.67$

M1 For a correct attempt to find **one of the first two positive** values of t using **their** '-0.67'.

So for $\sin\left(\frac{\pi t}{6}\right) = -\frac{2}{3}$ score for either $t = \frac{6 \times \left(2\pi - \arcsin\left(\frac{2}{3}\right)\right)}{\pi}$ or $\frac{6 \times \left(\pi + \arcsin\left(\frac{2}{3}\right)\right)}{\pi}$

For $\sin\left(\frac{\pi t}{6}\right) = \frac{2}{3}$ score for either $t = \frac{6 \times \arcsin\left(\frac{2}{3}\right)}{\pi} = \text{awrt } 1.4$ or $\frac{6 \times \left(\pi - \arcsin\left(\frac{2}{3}\right)\right)}{\pi} = \text{awrt } 4.6$

A relatively common answer following $\sin\left(\frac{\pi t}{6}\right) = -\frac{2}{3}$ is $t = -1.39$. This scores M0 as it not one of the first two positive values for a rhs of $-\frac{2}{3}$.

A1 One of t awrt 7.4 or awrt 10.6 One of these values would probably imply the previous M mark Can be implied one correct time 7:23/7:24 am (0724) or 10:36/10:37 am (1036).

Can be implied by one correct number of minutes 443/444 minutes and 636/637 minutes

M1 For a (correct) attempt to find **both of the first two positive** values of t using **their** '-0.67'.

A1 Times are required for this mark only

Accept both 7:23/7:24 am and 10:36/10:37 am in 12 hour times

Allow 0723/ 0724 and 1036/1037 in 24 hour times.

Allow 07:23/ 07:24 and 10:36/1037 in 24 hour times

07.23/ 07.24 and 10.36/10.37 in 24 hour times. (The 0 must be present in 07.23/ 07.24)

Some candidates may choose to do part (c) using degrees. This is fine and the scheme can be applied as long as $\pi = 180^\circ$ and the candidates are consistently using degrees

M1 A1 $\sin(30t) = -\frac{2}{3}$

M1 Attempts to find one value of t (but the units must be consistent)

$$\sin(30t) = -\frac{2}{3} \Rightarrow 30t = 221.81 \text{ or } 318.19 \Rightarrow t =$$

