Write your name here

| Surname | Other names |
|---|---|
| | |

**Pearson Edexcel**
**International**
**Advanced Level**

Centre Number | | | | | |

Candidate Number | | | | |

# Core Mathematics C34

## Advanced

Tuesday 19 June 2018 – Afternoon
**Time: 2 hours 30 minutes**

Paper Reference
**WMA02/01**

**You must have:**
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

P50803A



1/1/1/1/

P 5 0 8 0 3 A 0 1 5 2

Pearson

Leave blank

**1.** (i) Find

$$\int \frac{2x^2 + 5x + 1}{x^2} \mathrm{d}x, \quad x > 0$$

**(3)**

(ii) Find

$$\int x \cos 2x \, \mathrm{d}x$$

**(3)**

Leave blank

**Question 1 continued**

**Q1**

**(Total 6 marks)**

Leave blank

**2.** A curve $C$ has parametric equations

$$x = \frac{3}{2}t - 5, \quad y = 4 - \frac{6}{t} \qquad t \neq 0$$

(a) Find the value of $\frac{dy}{dx}$ at $t = 3$, giving your answer as a fraction in its simplest form.

**(3)**

(b) Show that a cartesian equation of $C$ can be expressed in the form

$$y = \frac{ax + b}{x + 5} \qquad x \neq k$$

where $a$, $b$ and $k$ are integers to be found.

**(4)**

Leave blank

**Question 2 continued**

Leave blank

**Question 2 continued**

Leave blank

**Question 2 continued**

**Q2**

**(Total 7 marks)**

Leave blank

**3.** $f(x) = 2^{x-1} - 4 + 1.5x \qquad x \in \mathbb{R}$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \frac{1}{3}(8 - 2^x)$$

**(2)**

The equation $f(x) = 0$ has a root $\alpha$, where $\alpha = 1.6$ to one decimal place.

(b) Starting with $x_0 = 1.6$, use the iteration formula

$$x_{n+1} = \frac{1}{3}(8 - 2^{x_n})$$

to calculate the values of $x_1$, $x_2$ and $x_3$, giving your answers to 3 decimal places.

**(3)**

(c) By choosing a suitable interval, prove that $\alpha = 1.633$ to 3 decimal places.

**(2)**

Leave blank

**Question 3 continued**

**Q3**

**(Total 7 marks)**

Leave blank

**4.** (a) Find the binomial expansion of

$$(1 + px)^{-4}, \quad |px| < 1$$

in ascending powers of $x$, up to and including the term in $x^3$, giving each coefficient as simply as possible in terms of the constant $p$.

**(3)**

$$f(x) = \frac{3 + 4x}{(1 + px)^4} \qquad |px| < 1$$

where $p$ is a positive constant.

In the series expansion of $f(x)$, the coefficient of $x^2$ is twice the coefficient of $x$.

(b) Find the value of $p$.

**(5)**

(c) Hence find the coefficient of $x^3$ in the series expansion of $f(x)$, giving your answer as a simplified fraction.

**(2)**

Leave blank

**Question 4 continued**

Leave blank

**Question 4 continued**

Leave blank

**Question 4 continued**

**Q4**

**(Total 10 marks)**

Leave blank

**5.** (i) The functions f and g are defined by

$$f : x \to e^{2x} - 5, \qquad x \in \mathbb{R}$$

$$g : x \to \ln(3x - 1), \qquad x \in \mathbb{R},\ x > \frac{1}{3}$$

(a) Find $f^{-1}$ and state its domain.

**(3)**

(b) Find fg(3), giving your answer in its simplest form.

**(2)**

(ii) (a) Sketch the graph with equation

$$y = |4x - a|$$

where $a$ is a positive constant. State the coordinates of each point where the graph cuts or meets the coordinate axes.

**(2)**

Given that

$$|4x - a| = 9a$$

where $a$ is a positive constant,

(b) find the possible values of

$$|x - 6a| + 3|x|$$

giving your answers, in terms of $a$, in their simplest form.

**(5)**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

**(Total 12 marks)**

**Q5**



Leave blank

**6.** (a) Express $\sqrt{5}\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

State the value of $R$ and give the value of $\alpha$ to 4 significant figures.

**(3)**

(b) Solve, for $-\pi < \theta < \pi$,

$$\sqrt{5}\cos\theta - 2\sin\theta = 0.5$$

giving your answers to 3 significant figures.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

**(4)**

$$f(x) = A\left(\sqrt{5}\cos\theta - 2\sin\theta\right) + B \qquad \theta \in \mathbb{R}$$

where $A$ and $B$ are constants.

Given that the range of f is

$$-15 \leqslant f(x) \leqslant 33$$

(c) find the value of $B$ and the possible values of $A$.

**(4)**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

**(Total 11 marks)**

**Q6**

Leave blank

**7.**

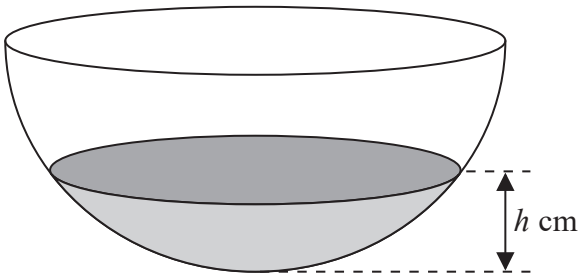


**Figure 1**

Figure 1 shows a hemispherical bowl.

Water is flowing into the bowl at a constant rate of $180\,\text{cm}^3\text{s}^{-1}$.

When the height of the water is $h$ cm, the volume of water $V\,\text{cm}^3$ is given by

$$V = \frac{1}{3}\pi h^2(90 - h), \qquad 0 \leqslant h \leqslant 30$$

Find the rate of change of the height of the water, in $\text{cm}\,\text{s}^{-1}$, when $h = 15$
Give your answer to 2 significant figures.

**(5)**

Leave blank

**Question 7 continued**

**Q7**

**(Total 5 marks)**



Leave blank

**8.** With respect to a fixed origin $O$, the lines $l_1$ and $l_2$ are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

where $\lambda$ and $\mu$ are scalar parameters.

(a) Show that $l_1$ and $l_2$ do not meet.

**(4)**

The point $P$ is on $l_1$ where $\lambda = 0$, and the point $Q$ is on $l_2$ where $\mu = -1$

(b) Find the acute angle between the line segment $PQ$ and $l_1$, giving your answer in degrees to 2 decimal places.

**(5)**

(c) Find the shortest distance from the point $Q$ to the line $l_1$, giving your answer to 3 significant figures.

**(2)**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

**(Total 11 marks)**

**Q8**

Leave blank

**9.**

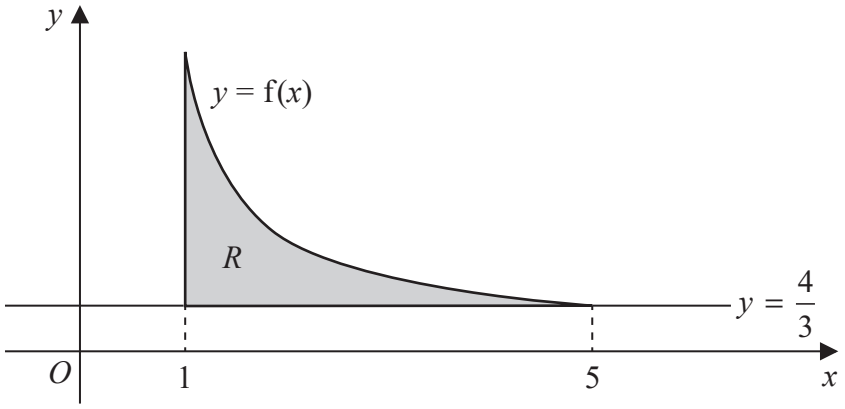


Diagram not drawn to scale

**Figure 2**

(a) Find

$$\int \frac{1}{(2x-1)^2}\,dx$$

**(2)**

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \frac{12}{(2x-1)} \qquad 1 \leqslant x \leqslant 5$$

The finite region $R$, shown shaded in Figure 2, is bounded by the line with equation $x = 1$, the curve with equation $y = f(x)$ and the line with equation $y = \frac{4}{3}$.

The region $R$ is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(b) Find the exact value of the volume of the solid generated, giving your answer in its simplest form.

**(6)**

Leave blank

**Question 9 continued**

Leave blank

**Question 9 continued**

Leave blank

**Question 9 continued**

**Q9**

**(Total 8 marks)**

Leave blank

**10.** The curve $C$ satisfies the equation

$$x\,e^{5-2y} - y = 0 \qquad x > 0, \quad y > 0$$

The point $P$ with coordinates $(2e^{-1}, 2)$ lies on $C$.

The tangent to $C$ at $P$ cuts the $x$-axis at the point $A$ and cuts the $y$-axis at the point $B$.

Given that $O$ is the origin, find the exact area of triangle $OAB$, giving your answer in its simplest form.

**(7)**

Leave blank

**Question 10 continued**

Leave blank

**Question 10 continued**

Leave blank

**Question 10 continued**

**Q10**

**(Total 7 marks)**

Leave blank

**11.**

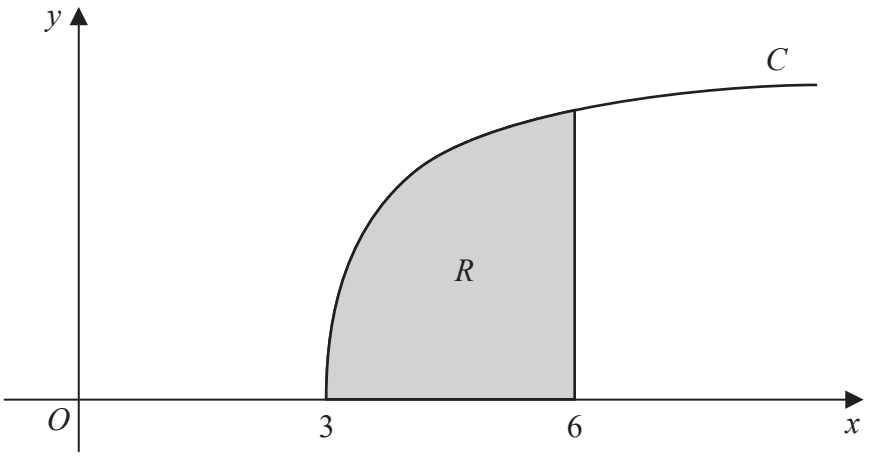


**Figure 3**

(a) By writing $\sec\theta$ as $\dfrac{1}{\cos\theta}$, show that when $x = 3\sec\theta$,

$$\frac{dx}{d\theta} = 3\sec\theta\tan\theta$$

**(2)**

Figure 3 shows a sketch of part of the curve $C$ with equation

$$y = \frac{\sqrt{x^2 - 9}}{x} \qquad x \geqslant 3$$

The finite region $R$, shown shaded in Figure 3, is bounded by the curve $C$, the $x$-axis and the line with equation $x = 6$

(b) Use the substitution $x = 3\sec\theta$ to find the exact value of the area of $R$.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

**(7)**

Leave blank

**Question 11 continued**

Leave blank

**Question 11 continued**

Leave blank

**Question 11 continued**

**Q11**

**(Total 9 marks)**

Leave blank

**12.** (a) Show that

$$\cot x - \tan x \equiv 2\cot 2x, \qquad x \neq 90n°,\ n \in \mathbb{Z}$$

**(4)**

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$

$$5 + \cot(\theta - 15°) - \tan(\theta - 15°) = 0$$

giving your answers to one decimal place.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

**(5)**

Leave blank

**Question 12 continued**

Leave blank

**Question 12 continued**

Leave blank

**Question 12 continued**

**Q12**

**(Total 9 marks)**

Leave blank

**13.** (a) Express $\dfrac{1}{(4-x)(2-x)}$ in partial fractions.

**(2)**

The mass, $x$ grams, of a substance at time $t$ seconds after a chemical reaction starts is modelled by the differential equation

$$\frac{dx}{dt} = k(4-x)(2-x), \qquad t \geqslant 0,\ 0 \leqslant x < 2$$

where $k$ is a constant.

Given that when $t = 0$, $x = 0$

(b) solve the differential equation and show that the solution can be written as

$$x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$$

**(7)**

Given that $k = 0.1$

(c) find the value of $t$ when $x = 1$, giving your answer, in seconds, to 3 significant figures.

**(2)**

Leave blank

**Question 13 continued**

Leave blank

**Question 13 continued**

Leave blank

**Question 13 continued**

**Q13**

**(Total 11 marks)**

Leave blank

**14.** Given that

$$y = \frac{(x^2 - 4)^{\frac{1}{2}}}{x^3} \qquad x > 2$$

(a) show that

$$\frac{dy}{dx} = \frac{Ax^2 + 12}{x^4(x^2 - 4)^{\frac{1}{2}}} \qquad x > 2$$

where $A$ is a constant to be found.

**(6)**

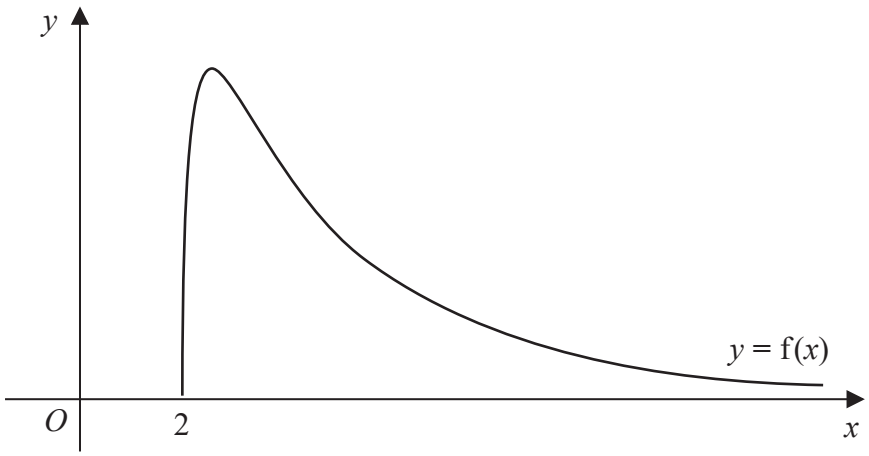


**Figure 4**

Figure 4 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = \frac{24(x^2 - 4)^{\frac{1}{2}}}{x^3} \qquad x > 2$$

(b) Use your answer to part (a) to find the range of f.

**(5)**

(c) State a reason why $f^{-1}$ does not exist.

**(1)**

Leave blank

**Question 14 continued**

Leave blank

**Question 14 continued**

Leave blank

**Question 14 continued**

Leave blank

**Question 14 continued**

**Q14**

**(Total 12 marks)**

**TOTAL FOR PAPER: 125 MARKS**

**END**