Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Core Mathematics C34

Advanced

Tuesday 20 June 2017 – Afternoon
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

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1. A curve C has equation

$$3x^2 + 2xy - 2y^2 + 4 = 0$$

Find an equation for the tangent to C at the point (2, 4), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

Question Number	Scheme	Marks
1	$3x^2+2xy-2y^2+4=0\Rightarrow 6x+\overline{\overline{2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y}}\underline{-4y\frac{\mathrm{d}y}{\mathrm{d}x}}=0$	$\overline{\overline{\text{B1}}}$ $\underline{\text{M1}}$ A1
	Sets $x=2, y=4\Rightarrow 12+4\frac{\mathrm{d}y}{\mathrm{d}x}+8-16\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{3}$	M1
	Uses $x=2, y=4$ and their $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{3}\Rightarrow(y-4)=\frac{5}{3}(x-2)$	M1
	$5x-3y+2=0$	A1
		(6 marks)

B1: $2xy$ differentiated correctly to give $2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$ or any equivalent correct expression.

M1: Attempts to apply the chain rule to $-2y^2$ to give an expression of the form $Ay\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: Fully correct differentiation of $3x^2-2y^2+4$ to give $6x-4y\frac{\mathrm{d}y}{\mathrm{d}x}$ and "= 0" which may be implied by subsequent work. "= 0" may also be implied if the candidate rearranges the given equation first.
Allow the candidate to start with $\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$ for all the above marks but if **this** $\frac{\mathrm{d}y}{\mathrm{d}x}$ is used to find the gradient, the next mark would be withheld as the two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms must come from the $2xy$ and $2y^2$ terms – see below.

Note: If $6x\mathrm{d}x+2x\mathrm{d}y+2y\mathrm{d}x-4y\mathrm{d}y=0\Rightarrow 6x+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y-4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$ is seen, score B1 for $2x\mathrm{d}y+2y\mathrm{d}x$ and M1 for $6x\mathrm{d}x-4y\mathrm{d}y=0$ then A1 for $6x+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y-4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$

M1: Substitutes $x=2$ and $y=4$ and attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ (this may be implied e.g. they may rearrange their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find $-\frac{\mathrm{d}x}{\mathrm{d}y}$ and then substitute). This is not formally dependent on the first M but is dependent upon them having two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms in their derivative. One coming from $2xy$ and one coming from $2y^2$.

M1: Uses $x=2$ and $y=4$ and their numerical value of $\frac{\mathrm{d}y}{\mathrm{d}x}\left(=\frac{20}{12}=\frac{5}{3}\right)$ to find an equation of a <u>tangent</u> (not a normal). If $y = mx + c$ is used they much reach as far as finding a value for c.
A1: Accept $5x-3y+2=0$ or any integer multiple of this equation.

2. Use integration by parts to find the exact value of $\int_1^{e} \frac{\ln x}{x^2}\,dx$

Write your answer in the form $a + \frac{b}{e}$, where a and b are integers.

(6)

Question Number	Scheme	Marks
2	$\int \frac{\ln x}{x^2}\mathrm{d}x = \int x^{-2}\ln x\,\mathrm{d}x = \frac{x^{-1}}{-1}\ln x - \int \frac{x^{-1}}{-1}\times\frac{1}{x}\,\mathrm{d}x$	M1A1
	$= \frac{x^{-1}}{-1}\ln x + \int x^{-2}\,\mathrm{d}x$	
	$= \frac{x^{-1}}{-1}\ln x + \frac{x^{-1}}{-1}(+c)$	M1A1
	$\int_1^{\mathrm{e}} \frac{\ln x}{x^2}\mathrm{d}x = \left[\frac{-1}{x}\ln x - \frac{1}{x}\right]_1^{\mathrm{e}} = \left(\frac{-1}{\mathrm{e}}\ln \mathrm{e} - \frac{1}{\mathrm{e}}\right) - \left(\frac{-1}{1}\ln 1 - \frac{1}{1}\right)$	M1
	$= 1 - \frac{2}{\mathrm{e}}$	A1
		(6)
	Alternative by substitution:	
	$u = \ln x \Rightarrow \int \frac{\ln x}{x^2}\mathrm{d}x = \int \frac{u}{\mathrm{e}^{2u}}\mathrm{e}^{u}\,\mathrm{d}u = \int u\mathrm{e}^{-u}\,\mathrm{d}u$	
	$\int u\mathrm{e}^{-u}\,\mathrm{d}u = -u\mathrm{e}^{-u} - \int -\mathrm{e}^{-u}\,\mathrm{d}u$	M1A1
	$\int u\mathrm{e}^{-u}\,\mathrm{d}u = -u\mathrm{e}^{-u} - \mathrm{e}^{-u}\,(+c)$	M1A1
	$\int_1^{\mathrm{e}} \frac{\ln x}{x^2}\mathrm{d}x = \left[-u\mathrm{e}^{-u} - \mathrm{e}^{-u}\right]_0^1 = \left(-\frac{1}{\mathrm{e}} - \frac{1}{\mathrm{e}}\right) - (0-1)$	M1
	$= 1 - \frac{2}{\mathrm{e}}$	A1

(Condone the lack of "dx" throughout)

M1: An application of integration by parts the right way around.
If the rule is quoted it must be correct. (A version appears in the formula booklet)

Must see an expression of the form $Ax^{-1}\ln x \pm B\int x^{-1}\times\frac{1}{x}\,\mathrm{d}x$ **for this mark**

A1: A correct un-simplified (or simplified) expression e.g. $\frac{x^{-1}}{-1}\ln x - \int \frac{x^{-1}}{-1}\times\frac{1}{x}\,\mathrm{d}x$, $\left[-\frac{1}{x}\ln x\right]_1^{\mathrm{e}} + \int \frac{1}{x^2}\,\mathrm{d}x$

M1: It is for 'combining' their two terms in x correctly and integrating their resulting term by adding one to the power.

A1: A completely correct integral (simplified or un-simplified)

For students who substitute in limits early, look for e.g. $\left(\frac{\mathrm{e}^{-1}}{-1}\ln \mathrm{e}\right) - \left(\frac{1^{-1}}{-1}\ln 1\right) + \left[\frac{x^{-1}}{-1}\right]_1^{\mathrm{e}}$

M1: It is for substituting in the limits 1 and e (either way round) and subtracting.

A1: cso and cao for $1-\frac{2}{e}$ or $-\frac{2}{e}+1$. Allow e^1 for e. Accept $1+\frac{-2}{e}$.

Alt 1

M1: An application of integration by parts the right way around.
If the rule is quoted it must be correct. (A version appears in the formula booklet)
Must see an expression of the form $\int ue^{-u}\,\mathrm{d}u = Aue^{-u} \pm \int e^{-u}\,\mathrm{d}u$ **for this mark**

A1: A correct un-simplified (or simplified) expression e.g. $\int ue^{-u}\,\mathrm{d}u = -ue^{-u} - \int -e^{-u}\,\mathrm{d}u$

M1: It is for $\int e^{-u}\,\mathrm{d}u \to e^{-u}$

A1: A completely correct integral (simplified or un-simplified)

M1: It is for substituting in the limits 0 and 1 (either way round) and subtracting.

A1: A1: cso and cao for $1-\frac{2}{e}$ or $-\frac{2}{e}+1$. Allow e^1 for e. Accept $1+\frac{-2}{e}$.

3. The function g is defined by

$$g(x) = \frac{6x}{2x + 3} \qquad x > 0$$

(a) Find the range of g.
(2)

(b) Find $g^{-1}(x)$ and state its domain.
(3)

(c) Find the function gg(x), writing your answer as a single fraction in its simplest form.
(3)

Question Number	Scheme	Marks
3 (a)	$0<\text{g}<3$	M1A1
		(2)
(b)	$y=\dfrac{6x}{2x+3}\Rightarrow 2xy+3y=6x\Rightarrow(6-2y)x=3y\Rightarrow x=\dfrac{3y}{(6-2y)}$	M1A1
	$\Rightarrow \text{g}^{-1}\left(x\right)=\dfrac{3x}{(6-2x)}\qquad 0<x<3$	A1ft
		(3)
(c)	$\text{gg}(x)=\text{g}\left(\dfrac{6x}{2x+3}\right)=\dfrac{6\times\dfrac{6x}{2x+3}}{2\times\dfrac{6x}{2x+3}+3}$	M1
	$=\dfrac{6\times 6x}{2\times 6x+3(2x+3)}$	dM1
	$=\dfrac{36x}{18x+9}=\dfrac{4x}{2x+1}$	A1
		(3)
		(8 marks)

(a)

M1: For one 'end' fully correct $\text{g}(x)>0$ (**not** $x > 0$) or $\text{g}(x)<3$ (**not** $x < 3$) or both ends (incorrect) eg. accept $0\leqslant\text{g}\leqslant 3$. Accept incorrect notation such as $0<x<3$ for this mark but **not** $x > 0$ or $x < 3$ **on their own**.

Allow use of f rather than g for the M mark but not the A mark.

A1: Accept $0<\text{g}<3$, $0<y<3$, $\text{g}(x)>0$ and $\text{g}(x)<3$, $\left(0,3\right)$

(b)

M1: An attempt to make x or a replaced y the subject of the formula. The minimum expectation is that there is an attempt to cross multiply, expand and collect/factorise terms in x or a replaced y and obtain $x=\dfrac{\pm 3y}{(\pm 6\pm 2y)}$ or equivalent i.e. sign errors only on their algebra.

A1: $x=\dfrac{3y}{(6-2y)}$ or $\dfrac{-3y}{(2y-6)}$ or $y=\dfrac{3x}{(6-2x)}$ or $\dfrac{-3x}{(2x-6)}$ or $-\dfrac{3}{2}-\dfrac{9}{2(x-3)}$ etc. Allow $2(x-3)$ for $(2x-6)$.

A1ft: $\text{g}^{-1}\left(x\right)=\dfrac{3x}{(6-2x)}\left(\text{or}\dfrac{-3x}{(2x-6)}\right)$ **and** $0<x<3$. You can follow through on any range from part (a) but the domain must be in terms of x not in terms of e.g. g(x) or g^{-1}(x). Do not allow $x\in\mathbb{R}$

Accept $y=\dfrac{3x}{(6-2x)}\left(\text{or}\dfrac{-3x}{(2x-6)}\right)\qquad 0<x<3$. Allow $2(x-3)$ for $(2x-6)$.

(c)

M1: Attempts to find $\text{gg}(x)$ by finding $\text{g}\left(\dfrac{6x}{2x+3}\right)$

dM1: Correct processing to obtain a single fraction of the form $\dfrac{a}{b}$. Achieved by,

- multiplying both numerator and denominator by $(2x+3)$ (must multiply both terms in the denominator)

- attempting to write the denominator as a single fraction followed by the multiplication of the numerator by an inverted denominator to obtain a single fraction of the form $\frac{a}{b}$
- attempting to write the denominator as a single fraction followed the cancellation of the same denominators

A1: $\frac{4x}{2x+1}$ cao. Ignore the presence or absence of a domain and isw once the correct answer is seen.

4.

$$f(x) = \frac{27}{(3-5x)^2} \qquad |x| < \frac{3}{5}$$

(a) Find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(5)

Use your answer to part (a) to find the series expansion in ascending powers of x, up to and including the term in x^3, of

(b) $g(x) = \frac{27}{(3+5x)^2} \qquad |x| < \frac{3}{5}$

(1)

(c) $h(x) = \frac{27}{(3-x)^2} \qquad |x| < 3$

(2)

Question Number	Scheme	Marks
4(a)	$27(3-5x)^{-2}=27\times\frac{1}{9}\left(1-\frac{5}{3}x\right)^{-2}$	B1, B1
	$=3\left(1+(-2)\left(-\frac{5}{3}x\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5}{3}x\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(-\frac{5}{3}x\right)^3+...\right)$	M1
	$=3+10x+25x^2+\frac{500}{9}x^3+...$	A1, A1
		(5)
(b)	$27(3+5x)^{-2}=27(3+5x)^{-2}$	
	$=3-10x+25x^2-\frac{500}{9}x^3+...$	B1ft
		(1)
(c)	$27(3-x)^{-2}=3+\frac{10}{5}x+\frac{25}{5^2}x^2+\frac{500}{9\times5^3}x^3$	M1
	$=3+2x+x^2+\frac{4}{9}x^3$	A1
		(2)
		(8 marks)
4(a) alt	$27(3-5x)^{-2}=27\left(\begin{array}{l}3^{-2}+(-2)\times3^{-3}\times(-5x)+\frac{(-2)(-3)}{2}\times3^{-4}\times(-5x)^2\\+\frac{(-2)(-3)(-4)}{3!}(3)^{-5}(-5x)^3\end{array}\right)$ $=27\left(\frac{1}{9}+\frac{10x}{27}+\frac{25x^2}{27}+\frac{500x^3}{243}+..\right)$ $=3+10x+25x^2+\frac{500}{9}x^3+...$	B1 B1 M1 A1 A1

(a)

B1: Writes down $(3-5x)^{-2}$ or uses a power of -2

B1: Takes out a factor of 3^{-2} which can be implied by $\frac{1}{9}$ or $3\times(....)$ or a first term of 3

M1: Expands $(1+kx)^{-2}$, $k\neq\pm1$ with the structure for at least 2 terms correct (not including the "1"), from $\left(1+(-2)kx+\frac{(-2)(-3)}{2}(kx)^2+\frac{(-2)(-3)(-4)}{3!}(kx)^3+...\right)$ with or without the bracket around the kx

A1: Two of the four terms correct and simplified **but the method mark must have been awarded!**

A1: Fully correct simplified expansion $3+10x+25x^2+\frac{500}{9}x^3+...$all on one line but isw once a correct expansion is seen.

Alternative for (a):

B1: Writes down $(3-5x)^{-2}$ or uses a power of -2

B1: For a first term of 3^{-2} in the bracket

M1: For correct structure for two of $(-2)\times 3^{-3}\times(-5x)+\frac{(-2)(-3)}{2}\times 3^{-4}\times(-5x)^2+\frac{(-2)(-3)(-4)}{3!}(3)^{-5}(-5x)^3$

A1A1: As defined in main scheme

Allow a re-start in (b) and (c):
(b)

B1ft: The correct answer $3-10x+25x^2-\frac{500}{9}x^3+...$ or if (a) was incorrect, follow through on their (a) i.e.

$A+Bx+Cx^2+Dx^3\to A-Bx+Cx^2-Dx^3$. **There must be 4 non-zero terms.** Allow follow through on an un-simplified or "un-expanded" part (a).

(c)

M1: An attempt to divide the coefficient of *x* by 5, the coefficient of x^2 by 5^2 and the coefficient of x^3 by 5^3, seen in at least two cases on an expansion consisting of at least 3 terms.

A1: The correct answer $3+2x+x^2+\frac{4}{9}x^3$

Or:

M1: Expands $(1+kx)^{-2}$, $k\neq\pm1$ with the structure for at least 2 terms correct (not including the "1"), from $\left(1+(-2)kx+\frac{(-2)(-3)}{2}(kx)^2+\frac{(-2)(-3)(-4)}{3!}(kx)^3+...\right)$ with or without the bracket around the kx

NB *k* should be $-\frac{1}{3}$

A1: The correct answer $3+2x+x^2+\frac{4}{9}x^3$

If (c) is attempted using the alternative binomial method, send to review.

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5.

$$\frac{6-5x-4x^2}{(2-x)(1+2x)} \equiv A + \frac{B}{(2-x)} + \frac{C}{(1+2x)}$$

(a) Find the values of the constants *A*, *B* and *C*.

(4)

$$f(x) = \frac{6-5x-4x^2}{(2-x)(1+2x)} \qquad x > 2$$

(b) Using part (a), find $f'(x)$.

(3)

(c) Prove that $f(x)$ is a decreasing function.

(1)

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Question Number	Scheme	Marks
5(a)	$\frac{6-5x-4x^2}{(2-x)(1+2x)} = A + \frac{B}{(2-x)} + \frac{C}{(1+2x)}$	
	$6-5x-4x^2 = A(2-x)(1+2x)+B(1+2x)+C(2-x)$	M1
	Coefficients of $x^2 \Rightarrow A=2$	B1
	Sub $x=2 \Rightarrow -20=5B \Rightarrow B=-4$, $x=-\frac{1}{2} \Rightarrow 7.5=2.5C \Rightarrow C=3$	dM1A1
		(4)
(b)	$\text{f}(x)=2-\frac{4}{(2-x)}+\frac{3}{(1+2x)} \Rightarrow \text{f}'(x) = -\frac{4}{(2-x)^2}-\frac{6}{(1+2x)^2}$	M1A1ftA1
		(3)
(c)	As $(2-x)^2>0$ and $(1+2x)^2>0 \Rightarrow \text{f}'(x)<0$	B1
		(1)
		(8 marks)
5 (a) alt	$\begin{array}{r} 2 \\ -2x^2+3x+2\overline{\smash{)}-4x^2-5x+6} \\ \underline{-4x^2+6x+4} \\ -11x+2 \end{array}$ $\frac{-11x+2}{(2-x)(1+2x)} = \frac{B}{(2-x)}+\frac{C}{(1+2x)}$	B1 M1
	$-11x+2 = B(1+2x)+C(2-x)$ Sub $x=2 \Rightarrow -20=5B \Rightarrow B=-4$, $x=-\frac{1}{2} \Rightarrow 7.5=2.5C \Rightarrow C=3$	dM1 A1 **(4)**

(a)

M1: Writes $6-5x-4x^2 = A(2-x)(1+2x)+B(1+2x)+C(2-x)$ and makes an attempt to find any constant.

In the alternative it is for dividing first to obtain a quotient of ±2 and a remainder *px* + *q*, *p*, *q* ≠ 0 and then writing the remainder in the form $\frac{B}{(2-x)}+\frac{C}{(1+2x)}$

B1: For the $2+....$ OR $A=2$ or quotient 2

dM1: Dependent upon previous M. It is for a correct method of finding any of the constants *B* or *C* by either substitution or correctly equating coefficients and solving simultaneous equations. In the alternative, it is for writing $px+q=B(1+2x)+C(2-x)$ and attempting to find any of the constants *B* or *C* by either substitution or equating coefficients and solving simultaneous equations

A1: For the $\frac{-4}{(2-x)}+\frac{3}{(1+2x)}$ or the values of the constants stated correctly.

(b)

M1: For an application of the chain rule. Award for $\frac{\cdot\cdot}{(2-x)} \to \frac{\cdot\cdot}{(2-x)^2}\ OR\ \frac{\cdot\cdot}{(1+2x)} \to \frac{\cdot\cdot}{(1+2x)^2}$ where … is a constant.

A1ft: One of the two terms correct or one of the two terms correct following through from their constants i.e.
$\frac{B}{(2-x)} \to \frac{B}{(2-x)^2}$ OR $\frac{C}{(1+2x)} \to \frac{-2C}{(1+2x)^2}$

A1: Fully correct $\mathrm{f}'(x) = -\frac{4}{(2-x)^2} - \frac{6}{(1+2x)^2}$. **Allow full marks in (b) even if *A* is incorrect in (a).**

(c)

B1: This mark depends on a fully correct derivative in (b) and a minimum of, squares are always positive, hence $\mathrm{f}'(x) < 0$

Attempts at part (c) based on f(*x*) alone should be sent to review.

Special case: If the candidate goes back to the original function for parts (b) and (c)
(b) score for use of the quotient rule.

M1: Look for $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{6-5x-4x^2}{(2-x)(1+2x)}\right) = \frac{(2-x)(1+2x)(P+Qx)-(6-5x-4x^2)(L+Mx)}{((2-x)(1+2x))^2}$

A1 A1: $\frac{(2-x)(1+2x)(-5-8x)-(6-5x-4x^2)(3-4x)}{((2-x)(1+2x))^2}$

(c) It is very unlikely to be correct from the quotient rule.

It would require f '(*x*) to be put in a form $\mathrm{f}'(x) = \frac{-28\left(\left(x-\frac{4}{7}\right)^2 + \frac{54}{49}\right)}{((2-x)(1+2x))^2}$

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6. The line l_1 has vector equation $\mathbf{r} = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} + \lambda\begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has vector equation $\mathbf{r} = \begin{pmatrix} 10 \\ 5 \\ -3 \end{pmatrix} + \mu\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Justify, giving reasons in each case, whether the lines l_1 and l_2 are parallel, intersecting or skew.

(6)

Question Number	Scheme	Marks
6.	Not parallel as $\begin{pmatrix}6\\3\\-1\end{pmatrix}$ is not equal (or a multiple of) $\begin{pmatrix}3\\1\\2\end{pmatrix}$	B1
	Sets $\begin{pmatrix}5\\-2\\4\end{pmatrix}+\lambda\begin{pmatrix}6\\3\\-1\end{pmatrix}=\begin{pmatrix}10\\5\\-3\end{pmatrix}+\mu\begin{pmatrix}3\\1\\2\end{pmatrix}\Rightarrow\begin{matrix}5+6\lambda=10+3\mu\\-2+3\lambda=5+1\mu\\4-1\lambda=-3+2\mu\end{matrix}$ Two of three	M1
	Full method to solve any two Eg (2) and (3) $\mu=2,\lambda=3$	M1, A1
	Sub into both sides of the other eqn. Eg (1) $5+6\times3$ **and** $10+3\times2$	M1
	$23\neq16$ and states that as they are not equal or do not intersect (or lines are SKEW)	A1 cso
		(6)
		(6 marks)

B1: States that lines are not parallel with a valid reason e.g.

- The direction vectors $\begin{pmatrix}6\\3\\-1\end{pmatrix}$ and $\begin{pmatrix}3\\1\\2\end{pmatrix}$ are not equal (or multiples of each other)
- $6:3:-1\neq3:1:2$
- $\sqrt{6^2+3^2+(-1)^2}\sqrt{3^2+1^2+2^2}\neq6\times3+3\times1+2\times(-1)$ or shows $\cos\theta\neq1$ (Note $\cos\theta=0.7489$ and $\theta=41.5...^\circ$)
- $6\div3=2$ and $3\div1=3$

M1: Equates the lines. Evidence will be two of the three equations. Allow slips provided the intention to equate components is clear.

M1: Full method to solve two of the three equations to obtain values for λ *and* μ or numerical expressions for λ *and* μ

A1: Correct values of either λ *or* μ for their two equations.

Note (1) and (2) give $\lambda=\frac{16}{3}$ ($awrt\,5.3$) and $\mu=9(awrt\,9.0)$,

(1) and (3) give $\lambda=\frac{31}{15}$ ($awrt\,2.1$) and $\mu=\frac{37}{15}$ ($awrt\,2.5$)

M1: Substitutes both values into the third equation. Alternatively uses the value of one variable, expresses the other variable in terms of this and substitutes both into the third equation.

A1: Requires all values correct (and exact if necessary) and a statement that the lines do not intersect (or are skew). The substitution into their 3rd equation does need to be made but not fully evaluated if the results are clearly not equal. There is no need for candidates to use the word SKEW – not intersecting (or equivalent) is sufficient.

Note that a score of 011111 is possible.

Leave blank

7. (a) Prove that

$$\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x, \qquad x \neq (2n+1)90°,\ n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve, for $-90° < \theta < 90°$,

$$\frac{2-2\cos 2\theta}{1+\cos 2\theta} - 2 = 7\sec\theta$$

Give your answers in degrees to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Question Number	Scheme	Marks
	Examples:	
7(a)	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\left(1-2\sin^2 x\right)}{1+\left(2\cos^2 x-1\right)}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\left(1-2\sin^2 x\right)}{1+\left(2\cos^2 x-1\right)}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M0dM0A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}$ $=\frac{\cos^2 x+\sin^2 x-\cos^2 x+\sin^2 x}{2\cos^2 x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
		(3)
(b)	$\frac{2-2\cos 2\theta}{1+\cos 2\theta}-2=7\sec\theta$	
	$2\left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right)-2=7\sec\theta \Rightarrow 2\tan^2\theta-2=7\sec\theta$	M1
	$\Rightarrow 2\left(\sec^2\theta-1\right)-2=7\sec\theta$	M1
	$\Rightarrow 2\sec^2\theta-7\sec\theta-4=0$	A1
	$\Rightarrow (2\sec\theta+1)(\sec\theta-4)=0$	
	$\Rightarrow \sec\theta=-\frac{1}{2},4$	
	$\Rightarrow \cos\theta=-2,\frac{1}{4}\Rightarrow\theta=...$	M1
	$\Rightarrow \theta=75.5°,-75.5°$	A1,A1
		(6)
		(9 marks)
7(a) alt1	$\tan^2 x=\frac{\sin^2 x}{\cos^2 x}=\frac{\frac{1}{2}\left(1-\cos 2x\right)}{\frac{1}{2}(1+\cos 2x)}=\frac{\left(1-\cos 2x\right)}{(1+\cos 2x)}$	M1dM1A1
7(a) alt2	$\frac{1-\cos 2x}{1+\cos 2x}=\tan^2 x\Rightarrow 1-\cos 2x=\tan^2 x\left(1+\cos 2x\right)$ $1-\left(1-2\sin^2 x\right)=\tan^2 x\left(1+2\cos^2 x-1\right)$ $2\sin^2 x=\frac{\sin^2 x}{\cos^2 x}\left(2\cos^2 x\right)$ $2\sin^2 x=2\sin^2 x$	M1dM1A1

Question Number	Scheme	Marks
	Examples:	
	Which is true*	

7 (b)alt1	$2\left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right)-2=7\sec\theta \Rightarrow 2\tan^2\theta-2=7\sec\theta$	M1
	$\Rightarrow 2\frac{\sin^2\theta}{\cos^2\theta}-2=\frac{7}{\cos\theta}$ $\Rightarrow 2\sin^2\theta-2\cos^2\theta=7\cos\theta$ $\Rightarrow 2(1-\cos^2\theta)-2\cos^2\theta=7\cos\theta$	M1
	$\Rightarrow 4\cos^2\theta+7\cos\theta-2=0$	A1
	$\Rightarrow (4\cos\theta-1)(\cos\theta+2)=0$	
	$\Rightarrow \cos\theta=-2,\frac{1}{4}\Rightarrow\theta=...$	M1
	$\Rightarrow \theta=75.5°,-75.5°$	A1A1
		(6)
7 (b)alt2	$2\left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right)-2=7\sec\theta \Rightarrow 2\tan^2\theta-2=7\sec\theta$	M1
	$2\tan^2\theta-2=7\sqrt{1+\tan^2\theta}$ $\left(2\tan^2\theta-2\right)^2=\left(7\sqrt{1+\tan^2\theta}\right)^2\Rightarrow 4\tan^4\theta-8\tan^2\theta+4=49\left(1+\tan^2\theta\right)$	M1
	$4\tan^4\theta-57\tan^2\theta-45=0$	A1
	$\left(4\tan^2\theta+3\right)\left(\tan^2\theta-15\right)=0\Rightarrow\tan^2\theta=15$	
	$\tan\theta=\sqrt{15}\Rightarrow\theta=...$	M1
	$\Rightarrow \theta=75.5°,-75.5°$	A1A1
		(6)

(a)
M1: Uses a correct double angle identity on the numerator or denominator **and applies this to the fraction**.
dM1: Uses correct double angle identities in the numerator and denominator leading to an expression of the form $\frac{a\sin^2 x}{a\cos^2 x}$
A1*: Completely correct solution. The variables must be consistent and do not accept expressions of the form '$\frac{\sin^2}{\cos^2}=\tan^2$' within the proof. If their working necessitates the appearance of the 2’s in the numerator and denominator and they are not shown, this mark can be withheld – see examples.
(a) Alt1:
M1: Uses the identity $\tan^2 x=\frac{\sin^2 x}{\cos^2 x}$
dM1: Uses any two correct double angle identities.

A1*: Completely correct solution. The variables must be consistent and do not accept expressions of the form '$\frac{\sin^2}{\cos^2} = \tan^2$' within the proof.

(a) Alt 2:

M1: Multiplies both sides by the denominator of the lhs and uses any two correct double angle identities
dM1: Uses any two correct double angle identities.
A1: Obtains a correct identity and makes a conclusion.
See main scheme for some other varieties and the marks to award

(b) **Inc. Alt 1**

M1: Obtains an equation of the form $A\tan^2\theta - 2 = 7\sec\theta$ or $A\tan^2\theta - 2 = \frac{7}{\cos\theta}$

M1: Attempts to use the trig identity $\tan^2\theta = \pm\sec^2\theta \pm 1$ to produce a quadratic equation in $\sec\theta$ or attempts to use $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$ and $\sin^2\theta = \pm 1 \pm \cos^2\theta$ to produce a quadratic in $\cos\theta$.

A1: Correct 3TQ = 0. Either $2\sec^2\theta - 7\sec\theta - 4 = 0$ or $4\cos^2\theta + 7\cos\theta - 2 = 0$ or equivalent

M1: Correct method of solving 3TQ = 0 in either $\sec\theta$ or $\cos\theta$ **AND** using arccos in producing at least one answer for θ. You may need to check the roots of their quadratic if no working is seen and if the roots are incorrect and no working is shown, score M0.

A1: One of awrt $\theta = 75.5°, –75.5°$

A1: Both of awrt $\theta = 75.5°, –75.5°$

In an otherwise correct solution, deduct the final mark for extra answers in range. Ignore answers outside the range.

(b) **Alt 2**

M1: Obtains an equation of the form $A\tan^2\theta - 2 = 7\sec\theta$ or $A\tan^2\theta - 2 = \frac{7}{\cos\theta}$

M1: Attempts to use the trig identity $\tan^2\theta = \pm\sec^2\theta \pm 1$ and squares to produce a quadratic equation in $\tan^2\theta$

A1: Correct 3TQ = 0. $4\tan^4\theta - 57\tan^2\theta - 45 = 0$ or equivalent

M1: Correct method of solving 3TQ = 0 **AND** using arctan after square root in producing at least one answer for θ. You may need to check the roots of their quadratic if no working is seen and if the roots are incorrect and no working is shown, score M0.

A1: One of awrt $\theta = 75.5°, –75.5°$

A1: Both of awrt $\theta = 75.5°, –75.5°$

For answers in radians (awrt 1.3, -1.3) deduct the final A mark.

In an otherwise correct solution, deduct the final mark for extra answers in range. Ignore answers outside the range.

Part (b) Note:

If the quadratic (in sec or cos) is incorrect but fortuitously leads to the correct answers e.g. from factors of $(\sec\theta - 4)$ or $(4\cos\theta - 1)$ then the final A mark can be withheld.

If the quadratic (in sec or cos) is correct but in their factorisation the $(\sec\theta - 4)$ or $(4\cos\theta - 1)$ is correct and the other factor incorrect then the final A mark can be withheld if they proceed to obtain the correct angles.

M1: Obtains an equation of the form $A\tan^2\theta - 2 = 7\sec\theta$ or $A\tan^2\theta - 2 = \frac{7}{\cos\theta}$ (or a method using identities (allow sign errors) to obtain an equation in terms of single angles)
M1: Uses identities (allow sign errors) to produce an equation in terms of a single trig. function.
A1: Correct equation
M1: Solves to obtain at least one value
A1: One of awrt $\theta = 75.5°, –75.5°$
A1: Both of awrt $\theta = 75.5°, –75.5°$

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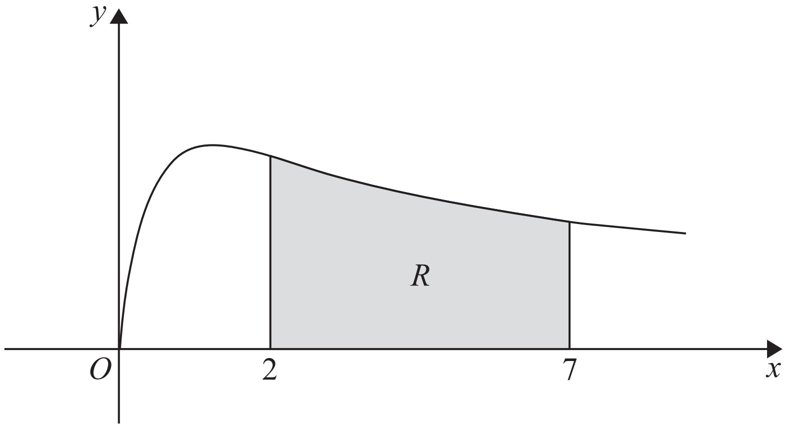

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{\dfrac{x}{x^2+1}}, \quad x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 7$

The table below shows corresponding values of x and y for $y = \sqrt{\dfrac{x}{x^2+1}}$

x	2	3	4	5	6	7
y	0.6325	0.5477	0.4851	0.4385	0.4027	0.3742

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for the area of R, giving your answer to 3 decimal places.

(3)

The region R is rotated 360° about the x-axis to form a solid of revolution.

(b) Use calculus to find the exact volume of the solid of revolution formed. Write your answer in its simplest form.

(4)

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Question Number	Scheme	Marks
8 (a)	Strip Width =1	B1
	$\text{Area} \approx \frac{1}{2}(0.6325+0.3742+2\times(0.5477+0.4851+0.4385+0.4027))$ $\left(=\frac{1}{2}\times 4.7547\right)$	M1
	Awrt =2.377	A1
		(3)
(b)	$\text{Volume} = (\pi)\int_2^7 \frac{x}{x^2+1}\text{d}x = (\pi)\left[\frac{1}{2}\ln\left(x^2+1\right)\right]_2^7$	M1A1
	$=\frac{(\pi)}{2}(\ln 50 - \ln 5)$	dM1
	$=\frac{\pi}{2}\ln 10$	A1
		(4)
		(7 marks)

(a)

B1: Strip width = 1 which may be implied by the $\frac{1}{2}$ in the trapezium rule

M1: For a correct attempt at using the trapezium rule.

Look for $\frac{1}{2}h((y \text{ at } x=2)+(y \text{ at } x=7)+2(\text{sum of other } y \text{ values}))$. Must be correct with no missing values and no extra values. (May be implied by a correct answer)

A1: Awrt = 2.377

Note: h = 5/6 gives Area 1.981125 and h = 5 gives 11.88675 and will probably just score the M1

Note that $\frac{1}{2}\times 1\times 0.6325+0.3742+2\times(0.5477+0.4851+0.4385+0.4027)$ scores B1 only unless the missing brackets are implied by a correct answer.

(b)

M1: Attempts to find $C\int\frac{x}{x^2+1}\text{d}x$ to give an expression of the form $D\left[\ln k\left(x^2+1\right)\right]$

A1: $\text{Volume} = (\pi)\int\frac{x}{x^2+1}\text{d}x = \frac{(\pi)}{2}\left[\ln\left(x^2+1\right)\right]$. Correct expression **<u>with or without</u>** π. **Ignore any limits.**

Do not allow the brackets around the $x^2 + 1$ to be missing unless their presence is implied by later work.

dM1: Dependent upon previous M. It is for substituting $x = 7$ and $x = 2$ and subtracting either way round. Following correct work, this mark may be implied by awrt 3.62.

A1: $V = \frac{\pi}{2}\ln 10$ or exact equivalent e.g. $\pi\ln\sqrt{10}$ and isw once the correct answer is seen. Allow $V = \frac{\pi}{2}\ln|10|$

<u>By substitution 1:</u>

M1: Uses $u = x^2$ Attempts to find $C\int\frac{x}{x^2+1}\text{d}x$ to give an expression of the form $D[\ln k(u+1)]$

A1: $\text{Volume} = (\pi)\int\frac{x}{u+1}\frac{1}{2x}\text{d}u = \frac{(\pi)}{2}[\ln(u+1)]$. Correct expression **<u>with or without</u>** π. **Ignore any limits.**

Do not allow the brackets around the $u + 1$ to be missing unless their presence is implied by later work.

dM1: Dependent upon previous M. It is for substituting $x = 7^2$ and $x = 2^2$ and subtracting either way round or changing back to x and substituting $x = 7$ and $x = 2$ and subtracting either way round.

A1: $V=\frac{\pi}{2}\ln 10$ or exact equivalent e.g. $\pi\ln\sqrt{10}$ and isw once the correct answer is seen. Allow $V=\frac{\pi}{2}\ln|10|$

By substitution 2:

M1: Uses $u=x^2+1$Attempts to find $C\int\frac{x}{x^2+1}\mathrm{d}x$ to give an expression of the form $D[\ln ku]$

A1: $\text{Volume}=(\pi)\int\frac{x}{u}\frac{1}{2x}\mathrm{d}u=\frac{(\pi)}{2}[\ln u]$. Correct expression **with or without** π. **Ignore any limits.**

dM1: Dependent upon previous M. It is for substituting $x=7^2+1$ and $x=2^2+1$ and subtracting either way round or changing back to x and substituting $x=7$ and $x=2$ and subtracting either way round.

A1: $V=\frac{\pi}{2}\ln 10$ or exact equivalent e.g. $\pi\ln\sqrt{10}$ and isw once the correct answer is seen. Allow $V=\frac{\pi}{2}\ln|10|$

9.

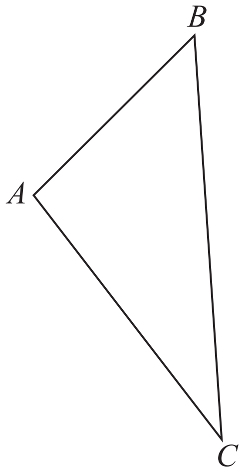


Figure 2

Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{AC} = 5\mathbf{i} - 6\mathbf{j} + \mathbf{k}$,

(a) find the size of angle *CAB*, giving your answer in degrees to 2 decimal places,
(3)

(b) find the area of triangle *ABC*, giving your answer to 2 decimal places.
(2)

Using your answer to part (b), or otherwise,

(c) find the shortest distance from *A* to *BC*, giving your answer to 2 decimal places.
(3)

Question Number	Scheme	Marks
9(a)	Attempts $\begin{pmatrix}2\\3\\-2\end{pmatrix}.\begin{pmatrix}5\\-6\\1\end{pmatrix}=\sqrt{\left(2^2+3^2+(-2)^2\right)}\sqrt{\left(5^2+(-6)^2+1^2\right)}\cos\left(\angle CAB\right)$	M1
	$\angle CAB=\arccos\left(-\frac{10}{\sqrt{17}\sqrt{62}}\right)=107.94°$	dM1A1
		(3)
(b)	Area $=\frac{1}{2}\sqrt{17}\sqrt{62}\sin\left(107.94°\right)=15.44$	M1A1
		(2)
(c)	Calculates $\lvert BC\rvert=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$	M1
	Uses Area $=\frac{1}{2}\lvert BC\rvert\times\lvert AD\rvert\ \Rightarrow 15.44=\frac{1}{2}\times\sqrt{99}\times\lvert AD\rvert\Rightarrow\lvert AD\rvert=3.10$	M1A1
		(3)
		(8 marks)
ALT (a)	Calculates $\lvert BC\rvert=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$	M1
	Uses cosine rule $\cos\angle CAB=\frac{17+62-99}{2\times\sqrt{17}\times\sqrt{62}}$	dM1
	$\Rightarrow\angle CAB=\arccos\left(-\frac{20}{2\sqrt{17}\sqrt{62}}\right)=107.94°$	A1
		(3)
ALT (b)	Area = $\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{\left(\frac{\sqrt{17}+\sqrt{62}+\sqrt{99}}{2}\right)\left(\frac{\sqrt{17}-\sqrt{62}+\sqrt{99}}{2}\right)\left(\frac{\sqrt{17}+\sqrt{62}-\sqrt{99}}{2}\right)\left(\frac{\sqrt{62}+\sqrt{99}-\sqrt{17}}{2}\right)}$	M1A1
Alt (b) ii	Area = $\frac{1}{2}\sqrt{\lvert AB\rvert^2\lvert AC\rvert^2-\left(\overrightarrow{AB}.\overrightarrow{AC}\right)^2}=\frac{1}{2}\sqrt{17\times62-100}=\frac{3}{2}\sqrt{106}$	M1A1
ALT (b)	**Area =** $\frac{1}{2}\lvert a\times b\rvert=\frac{1}{2}\begin{vmatrix}i&j&k\\2&3&-2\\5&-6&1\end{vmatrix}=\frac{1}{2}\lvert -9i-12j-27k\rvert=\frac{1}{2}\sqrt{9^2+12^2+27^2}=15.44$	M1A1
ALT (c)	Calculates $\lvert BC\rvert=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$	M1
	$\frac{\sin(\text{"}107.94\text{"})}{BC}=\frac{\sin B}{\text{"}\sqrt{62}\text{"}}\Rightarrow B=48.84...\Rightarrow AD=\text{"}\sqrt{17}\text{"}\sin B=3.10$	M1A1

(a)

M1: Attempts to use $\mathbf{a}.\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ where $\mathbf{a}=\pm\left(2\mathbf{i}+3\mathbf{j}-2\mathbf{k}\right)$, $\mathbf{b}=\pm\left(5\mathbf{i}-6\mathbf{j}+\mathbf{k}\right)$ I.e. correct use of Pythagoras to find and multiply the lengths together and multiplies and adds components (allow arithmetic slips) for dot product.

dM1: Dependent upon previous M1. For continuing to find $\angle CAB$ using invcos. Allow $\arccos\left(\frac{\pm10}{\sqrt{17}\sqrt{62}}\right)$

Implied by previous M1 + angle rounding to $108°$ or $72.1°$ (NB $\cos\theta$= -0.308…)

A1: awrt $107.94°$ only (Do **not** isw, so 107.94.. followed by 180 – 107.94… = 72.06…. scores A0

Note that the correct answer in radians is 1.8839… and scores M1dM1A0 following correct work

(b)

M1: Uses Area= $'\frac{1}{2}|\mathbf{a}||\mathbf{b}|\sin C'$ with $\mathbf{a}=2\mathbf{i}+3\mathbf{j}-2\mathbf{k},\ \ \mathbf{b}=5\mathbf{i}-6\mathbf{j}+\mathbf{k}$ and $C=\text{their}\,107.94°$

A1: Area $=\text{awrt}\,15.44$ (Allow if $72.06°$ is used for angle *CAB*)

(c)

M1: Attempts to find the length of *BC* or BC^2 e.g. $|BC|=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$

Alternatively uses the cosine rule $BC^2="17"+"62"-2"\sqrt{17}\sqrt{62}"\cos"107.9"$

NB *BC* = √99 or 3√11

This may be seen in (a) or (b) but must be <u>used</u> in (c) to score this mark.

M1: Attempts to use Area $=\frac{1}{2}|BC|\times|AD|$ using their area from (b) and their $|BC|$. This may be implied by their working.

A1: $|AD|=\text{awrt}\,3.10$ (Not 3.1)

Note that assuming *AD* bisects *BAC* without finding *BC* generally scores no marks.

10. (a) Write $2\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha \leqslant 90°$. Give the exact value of R and give the value of α to one decimal place.
(3)

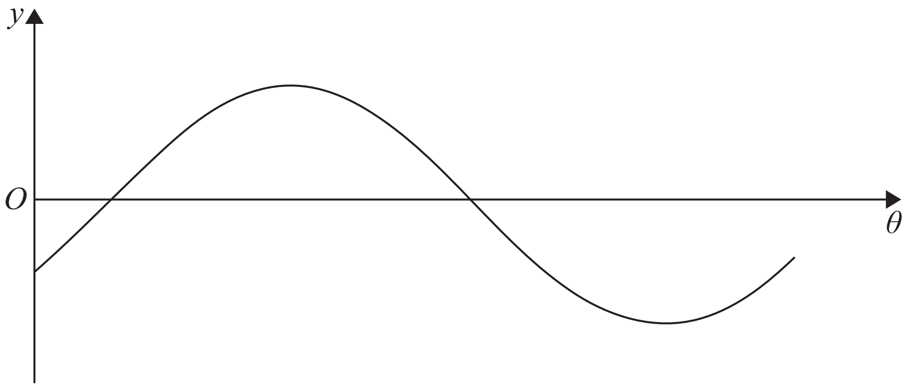


Figure 3

Figure 3 shows a sketch of the graph with equation $y = 2\sin\theta - \cos\theta, \quad 0 \leqslant \theta < 360°$

(b) Sketch the graph with equation

$$y = |2\sin\theta - \cos\theta|, \qquad 0 \leqslant \theta < 360°$$

stating the coordinates of all points at which the graph meets or cuts the coordinate axes.
(3)

The temperature of a warehouse is modelled by the equation

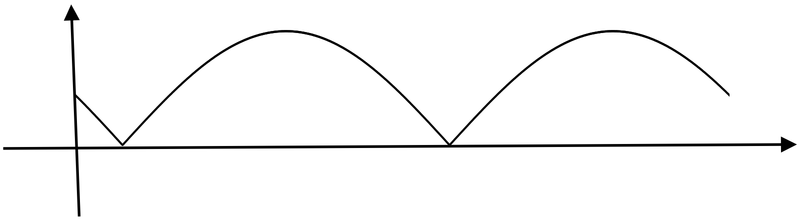
$$\mathrm{f}(t) = 5 + |2\sin(15t)° - \cos(15t)°|, \qquad 0 \leqslant t < 24$$

where $\mathrm{f}(t)$ is the temperature of the warehouse in degrees Celsius and t is the time measured in hours from midnight.

State

(c) (i) the maximum value of $\mathrm{f}(t)$,

(ii) the largest value of t, for $0 \leqslant t < 24$, at which this maximum value occurs. Give your answer to one decimal place.
(3)

Question Number	Scheme	Marks
10(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow(\alpha=)26.6°$	M1,A1
		(3)
(b)	 Shape $(0,1)$	B1 B1
	$("26.6",0)$ and $("206.6",0)$ (Allow in radians i.e. their α and π+α	B1ft
		(3)
(c)(i)	$5+'R'=5+\sqrt{5}$	B1ft
(c)(ii)	$15t-'26.6'=270\Rightarrow t=19.8$	M1,A1
		(3)
		(9 marks)

(a)

B1: $R=\sqrt{5}$

M1: For $\tan\alpha=\pm\frac{1}{2}$ or $\tan\alpha=\pm\frac{2}{1}$ or $\sin\alpha=\pm\frac{1}{"\sqrt{5}"}$ or $\cos\alpha=\pm\frac{2}{"\sqrt{5}"}$

A1: Awrt $\alpha=26.6°$

(b)

B1: Correct shape including cusps. A curve that starts downwards from the positive *y*-axis with two maxima. This mark is essentially for realising that the parts of the curve under the *x*-axis are reflected in the *x*-axis and for cusps that look “pointed” and not rounded.

B1: $(0,1)$ may be seen on the diagram or in the body of the script as coordinates or seen as *x* = 0, *y* = 1. If there is any ambiguity, the sketch takes precedence. Allow (1, 0) as long as it is marked in the correct place on the sketch.

B1**ft**: $(26.6,0)$ and $(206.6,0)$ or their 26.6 and 180 + their 26.6. May be seen on the sketch or in the body of the script as coordinates or seen as *y* = 0, $\theta(\text{or }x)=26.6,\ \ \theta(\text{or }x)=206.6$. If there is any ambiguity, the sketch takes precedence. Allow awrt 26.6 and awrt 207 or their ft values.

(c)(i)

B1ft: Follow through on $5+'R'$ including decimal answers (NB 5 + √5 = 7.24…)

(c)(ii)

M1: Attempts $15t-"26.6"=90\text{ or }270\Rightarrow t=...$ (Allow π/2, 3π/2 for 90, 270 if working in radians)

A1: $t=19.8$ **only**

(c)(ii) Alternative:

$\text{f}(t)=5+2\sin(15t)-\cos(15t)\Rightarrow \text{f}'(t)=30\cos(15t)+15\sin(15t)$

M1: Attempts $\text{f}'(t)=0\Rightarrow 15t=180-63.43...\text{ or }\ 360-63.43$

A1: $t=19.8$ **only**

11. $$y = (2x^2 - 3)\tan\left(\frac{1}{2}x\right), \qquad 0 < x < \pi$$

(a) Find the exact value of x when $y = 0$

(1)

Given that $\frac{dy}{dx} = 0$ when $x = \alpha$,

(b) show that

$$2\alpha^2 - 3 + 4\alpha \sin \alpha = 0$$

(6)

The iterative formula

$$x_{n+1} = \frac{3}{\left(2x_n + 4\sin x_n\right)}$$

can be used to find an approximation for α.

(c) Taking $x_1 = 0.7$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

(d) By choosing a suitable interval, show that $\alpha = 0.7283$ to 4 decimal places.

(2)

Question Number	Scheme	Marks
11.(a)	$\sqrt{\frac{3}{2}}$ or $\frac{\sqrt{3}}{\sqrt{2}}$ or $\sqrt{1.5}$ or $\frac{\sqrt{6}}{2}$	B1
		(1)
(b)	$y=(2x^2-3)\tan\left(\frac{1}{2}x\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=4x\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$	M1A1A1
	When $x=\alpha$ $\quad 4\alpha\tan\left(\frac{1}{2}\alpha\right)+(2\alpha^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}\alpha\right)=0$	
	$8\alpha\frac{\sin\left(\frac{1}{2}\alpha\right)}{\cos\left(\frac{1}{2}\alpha\right)}+(2\alpha^2-3)\times\frac{1}{\cos^2\left(\frac{1}{2}\alpha\right)}=0$	M1
	$8\alpha\sin\left(\frac{1}{2}\alpha\right)\cos\left(\frac{1}{2}\alpha\right)+(2\alpha^2-3)=0$	
	$4\alpha\sin\alpha+(2\alpha^2-3)=0$	dM1
	$2\alpha^2-3+4\alpha\sin\alpha=0$	A1*
		(6)
(c)	$x_2=\frac{3}{\left(2\times0.7+4\sin0.7\right)}$	M1
	$x_2=0.7544,\ x_3=0.7062$	A1
		(2)
(d)	Chooses interval $\left[0.72825, 0.72835\right]$	M1
	$2\times0.72825^2-3+4\times0.72825\sin0.72825=-0.0005<0$ $2\times0.72835^2-3+4\times0.72835\sin0.72835=0.00026>0$ + Reason +conclusion	A1
		(2)
		(11 marks)

(a)

B1: $x=\sqrt{\frac{3}{2}}$ or exact equivalent and no others **inside** the range. Ignore any solution outside the range so allow e.g. $x=\pm\sqrt{\frac{3}{2}}$. $\sqrt{\frac{3}{2}}$ seen unless seen in an incorrect statement e.g. $x^2=\sqrt{\frac{3}{2}}$.

(b)

M1: Attempts product rule on $y=(2x^2-3)\tan\left(\frac{1}{2}x\right)$ or $y=2x^2\tan\left(\frac{1}{2}x\right)$ if they multiply out first so look for

$\frac{\mathrm{d}(2x^2-3)}{\mathrm{d}x}\times\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{\mathrm{d}\tan\left(\frac{1}{2}x\right)}{\mathrm{d}x}$ or $\frac{\mathrm{d}\left(2x^2\right)}{\mathrm{d}x}\times\tan\left(\frac{1}{2}x\right)+2x^2\times\frac{\mathrm{d}\tan\left(\frac{1}{2}x\right)}{\mathrm{d}x}$ or e.g.

$Ax\tan\left(\frac{1}{2}x\right)+Bx^2\sec^2\frac{1}{2}x$

A1: One term correct: of $4x\tan\left(\frac{1}{2}x\right)$ or $+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ or $+(2x^2)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=4x\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$. A fully correct un-simplified or simplified derivative.

M1: They need to have a $\frac{\mathrm{d}y}{\mathrm{d}x}$ with both $\tan\left(\frac{1}{2}x\right)$ and $\sec^2\left(\frac{1}{2}x\right)$. It is for using $\tan\left(\frac{1}{2}x\right)=\frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$ and

$\sec^2\left(\frac{1}{2}x\right)=\frac{1}{\cos^2\left(\frac{1}{2}x\right)}$ or $\sec^2\left(\frac{1}{2}x\right)=1+\tan^2\left(\frac{1}{2}x\right)=1+\frac{\sin^2\left(\frac{1}{2}x\right)}{\cos^2\left(\frac{1}{2}x\right)}$ and setting $\frac{\mathrm{d}y}{\mathrm{d}x}=0$

This mark is for converting to an equation in sin and cos using the correct identities.

dM1: **Dependent upon both previous M's**. It is for multiplying by $\cos^2\left(\frac{1}{2}x\right)$ and using the correct identity $2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)=\sin x$. This may be implied by their work but if the identity $\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)=\sin x$ is suggested, score M0.

A1*: $2\alpha^2-3+4\alpha\sin\alpha=0$ (Allow $4\alpha\sin\alpha+2\alpha^2-3=0$) . This is a printed answer so there must have been no errors, including bracketing errors. (May work in *x* or α or *a* but must be α or *a* for final A1)
(c)
M1: For substituting $x_1=0.7$, into the iterative formula to find x_2. It may be implied by the sight of

$x_2=\frac{3}{(2\times0.7+4\sin0.7)}$, $x_2=$ awrt 0.75 and also (if degrees were used) $x_2=$ awrt 2.1

A1: $x_2=0.7544$, $x_3=0.7062$ (awrt 4 dp)

(d)
M1: Chooses interval $[0.72825, 0.72835]$ or a smaller interval containing the root
A1: Substitutes both values into a suitable function, calculates both and follows with a reason and a conclusion.

Accept as suitable functions, $\pm\left(2\alpha^2-3+4\sin\alpha\right)$, $\pm\left(x-\frac{3}{(2x+4\sin x)}\right)$

NB: $0.72825-\frac{3}{(2\times0.72825+4\sin0.72825)}=-0.0001$, $0.72835-\frac{3}{(2\times0.72835+4\sin0.72835)}=0.00006$

Requires calculation correct to 1 sf rounded or truncated, reason (change in sign) and a minimal conclusion such as root/ turning point/ proven, hence suitable interval.

12.

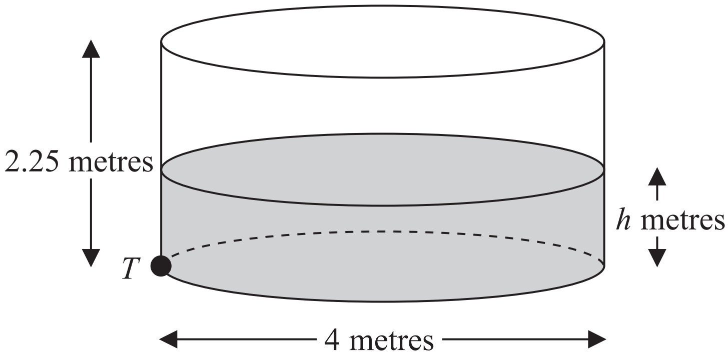


Figure 4

Figure 4 shows a right cylindrical water tank. The diameter of the circular cross section of the tank is 4 m and the height is 2.25 m. Water is flowing into the tank at a constant rate of $0.4\pi \text{ m}^3 \text{min}^{-1}$. There is a tap at a point T at the base of the tank. When the tap is open, water leaves the tank at a rate of $0.2\pi\sqrt{h} \text{ m}^3 \text{min}^{-1}$, where h is the height of the water in metres.

(a) Show that at time t minutes after the tap has been opened, the height h m of the water in the tank satisfies the differential equation

$$20\frac{\mathrm{d}h}{\mathrm{d}t} = 2 - \sqrt{h}$$

(5)

At the instant when the tap is opened, $t = 0$ and $h = 0.16$

(b) Use the differential equation to show that the time taken to fill the tank to a height of 2.25 m is given by

$$\int_{0.16}^{2.25} \frac{20}{2-\sqrt{h}} \,\mathrm{d}h$$

(2)

Using the substitution $h = (2 - x)^2$, or otherwise,

(c) find the time taken to fill the tank to a height of 2.25 m.

Give your answer in minutes to the nearest minute.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

Question Number	Scheme	Marks
12 (a)	States or uses $\frac{\mathrm{d}V}{\mathrm{d}t}=0.4\pi-0.2\pi\sqrt{h}$	B1
	States or uses $V=4\pi h\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}h}=4\pi$	M1A1
	Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow0.4\pi-0.2\pi\sqrt{h}=4\pi\times\frac{\mathrm{d}h}{\mathrm{d}t}$	M1
	$20\frac{dh}{dt}=2-\sqrt{h}$	A1*
		(5)
(b)	Separates the variables $\int 20\frac{\mathrm{d}h}{2-\sqrt{h}}=\int 1\mathrm{d}t$	M1
	$(t=)\int_{0.16}^{2.25}\frac{20}{2-\sqrt{h}}\mathrm{d}h$	A1*
		(2)
(c)	$h=(2-x)^2\Rightarrow\mathrm{d}h=-2(2-x)\mathrm{d}x$	B1
	$T=\int\frac{20}{2-\sqrt{h}}\mathrm{d}h=\int\frac{20}{2-(2-x)}\times-2(2-x)\mathrm{d}x$	M1
	$=\int-\frac{80}{x}+40\,\mathrm{d}x$ **(No need for limits here)**	**d**M1A1
	$T=\int_{0.16}^{2.25}\frac{20}{2-\sqrt{h}}\mathrm{d}h=\int_{1.6}^{0.5}-\frac{80}{x}+40\,\mathrm{d}x=\left[-80\ln x+40x\right]_{1.6}^{0.5}$	**dd**M1
	$=\left[-80\ln0.5+20\right]-\left[-80\ln1.6+64\right]=80\ln3.2-44=49\left(\text{minutes}\right)$	**ddd**M1A1
		(7)
		(14 marks)

(a)

B1: States or uses $\frac{\mathrm{d}V}{\mathrm{d}t}=0.4\pi-0.2\pi\sqrt{h}$. This may be embedded within the chain rule but must be identifiable as $\frac{\mathrm{d}V}{\mathrm{d}t}$

M1: Attempts $\frac{\mathrm{d}V}{\mathrm{d}h}$ from an equation for the volume of a cylinder. Accept $V=c\times\pi h\to\frac{\mathrm{d}V}{\mathrm{d}h}=c\pi$

A1: $\frac{\mathrm{d}V}{\mathrm{d}h}=4\pi$ This may be embedded within the chain rule

M1: Uses a correct form of the chain rule: Eg $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and $\frac{\mathrm{d}V}{\mathrm{d}t}$

Also accept forms such as $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$

A1*: $20\frac{dh}{dt}=2-\sqrt{h}$ This is a given answer so no errors must be seen e.g. bracketing errors.

(b)

M1: Separates variables, no need for limits or integral sign e.g. $\int 20\frac{\mathrm{d}h}{2-\sqrt{h}} = \int 1\mathrm{d}t, \int \frac{\mathrm{d}h}{2-\sqrt{h}} = \int \frac{1}{20}\mathrm{d}t$

Accept for this mark an equation where the dh is in the numerator and $2-\sqrt{h}$ in the denominator on one side and dt is on the other side, with or without the integral signs. The "20" may appear on either side but must be correctly placed.

A1*: cao $(t=)\int_{0.16}^{2.25} \frac{20}{2-\sqrt{h}}\mathrm{d}h$ Correct expression as printed.

(c)

B1: Accept $h=(2-x)^2 \Rightarrow \mathrm{d}h = -2(2-x)\mathrm{d}x$ or $\frac{\mathrm{d}h}{\mathrm{d}x} = -2(2-x)$ or $\frac{\mathrm{d}h}{\mathrm{d}x} = -2\sqrt{h}$

M1: Attempt to produce an integral just in x $\int \frac{20}{2-\sqrt{h}}\mathrm{d}h \to \int \frac{20}{2-(2-x)} \times -2(2-x)(\mathrm{d}x)$

For this to be scored $\mathrm{d}h$ cannot just be replaced by $\mathrm{d}x$

dM1: For an integral of the form $= \int \frac{A}{x} + B\,(\mathrm{d}x)$ (This may be implied by subsequent integration)

Dependent on the previous M mark

A1: $= \int -\frac{80}{x} + 40\,(\mathrm{d}x)$ (Allow $\int \frac{80}{x} - 40\,(\mathrm{d}x)$ if the limits have clearly been "reversed" correctly)

(May be implied by subsequent integration – beware that this may be done by e.g. integration by parts)

ddM1: $\int \frac{A}{x} + B\,(\mathrm{d}x) \to A\ln x + Bx$. **There is no need for limits.**

Dependent on both previous M marks.

dddM1: Substitutes 0.5 and 1.6 into $A\ln x + Bx$ and subtracts either way round.

Dependent on all previous M marks.

A1: Accept $80\ln 3.2 - 44$ (oe) or answers rounding to 49

Leave blank

13.

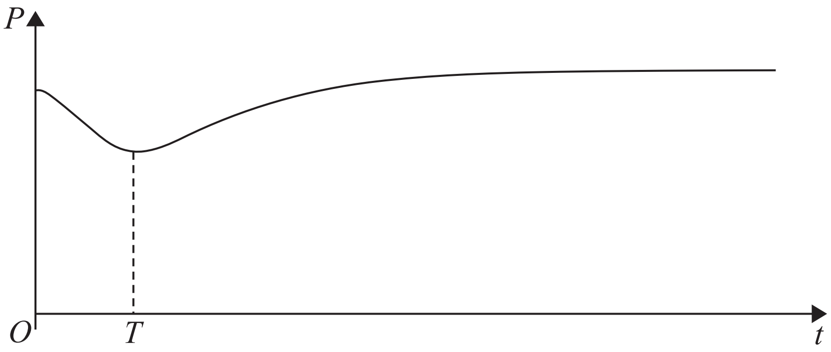


Figure 5

A colony of ants is being studied. The number of ants in the colony is modelled by the equation

$$P = 200 - \frac{160\mathrm{e}^{0.6t}}{15 + \mathrm{e}^{0.8t}} \qquad t \in \mathbb{R}, t \geqslant 0$$

where P is the number of ants, measured in thousands, t years after the study started. A sketch of the graph of P against t is shown in Figure 5

(a) Calculate the number of ants in the colony at the start of the study.

(2)

(b) Find $\frac{\mathrm{d}P}{\mathrm{d}t}$

(3)

The population of ants initially decreases, reaching a minimum value after T years, as shown in Figure 5

(c) Using your answer to part (b), calculate the value of T to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Question Number	Scheme	Marks
13.(a)	$t=0\Rightarrow\left(P=\right)200-\frac{160}{15+1}=190\Rightarrow 190\ 000$	M1A1
		(2)
(b)	$\mathrm{e}^{kt}\to a\mathrm{e}^{kt}$	M1
	$\frac{\mathrm{d}P}{\mathrm{d}t}=-\frac{(15+\mathrm{e}^{0.8t})\times 96\mathrm{e}^{0.6t}-160\mathrm{e}^{0.6t}\times 0.8\mathrm{e}^{0.8t}}{(15+\mathrm{e}^{0.8t})^{2}}$	M1A1
		(3)
(c)	Sets $\pm\frac{(15+e^{0.8t})\times 96e^{0.6t}-160e^{0.6t}\times 0.8e^{0.8t}}{(15+e^{0.8t})^{2}}=0\Rightarrow \mathrm{e}^{0.8t}=45$	M1A1
	$\Rightarrow T=\frac{\ln 45}{0.8}=4.76$	M1A1
		(4)
		(9 marks)

(a)

M1: Sets t = 0 in the top and bottom of the fraction, giving $\mathrm{e}^{0}=1$. Award if candidate attempts $200-\frac{160}{15+1}$ but not $\frac{200-160}{15+1}$ This can be awarded for a correct answer.

A1: Correct answer only. Accept 190 000 or (P =) 190 (ants).
The answer is an integer so do **not** allow awrt 190 or awrt 190 000 i.e. there should be no decimals.

(b)

M1: For showing that $\mathrm{e}^{kt}\to a\mathrm{e}^{kt}$ where a is a constant. This may be embedded within the product or quotient rule or their attempt to differentiate.

M1: For applying the quotient rule to obtain $\frac{\mathrm{d}P}{\mathrm{d}t}=\pm\frac{(15+\mathrm{e}^{0.8t})\times p\mathrm{e}^{0.6t}-q\mathrm{e}^{0.8t}\times \mathrm{e}^{0.6t}}{(15+\mathrm{e}^{0.8t})^{2}}$ or applying the product rule to obtain $\frac{\mathrm{d}P}{\mathrm{d}t}=\pm\left[A\mathrm{e}^{0.6t}(15+\mathrm{e}^{0.8t})^{-2}\times B\mathrm{e}^{0.8t}+(15+\mathrm{e}^{0.8t})^{-1}\times C\mathrm{e}^{0.6t}\right]$

Allow invisible brackets for this mark but not for the A mark below.

A1: A correct un-simplified or simplified $\frac{\mathrm{d}P}{\mathrm{d}t}$

Note $\frac{\mathrm{d}P}{\mathrm{d}t}=-\frac{(15+\mathrm{e}^{0.8t})\times 96\mathrm{e}^{0.6t}-160\mathrm{e}^{0.6t}\times 0.8\mathrm{e}^{0.8t}}{(15+\mathrm{e}^{0.8t})^{2}}$ or $-160\mathrm{e}^{0.6t}(15+\mathrm{e}^{0.8t})^{-2}\times 0.8\mathrm{e}^{0.8t}+(15+\mathrm{e}^{0.8t})^{-1}\times 128\mathrm{e}^{0.6t}$

(c) **Allow recovery here if the signs are reversed.**

M1: Sets their $\frac{\mathrm{d}P}{\mathrm{d}t}=0$ to obtain $p\mathrm{e}^{0.8t}=q$ or $A\mathrm{e}^{0.6t}=B\mathrm{e}^{1.4t}$ or equivalent.

A1: $\mathrm{e}^{0.8t}=45$ or $1440\mathrm{e}^{0.6t}=32\mathrm{e}^{1.4t}$ or equivalent correct equation.

M1: Having set their $\frac{\mathrm{d}P}{\mathrm{d}t}=0$ and obtained either $A\mathrm{e}^{\pm kt}=B$ (k may be incorrect) or $C\mathrm{e}^{\pm\alpha t}=D\mathrm{e}^{\pm\beta t}$ where $k,\alpha,\beta\neq 0$
it is awarded for the correct order of operations, taking ln's leading to $t=..$
It cannot be awarded from impossible equations Eg $\mathrm{e}^{0.8t}=-45$

A1: $T=\frac{\ln 45}{0.8}$ or equivalent or awrt $=4.76$

Leave blank

14.

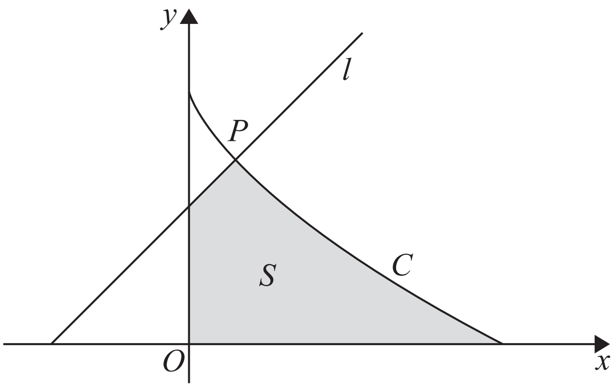


Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\cos^3\theta, \qquad y = 6\sin^2\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

Given that the point P lies on C and has parameter $\theta = \frac{\pi}{3}$

(a) find the coordinates of P.

(2)

The line l is the normal to C at P.

(b) Show that an equation of l is $y = x + 3.5$

(5)

The finite region S, shown shaded in Figure 6, is bounded by the curve C, the line l, the y-axis and the x-axis.

(c) Show that the area of S is given by

$$4 + 144\int_0^{\frac{\pi}{3}}(\sin\theta\cos^2\theta - \sin\theta\cos^4\theta)\,\mathrm{d}\theta$$

(6)

(d) Hence, by integration, find the exact area of S.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(3)

Question Number	Scheme	Marks
14 (a)	$(1,4.5)$	B1B1
		(2)
(b)	Attempts $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{12\sin\theta\cos\theta}{-24\cos^2\theta\sin\theta}=\left(-\frac{1}{2\cos\theta}\right)$	M1A1
	Subs $\theta=\frac{\pi}{3}$ into $\frac{dy}{dx}=(-1)$	M1
	Uses gradient of normal with $(1,4.5)\Rightarrow(y-4.5)=1(x-1)$	ddM1
	$y=x+3.5$	A1*
		(5)
(c)	Attempts $\int y\frac{dx}{d\theta}\,d\theta=\int 6\sin^2\theta\times-24\cos^2\theta\sin\theta\,d\theta$	M1A1
	Uses $\sin^2\theta=1-\cos^2\theta\Rightarrow\int y\frac{dx}{d\theta}\,d\theta=\int A\left(\cos^2\theta\sin\theta-\cos^4\theta\sin\theta\right)d\theta$	dM1
	Area of trapezium= $\frac{1}{2}(3.5+4.5)=4$	B1
	Attempts trapezium + area under curve $=\frac{1}{2}(3.5+4.5)-144\int_{\frac{\pi}{3}}^{0}\sin^3\theta\cos^2\theta\,d\theta$	ddM1
	$Area=4+144\int_0^{\frac{\pi}{3}}\left(\sin\theta\cos^2\theta-\sin\theta\cos^4\theta\right)d\theta$	A1*
		(6)
(d)	Area of S = $4+144\left[-\frac{\cos^3\theta}{3}+\frac{\cos^5\theta}{5}\right]_0^{\frac{\pi}{3}}=4+144\left(\left(-\frac{1}{24}+\frac{1}{160}\right)-\left(-\frac{1}{3}+\frac{1}{5}\right)\right)$	M1A1
	$=\frac{181}{10}$	A1
		(3)
		(16 marks)

(a)

B1: Either of $(1,4.5)$. Accept any exact equivalent for 4.5 e.g. 18/4, 9/2…(May be seen on the diagram)

B1: Both $(1,4.5)$. Accept any exact equivalent for 4.5 e.g. 18/4, 9/2…

(b)

M1: Attempts $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$ (Allow poor differentiation on *y* and/or *x* provided the functions are both "changed")

A1: $\frac{dy}{dx}=-\frac{12\sin\theta\cos\theta}{24\cos^2\theta\sin\theta}=\left(-\frac{1}{2\cos\theta}\right)$

M1: Subs $\theta=\frac{\pi}{3}$ into their $\frac{dy}{dx}=(-1)$

ddM1: **Dependent upon both previous M's**. It is for using the negative reciprocal of their $\frac{dy}{dx}$ with their $(1, 4.5)$ to produce an equation of a normal $\Rightarrow (y-4.5)=1(x-1)$. Need to be careful as m = 1 is easily identifiable from the given answer.

A1*|:cso $y=x+3.5$ (Allow $y=x+\frac{7}{2}$)

(c)

M1: Attempts to use area under a parametric curve $=\int y\frac{dx}{d\theta}\,d\theta=\int A\sin^3\theta\cos^2\theta(d\theta)$ (Allow omission of dθ and allow un-simplified)

A1: $=\int 6\sin^2\theta\times -24\cos^2\theta\sin\theta\,d\theta$

dM1: Uses the identity $\sin^2 x=1-\cos^2 x$ to produce an expression in an 'integrable form'

$$\int A\sin^3\theta\cos^2\theta\,d\theta=\int A\sin\theta\left(1-\cos^2\theta\right)\cos^2\theta\,d\theta=A\int\left(\sin\theta\cos^2\theta-\sin\theta\cos^4\theta\right)d\theta$$

Dependent on previous M mark.

B1: Area of trapezium= $\frac{1}{2}(3.5+4.5)$ or $\frac{1}{2}\times 1\times 8$ or $\frac{1}{2}\times 8$ alternatively

$$\int_0^1(x+3.5)dx=\left[\frac{x^2}{2}+3.5x\right]_0^1\left(=0.5+3.5\right)=4$$

Must see a calculation here – it is not acceptable just to state area of trapezium = 4 as this is effectively a given answer – must see a calculation.

ddM1: Attempts trapezium + area under curve. Look for $="\frac{1}{2}(3.5+4.5)"\pm A\int_0^{\frac{\pi}{3}}\sin^3\theta\cos^2\theta\,d\theta$

OR alternatively $="\frac{1}{2}(3.5+4.5)"\pm\ A\int_0^{\frac{\pi}{3}}\left(\sin\theta\cos^2\theta-\sin\theta\cos^4\theta\right)\,d\theta$

The correct limits must be seen either way around and they must be adding an attempt at the area of the trapezium.
Dependent on both previous M marks.

A1*: cso. Answer is given so **all previous marks must have been awarded** and no errors seen.

(d)

M1: $\int\sin\theta\cos^n\theta\,d\theta=\pm\frac{\cos^{n+1}\theta}{n+1}$ in either term.

Or by substitution e.g. $u=\cos\theta$ to give $\pm\int\left(u^2-u^4\right)du=\pm\frac{u^3}{3}\pm\frac{u^5}{5}$

A1: Any correct (un-simplified) answer $4+144\left[\left(-\frac{\cos^3\frac{\pi}{3}}{3}+\frac{\cos^5\frac{\pi}{3}}{5}\right)-\left(-\frac{\cos^3 0}{3}+\frac{\cos^5 0}{5}\right)\right]$ or appropriate limits if using substitution. If the (4 + 144) is bracketed then score A0 unless they recover.

A1: cso $=\frac{181}{10}$ (oe)

Correct answer with no working scores no marks.
Attempts at using Cartesian forms should be sent to review.