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Surname	Other	names
Pearson Edexcel nternational Advanced Level	Centre Number	Candidate Number
Coro Math	homatic	rc C ZA
Advanced	iematik	.5 С.5 Т
Advanced Tuesday 17 January 2017 - Time: 2 hours 30 minutes	- Morning	Paper Reference WMA02/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Find an equation of the tangent to the curve 1. $x^3 + 3x^2y + y^3 = 37$ at the point (1, 3). Give your answer in the form ax + by + c = 0, where a, b and c are integers. (6) 2 P 4 8 3 2 5 A 0 2 4 8

Qu Scheme Marks Differentiate wrt x $\underline{3x^2} + \underline{6xy} + \underline{3x^2}\frac{dy}{dx} + \underline{3y^2}\frac{dy}{dx} = \underline{0}$ 1 M1 <u>A1</u> B1 Substitutes (1, 3) **AND** rearranges to get $\frac{dy}{dr} \left(= -\frac{7}{10} \right)$ **M**1 $(y-3) = -\frac{7}{10}(x-1)$ so 7x + 10y - 37 = 0M1A1 (6) (6 marks) M1: Differentiates implicitly to include either $3x^2 \frac{dy}{dx}$ or $3y^2 \frac{dy}{dx}$ term Accept $3x^2 \frac{dy}{dx}$ appearing as $3x^2y'$ or $3y^2 \frac{dy}{dx}$ as $3y^2y'$ A1: Differentiates $y^3 \rightarrow 3y^2 \frac{dy}{dx}$ and $x^3 \rightarrow 3x^2$ and $37 \rightarrow 0$ **B1**: Uses the product rule to differentiate $3x^2y$ giving $6xy + 3x^2 \frac{dy}{dx}$ Do not penalise students who write $3x^2dx + 6xydx + 3x^2dy + 3y^2dy = 0$ M1: Substitutes x = 1, y = 3 into their expression (correctly each at least once) to find a 'numerical' value for $\frac{dy}{dx}$ (may be incorrect). Note that $\frac{dy}{dx} = \frac{-3x^2 - 6xy}{3x^2 + 3y^2}$ **M1**: Use of (y-3) = m(x-1) where *m* is their numerical value of $\frac{dy}{dx}$ Alternatively uses y = mx + c with (1,3) and their *m* as far as c = ...A1: Accept integer multiples of the answer i.e. 7kx + 10ky - 37k = 0 for example 21x + 30y - 111 = 0Note: If the gradient $-\frac{7}{10}$ just appears (from a graphical calculator) only M3 may be awarded

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2.

- $f(x) = x^3 5x + 16$
- (a) Show that the equation f(x) = 0 can be rewritten as

$$x = (ax + b)^{\frac{1}{3}}$$

giving the values of the constants *a* and *b*.

(2)

The equation f(x) = 0 has exactly one real root α , where $\alpha = -3$ to one significant figure.

(b) Starting with $x_1 = -3$, use the iteration

$$x_{n+1} = (ax_n + b)^{\frac{1}{3}}$$

with the values of *a* and *b* found in part (a), to calculate the values of x_2 , x_3 and x_4 , giving all your answers to 3 decimal places.

(3)

(c) Using a suitable interval, show that $\alpha = -3.17$ correct to 2 decimal places.

(2)





Qu	Scheme	Marks
2(a)	$f(x) = x^3 - 5x + 16 = 0 \text{ so } x^3 = 5x - 16$	M1
	$\Rightarrow x = \sqrt[3]{5x - 16}$	A1 (2)
(b)	$x_2 = \sqrt[3]{5 \times -3 - 16}$	M1
	$x_2 = -3.141$ awrt	A1
	$x_3 = -3.165$ awrt and $x_4 = -3.169$ awrt	A1
		(3)
(c)	f(-3.175) = -0.130984375, f(-3.165) = 0.120482875	
	Sign change (and as f (x) is continuous) therefore a root α lies in the interval [-3.175, -3.165] $\Rightarrow \alpha = -3.17$ (2 dp)	M1A1
		(2) (7 marks)
A1: con and $x =$ If a cano	npletely correct with all lines including $f(x) = 0$ stated or implied (see above), $x^3 = 5x - 16$ $\sqrt[3]{5x - 16}$ oe with or without $a = 5$, $b = -16$. Isw after a correct answer didate writes $x^3 = 5x - 16 \Rightarrow x = (5x - 16)^{\frac{1}{3}}$ then they can score 1 0 for a correct but incomplete	te solution.
Similarl Way 2: M1: sta A1: Con (b)	y if a candidate writes $x^3 - 5x + 16 = 0 \Rightarrow x = (5x - 16)^{\frac{1}{3}}$ rts with answer, cubes and reaches $a =, b = .$ mpletely correct reaching equation and stating hence $f(x) = 0$	
Ignore s M1: A	subscripts in this part, just mark as the first, second and third values given. In attempt to substitute $x_1 = -3$ into their iterative formula. E.g. Sight of $\sqrt[3]{-31}$, or can be	implied by
$x_2 = aw$ A1: x	n = 3.14	
A1: $x_3 =$	$= awrt - 3.165$ and $x_4 = awrt - 3.169$	
(c) M1: Cl be the v tighter r the cond	moose suitable interval for x, e.g. $[-3.175, -3.165]$ and at least one attempt to evaluate $f(x)$. If alues embedded within an expression or one value correct. A minority of candidates may ange which should include -3.1698 (alpha to 4dp). This would be acceptable for both mathematical for the A mark are met. Some candidates may use an adapted $f(x) = 0$, for example	Evidence would choose a rks, provided ple
g(x) = x	$x = \sqrt[3]{(5x-16)}$ This is also acceptable even if it is called f, but you must see it defined. For	r your
informa	tion $g(-3.175) = -0.004$, $g(-3.165) = (+)0.004$ If the candidate states an f (without define	ing it) it must
be assur	med to be $f(x) = x^3 - 5x + 16$	
A1: ne	teds (i) both evaluations correct to 1 sf, (either rounded or truncated) (ii) sign change stated (>0, <0 acceptable as would a negative product) and (iii) some form of conclusion which may be $\Rightarrow \alpha = -3.17$ or "so result shown" or qe equivalent	d or tick or

3. (a) Express $\frac{9+11x}{(1-x)(3+2x)}$ in partial fractions. (3) (b) Hence, or otherwise, find the series expansion of $\frac{9+11x}{(1-x)(3+2x)}$, $ x < 1$ in ascending powers of x, up to and including the term in x ³ . (6)	Winter 2017 Past Paper	7 www.mystudybro.com This resource was created and owned by Pearson Edexcel	Mathematics	s C34
(b) Hence, or otherwise, find the series expansion of $\frac{9+11x}{(1-x)(3+2x)}, x < 1$ in ascending powers of x, up to and including the term in x ³ . Give each coefficient as a simplified fraction. (6)	3. (a)	Express $\frac{9+11x}{(1-x)(3+2x)}$ in partial fractions.	(3)	Leave blank
	(b)	Hence, or otherwise, find the series expansion of		
in ascending powers of x, up to and including the term in x ³ . Give each coefficient as a simplified fraction. (6)		$\frac{9+11x}{(1-x)(3+2x)}, x < 1$		
		in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.		
			(6)	

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P 4 8 3 2 5 A 0 6 4 8

Qu	Scheme	Marks
3(a)	$\frac{9+11x}{(1-x)(3+2x)} = \frac{A}{1-x} + \frac{B}{3+2x}$ and attempt to find A or B A = 4, B = -3	M1 A1, A1 (3)
(b)	$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$	B1
	$(3+2x)^{-1} = \frac{1}{3} \times \left(1 + \left(-1\right)\left(\frac{2}{3}x\right) + \frac{(-1)(-2)}{2}\left(\frac{2}{3}x\right)^2 + \frac{(-1)(-2)(-3)}{6}\left(\frac{2}{3}x\right)^3 \dots\right)$	B1 M1
	Attempts $'4' \times () + '-3' \times ()$ = $3 + \frac{14}{3}x + \frac{32}{9}x^2 + \frac{116}{27}x^3$	M1 A1, A1 (6) (9 marks)
(a) M1: For coe A1: One A1: Bot You (b) B1: Cor	r expression in markscheme or $9 + 11 \ x = A \ (3+2x) + B \ (1-x)$ and use of substitution or conficients in an attempt to find A or B (Condone slips on the terms) e correct value (this implies the M1) h correct values (attached to the correct fraction). do not explicitly need to see the expression rewritten in PF form. rect expansion $(1-x)^{-1} = 1 + x + x^2 + x^3 +$ with or without working. Must be simplified	nparison of
B1 : For implied	taking out a factor of 3^{-1} Evidence would be seeing either 3^{-1} or $\frac{1}{3}$ before the bracket or c by the candidate multiplying their <i>B</i> by $\frac{1}{2}$.	could be
M1 : For	the form of the binomial expansion with $n = -1$ and a term of $\left(\pm \frac{2}{3}x\right)$.	
To s M1: Att atte A1: Two A1: All	score M1 it is sufficient to see just any two terms of the expansion. eg. $1 + \dots + \frac{(-1)(-2)}{2} \left(\pm \frac{2}{3}\right)$ empts to combine the two series expansions. Condone slips on signs but there must have been to combine terms (at least once) and to use both their coefficients the terms correct which may be unsimplified. four terms correct. (cao) Could be mixed number fraction form. ISW after a correct answer	$\left(\frac{2}{3}x\right)^2$ en some
Alternat B1: Fo M1: It	ive use of binomial in line 2 of scheme: ie. $3^{-1} + (-1)(3)^{-2}(2x)$ or seeing either 3^{-1} or $\frac{1}{3}$ as the first term is sufficient to see just the first two terms (unsimplified) then marks as before	
Way 2: Th Way 3:	Otherwise method: Use of $(9+11x)(1-x)^{-1}(3+2x)^{-1}$: B1 B1 M1 : as before then M1: Attempt to multiply three brackets and obtain $3 +$ A1: two terms correct A1: All for Use of $(9+11x)(3-(x+2x^2))^{-1}$ or alternatives is less likely – send to review	our correct

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4. Given that	
$f(x) = \frac{4}{3x+5}, x > 0$	
$g(x) = \frac{1}{x}, \qquad x > 0$	
(a) state the range of f,	(2)
(b) find $f^{-1}(x)$,	(3)
(c) find $fg(x)$.	(1)
(d) Show that the equation $fg(x) = gf(x)$ has no real solutions.	(4)
10	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	

Qu	Scheme	Marks
4. (a)	$0 < f(x) < \frac{4}{5}$	M1A1 (2)
(b)	$y = \frac{4}{3x+5} \qquad \Longrightarrow (3x+5)y = 4$	M1
	$\Rightarrow x = \frac{4 - 5y}{3y}$	dM1
	$f^{-1}(x) = \frac{4-5x}{3x} \qquad \left(0 < x < \frac{4}{5}\right)$	Alo.e.
		(3)
(c)	$fg(x) = \frac{4}{\frac{3}{x} + 5}$	B1 (1)
(d)	$\frac{3x+5}{4} = \frac{4}{\frac{3}{x}+5}$	M1
	$15x^2 + 18x + 15 = 0$	A1
	Uses $18^2 < 4 \times 15 \times 15$ and so deduce no real roots	M1 A1 (4) (10 marks)
		(

(a)

M1: One limit such as y > 0 or y < 0.8. Condone for this mark both limits but with x (not y) or with the boundary included. For example $[0, 0.8], 0 < x < 0.8, 0 \le y \le 0.8$

A1: Fully correct so accept $0 < f(x) < \frac{4}{5}$ and exact equivalents $0 < y < \frac{4}{5}$ (0,0.8)

(b)

M1: Set y = f(x) or x = f(y) and multiply both sides by denominator.

dM1:Make x (or a swapped y) the subject of the formula. Condone arithmetic slips

A1: o.e for example $y/f^{-1}(x) = \frac{1}{3}\left(\frac{4}{x}-5\right)$ or $y = \frac{\left(\frac{4}{x}-5\right)}{3}$ - do not need domain for this mark. ISW after a

correct answer.

(c) Mark parts c and d together

B1: $fg(x) = \frac{4}{\frac{3}{x} + 5}$ - allow any correct form then isw

(d)

M1: Sets fg(x) = gf(x) with **both sides correct** (but may be unsimplified) and forms a quadratic in x. Do not withhold this mark if fg or gf was originally correct but was "simplified" incorrectly and set equal to a correct gf A1: Correct 3TQ. It need not be all on one side of the equation. The =0 can be implied by later work M1: Attempts the discriminant or attempts the formula or attempts to complete the square.

A1: Completely correct work (cso) and conclusion. If $b^2 - 4ac$ has been found it must be correct (-576)

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Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = x \cos x, \quad x \in \mathbb{R}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve C and the x-axis

for $\frac{3\pi}{2} \leq x \leq \frac{5\pi}{2}$

(a) Complete the table below with the exact value of y corresponding to $x = \frac{7\pi}{4}$ and with

the exact value of y corresponding to $x = \frac{9\pi}{4}$

x	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$
у	0		2π		0

(b) Use the trapezium rule, with all five y values in the completed table, to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

(3)

(1)

(c) Find

$$\int x \cos x \, \mathrm{d}x$$

- (d) Using your answer from part (c), find the exact area of the region R.
- (2)



Qu	Scheme	Marks
5.(a)	$\frac{7\pi}{4\sqrt{2}}$ or equivalent e.g. $\frac{7\pi\sqrt{2}}{8}$ AND $\frac{9\pi}{4\sqrt{2}}$ or equivalent e.g. $\frac{9\pi\sqrt{2}}{8}$	B1 (1)
(b)	$\frac{1}{2} \times \frac{\pi}{4} \times \underbrace{\{\dots,\dots,\dots,\}}$	B1 oe
	$\frac{1}{2} \times h \times \left\{ 0 + 0 + 2 \left("\frac{7\pi}{4\sqrt{2}} " + 2\pi + "\frac{9\pi}{4\sqrt{2}} " \right) \right\}$	M1
	= 11.91 (only)	A1 (3)
(c)	$\int x \cos x dx = [x \sin x] - \int \sin x dx$ $= x \sin x + \cos x (+c)$	M1 dM1 A1 (3)
(d)	$\left[x\sin x + \cos x\right]_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} = \frac{5\pi}{2} + \frac{3\pi}{2} = 4\pi$	M1 A1 (2) (9 marks)
(a) B1: Bot	h correct (as above) Must be exact and not decimal	(>)
(b)	1π π	
B1: See	$\frac{1}{2} \times \frac{\pi}{4} \times \text{as part of trapezium rule or } h = \frac{\pi}{4}$ stated or used. This can be scored if 'h' is in an u	nsimplified
form. M1: Co	rrect structure of the bracket in the trapezium rule.	
Yo	u may not see the zero's Eg $2\left(\left\ \frac{7\pi}{4\sqrt{2}}\right\ +2\pi+\left\ \frac{9\pi}{4\sqrt{2}}\right\ \right)$	
A1: 11.9 (c)	91 only. This may be a result of using the decimal equivalents. Sight of 11.91 will score all 3 r	marks
M1: For	r a correct attempt at integration by parts to give an expression of the form $[\pm x \sin x] - \int \pm \sin x$	x dx
If you s	ee such an expression you would only withhold the mark if there is evidence of an incorrect f	ormula
(seen or	$ (mplied) Eg \int u dv = uv + \int v du $	
A1: cso	or $\perp x \sin x \perp \cos x$ following line one	
Allow a	Il three marks for candidate who just writes down the correct answer with no working $D I$	
Watch f	For candidates who write down methods like this. $\begin{array}{c} x & \cos x \\ 1 & \sin x \end{array}$ and then write $x \sin x - (-\cos x) = -\cos x$	(\mathbf{x}, \mathbf{x})
This is a answer if for the a	a commonly taught algorithm (They differentiate down the lh column and integrate on the rh c is found by $D1 \times I2 - D2 \times I3$ where D2 is the second entry in the D column. This can score inswer $x \sin x + \cos x$ but also pick up methods for slips.	olumn. The e full marks
If they a (d)	ttempt $D1 \times I2 + D2 \times I3$ it is M0 as they are implying an incorrect formula. Ask your TL if	unclear.
M1: Us	ing limits $\frac{5\pi}{2}$ and $\frac{3\pi}{2}$ correctly in their answer to part (c) - substituting (seen correctly in all t	terms or
implied A1: 4π	in all terms) and subtracting either way around τ or equivalent single term. CSO. It must have been derived from $x \sin x + \cos x$	

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6. (i) I	Differentiate $y = 5x^2 \ln 3x$, $x > 0$	
	Since that	(2)
(11) (viven that	
	$y = \frac{x}{\sin x + \cos x}, \qquad -\frac{x}{4} < x < \frac{3x}{4}$	
ç	how that	
5	dy $(1+x)\sin x + (1-x)\cos x$ $\pi = 3\pi$	
	$\frac{1}{\mathrm{d}x} = \frac{1}{1+\sin 2x}, \qquad -\frac{1}{4} < x < \frac{1}{4}$	
		(4)

P 4 8 3 2 5 A 0 1 8 4 8

Qu	Scheme	Marks		
6.(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 5x^2 \times \frac{3}{3x} + \ln(3x) \times 10x$	M1 A1		
		(2)		
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{(x + \cos x)^2}$	M1		
	$\frac{\mathrm{d}x}{\mathrm{d}x} = \frac{(\sin x + \cos x)}{(\sin x + \cos x)\mathbf{l} - x(\cos x - \sin x)} = \frac{(\sin x + \cos x)\mathbf{l} - x(\cos x - \sin x)}{(\sin x + \cos x)\mathbf{l} - x(\cos x - \sin x)}$	B1 B1		
	$dx (\sin^2 x + \cos^2 x) + (2\sin x \cos x) \qquad 1 + \sin 2x$ $dy (1+x)\sin x + (1-x)\cos x *$	A1 *		
	$\frac{dy}{dx} = \frac{(1+x)\sin x + (1-x)\cos x}{1+\sin 2x}$	(4)		
		(6 marks)		
(i)				
M1: A	pplies the Product rule to $y = 5x^2 \ln 3x$			
Exp	ect $\frac{dy}{dx} = Ax + Bx \ln(3x)$ for this mark (A, B positive constant)			
A1: ca	o- need not be simplified			
(ii)	r			
M1: A	M1: Applies the Quotient rule, a form of which appears in the formula book, to $y = \frac{x}{\sin x + \cos x}$			
Expect $\frac{dy}{dt} = \frac{(\sin x + \cos x) 1 - x(\pm \cos x \pm \sin x)}{(\sin x)^2}$ for M1				
Condone invisible brackets for the M and an attempted incorrect 'squared' term on the denominator				
Eg sin ² $x + \cos^2 x$				
B1: Denominator should be expanded to $\sin^2 x + \cos^2 x + \dots$ and $(\sin^2 x + \cos^2 x) \rightarrow 1$				
B1: Denominator should be expanded to + $k \sin x \cos x$ and $(k \sin x \cos x) \rightarrow \frac{\kappa}{2} \sin 2x$.				
For	For example sight of $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x = 1 + \sin 2x$ without the intermediate line on the			
A1: cs	o – answer is given. This mark is withheld if there is poor notation $\cos x \leftrightarrow \cos \sin^2 x \leftrightarrow \sin x^2$			
If the	only error is the omission of $(\sin^2 x + \cos^2 x) \rightarrow 1$ then this final A1* can be awarded.			
Use of product rule or implicit differentiation needs to be applied correctly with possible sign errors differentiating functions for M1, then other marks as before. If quoted the product rule must be correct				
Produc	et rule $\frac{dy}{dx} = (\sin x + \cos x)^{-1} \times 1 \pm x \times (\sin x + \cos x)^{-2} (\pm \cos x \pm \sin x)$			
Implicit differentiation $(\sin x + \cos x) y = x \Rightarrow (\sin x + \cos x) \frac{dy}{dx} + y(\pm \cos x \pm \sin x) = 1$				
To score the B's under this method there must have been an attempt to write $\frac{dy}{dx}$ as a single fraction				

Mathematics C34 www.mystudybro.com This resource was created and owned by Pearson Edexcel WMA02 Past Paper Leave blank 7. 0 1 x $\frac{-}{3}$ Figure 2 Figure 2 shows a sketch of the graph of y = f(x), $x \in \mathbb{R}$. The point $P\left(\frac{1}{3}, 0\right)$ is the vertex of the graph. The point Q(0, 5) is the intercept with the y-axis. Given that f(x) = |ax + b|, where *a* and *b* are constants, (a) (i) find all possible values for *a* and *b*, (ii) hence find an equation for the graph. (4) (b) Sketch the graph with equation $y = f\left(\frac{1}{2}x\right) + 3$ showing the coordinates of its vertex and its intercept with the y-axis. (3)



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Qu	Scheme	Marks	
7 (a)(i)	Substitute (0, 5) to give $ b = 5$ so $b = \pm 5$	B1	
	Substitute $\left(\frac{1}{3}, 0\right)$ to give $\left \frac{1}{3}a + b\right = 0$ so $a = \mp 15$	M1 A1	
(ii)	Gives equation as $y = -15x+5 $ or $y = 15x-5 $	B1 (1)	
(b)	y Q Q P at $\left(\frac{2}{3}, 3\right)$ Q at $(0, 8)$ Q at $(0, 8)$	(4) B1 B1 (3)	
		(7 marks)	
(a) (i) B1: For both $b = \pm 5$ not just $ b = 5$ M1: Substitute $(\frac{1}{3}, 0)$ to give $ \frac{1}{3}a + b = 0$. This mark is implied by $a = \pm 3 \times$ value of b A1: $a = 15$ corresponding to $b = -5$ and $a = -15$ corresponding to $b = 5$ If they write down an equation rather than giving values of a and b then Just $y = -15x+5 $ or $y = 15x-5 $ scores B0, M1, A0 Both $y = -15x+5 $ and $y = 15x-5 $ scores B1, M1, A1 Linear equations $y = -15x+5$ and/or $y = 15x-5($ without the modulus) only score B0 M1 A0 (ii) Note that this is an A1 mark on e-pen B1: $y = -15x+5 $ or $y = 15x-5 $ or allow equations such as for this mark only $f(x) = \begin{cases} \frac{15x-5}{-15x+5} & x < \frac{1}{3} \\ -15x+5 & x < \frac{1}{3} \end{cases}$ If candidates don't state (i), (ii) and write down just $y = -15x+5 $ they would score (i) B0 M1 A0 (ii) B1 (b) There must be a sketch to score any of these marks. B1: V shape the correct way up any position but not on the x - axis. Accept V's that don't have symmetry B1: P at $(\frac{2}{3}, 3)$ Score if the coordinates are stated within the text OR marked on the axes. If they appear in both then the graph takes precedence. B1: For crossing the y - axis at (0, 8). Accept 8 marked on the correct axis. Condone (8,0) marked on the correct axis			





$\tan(2x+x) = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ $2 \tan x$	M1		
$= \frac{\frac{1}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x}$	dM1		
$= \frac{2\tan x + \tan x(1 - \tan^2 x)}{1 - \tan^2 x - 2\tan^2 x} \text{OR} = \frac{\frac{2\tan x + \tan x - \tan^3 x}{1 - \tan^2 x}}{\frac{1 - \tan^2 x - 2\tan^2 x}{1 - \tan^2 x}} \text{oe}$	A1		
So $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} *$	A1*cso (4)		
Put $\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = 11\tan x$ so $3\tan x - \tan^3 x = 11\tan x(1 - 3\tan^2 x)$	M1		
$32\tan^3 x = 8\tan x$	A1		
So $\tan x = \pm \frac{1}{2}$ or $0 \Longrightarrow x =$	dM1		
$\Rightarrow x = \text{awrt } 26.6^{\circ} , -26.6^{\circ} , 0$	A1 A1		
	(5) (9 marks)		
(a) M1: Expands $\tan(2x+x) = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ condoning sign errors M1: Uses the correct double angle formula $2 \tan x$ hoth times within their expression for $\tan(2x+x)$			
$1 - \tan^2 x$)		
A1: Multiplies both numerator and denominator by $1 - \tan^2 x$ to obtain a correct intermediate line $Eg = \frac{2 \tan x + \tan x(1 - \tan^2 x)}{1 - \tan^2 x - 2 \tan^2 x}$ or similar. Alternatively they write both numerator and denominator as single correct fractions. They cannot just write down the final given answer for this mark A1*: Correct printed answer achieved with no errors and all of the lines in the markscheme (c.s.o.) Withhold the final A1 for candidates who use poor notation or mixed variables.			
Examples of poor notation would include $\tan \leftrightarrow \tan x \tan^2 x \leftrightarrow \tan x^2 \tan 2x = \frac{2 \tan \theta}{1 - \tan^2 \theta}$			
(b) M1: Attempts to use the given identity and multiplies $by 1-3 \tan^2 x$. Condone slips A1: Obtain $32\tan^3 x = 8\tan x$ or equivalent. Accept $32\tan^2 x = 8$ for this mark dM1: Obtains one value of x from $\tan x =$ using a correct method for their equation. The order of operations to find x must be correct but can be scored from $\tan x = 0 \Rightarrow x = 0$ A1: Either one of $x = 26.6^\circ$ or -26.6° or in radians ± 0.46 A1: CAO $x = awrt 26.6^\circ$, $awrt - 26.6^\circ$, 0 (do not need degrees symbol) with no extras within the range Note: Answers only scores 0 marks. Answers from a correct cubic/quadratic scores M1 A1 dM1 (implied) then			
	$\tan(2x + x) = \frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x}} + \tan x}$ $= \frac{2 \tan x + \tan x(1 - \tan^2 x)}{1 - \tan^2 x - 2 \tan^2 x} OR = \frac{\frac{2 \tan x + \tan x - \tan^3 x}{1 - \tan^2 x}}{\frac{1 - \tan^2 x}{1 - \tan^2 x}} e$ $= \frac{2 \tan x + \tan x(1 - \tan^2 x)}{1 - \tan^2 x - 2 \tan^2 x} OR = \frac{\frac{2 \tan x + \tan x - \tan^3 x}{1 - \tan^2 x}}{\frac{1 - \tan^2 x - 2 \tan^2 x}{1 - \tan^2 x}} e$ So $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$ Put $\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} = 11 \tan x$ so $3 \tan x - \tan^3 x = 11 \tan x(1 - 3 \tan^2 x)$ $32 \tan^3 x = 8 \tan x$ So $\tan x = \frac{\pm 1}{2}$ or $0 \Rightarrow x =$ $\Rightarrow x = \operatorname{awrt} 26.6^\circ, -26.6^\circ, 0$ adds $\tan(2x + x) = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ condoning sign errors s the correct double angle formula $\frac{2 \tan x}{1 - \tan^2 x}$ both times within their expression for $\tan(2x)$ plies both numerator and denominator by $1 - \tan^2 x$ to obtain a correct intermediate line $\frac{2 \tan x + \tan x(1 - \tan^2 x)}{1 - \tan^2 x}$ or similar. atively they write both numerator and denominator as single correct fractions. cannot just write down the final given answer for this mark exet printed answer achieved with no errors and all of the lines in the markscheme (c.s.o.) hold the final A1 for candidates who use poor notation or mixed variables. mples of poor notation would include $\tan \leftrightarrow \tan x \tan^2 x \leftrightarrow \tan x^2 \tan 2x = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ mpts to use the given identity and multiplies by $1 - 3\tan^2 x$. Condone slips an $32\tan^3 x = 8$ for this mark ains one value of x from $\tan x =$ using a correct method for their equation. The order o must be correct but can be scored from $\tan x = 0 \Rightarrow x = 0$ rom of $x = 26.6^\circ \text{ on } -26.6^\circ$, 0 (do not need degrees symbol) with no extras within the re were sonly scores 0 marks. Answers from a correct cubic/quadratic scores M1 A1 dM1 (in the re)		

Mathematics C34

WMA02



(a) By using the substitution u = 2x + 3, show that

$$\int_{0}^{12} \frac{x}{(2x+3)^2} \, \mathrm{d}x = \frac{1}{2} \ln 3 - \frac{2}{9} \tag{7}$$

The curve C has equation

26

 $y = \frac{9\sqrt{x}}{(2x+3)}, \quad x > 0$

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis and the line with equation x = 12. The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Use the result of part (a) to find the exact value of the volume of the solid generated. (2)



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Qu	Scheme	Marks	
9(a)	$\frac{\mathrm{d}u}{\mathrm{d}x} = 2$	B1	
	$\left(\int \frac{x}{(2x+3)^2} \mathrm{d}x\right) = \int \frac{u-3}{4u^2} \mathrm{d}u$	M1	
	$= \int \frac{1}{4} u^{-1} - \frac{3}{4} u^{-2} \mathrm{d}u$	dM1	
	$= \frac{1}{4} \ln u + \frac{3}{4} u^{-1}$	ddM1 A1	
	$\left[\frac{1}{4}\ln u + \frac{3}{4}u^{-1}\right]_{2}^{27} = \frac{1}{4}\ln 27 + \frac{3}{4} \times \frac{1}{27} - \left(\frac{1}{4}\ln 3 + \frac{3}{4} \times \frac{1}{3}\right) = \frac{1}{4}\ln 9 - \frac{8}{36}$	M1	
	$=\frac{1}{2}\ln 3 - \frac{2}{9}*$	A1*	
	a12 a	(7)	
(b)	$V = \pi \times \int_{0}^{12} \left(\frac{9\sqrt{x}}{2x+3}\right)^2 \mathrm{d}x$	M1	
	$= 81\pi \left(\frac{1}{2}\ln 3 - \frac{2}{9}\right)$	A1 (2)	
	(2 9)	(2) (9 marks)	
(a) B1: States or uses $\frac{du}{dx} = 2$ or equivalent such as $\frac{dx}{du} = 0.5$ or $dx = \frac{1}{2}du$			
M1: Expression of the form $k \int \frac{u-3}{u^2} du$ and allow missing du and/or missing integral sign			
d M1: Spi	lits into the form $u^{-1} \pmu^{-2}$ and again allow missing du and/or missing integral sign.		
Alternativ	ely they could use integration by parts at this stage $\int \frac{u-3}{4u^2} du = \pm a \frac{(u-3)}{u} \pm b \int \frac{1}{u} du$ A	small	
number o	of candidates will also use partial fractions that gives the same answer as the main (x, z)	scheme.	
ddM1: Fo	or $\ln u \pm u^{-1}$ or 'obviously $\ln 4u \pm u^{-1}$ or by parts $\pm a \frac{(u-3)}{u} \pm b \ln u$		
A1: $\frac{1}{4} \ln u$	$u + \frac{3}{4}u^{-1}$ This answer or equivalent such as $\frac{1}{4}\ln 4u + \frac{3}{4}u^{-1}$ or by parts $-\frac{(u-3)}{4} + \frac{1}{4}\ln u$		
M1: Applies limits of 27 and 3 to the result of integrating their function in u , subtracts the correct way around and combines the ln terms correctly. Alternatively using $u = 2x + 3$ applies the limits $x = 12$ and 0 to the result of their adapted function subtracts the correct way round and combines the ln terms correctly. A1*: given answer achieved correctly without errors. The only omission that would be allowed could be the dM1 line which could be implied. You need to see an intermediate step with correct ln work before the final answer is reached. (b)			
M1: Atter	npts to use part (a) to find the exact volume. Accept $\pi \times \int_{0}^{12} \left(\frac{9\sqrt{x}}{2x+3}\right)^2 dx$		
Condone only the omission of π or 81 or a bracket for this M mark so accept $81(\frac{1}{2}\ln 3 - \frac{2}{9})$ or $\pi(\frac{1}{2}\ln 3 - \frac{2}{9})$ or			
$\frac{1}{2}\ln 3 - \frac{2}{9} \times 81\pi$ as evidence			
A1: Any correct exact equivalent in terms of ln3 and π Accept for example $81\pi (\ln \sqrt{3}) - 18\pi$			

It is possible to do 9(a) by parts or via partial fractions **without** using the given substitution. This does not satisfy the demands of the question but should score some marks. A fully correct solution via either method scores 5 out of 7

Qu	Scheme		Marks
9(a)	By Parts	By Partial Fractions	В0
		$\int x(2x+3)^{-2} dx = \int \frac{\frac{1}{2}}{(2x+3)} + \frac{-\frac{3}{2}}{(2x+3)^{2}} dx$	
	$\int x (2x+3)^{-2} dx = \frac{x (2x+3)^{-1}}{-2} + \int \frac{(2x+3)^{-1}}{2} dx$	One term of $=\frac{1}{4}\ln(2x+3) + \frac{3(2x+3)^{-1}}{4}$	M1
	$=\frac{x(2x+3)^{-1}}{-2}+\frac{1}{4}\ln(2x+3)$	Both terms of $=\frac{1}{4}\ln(2x+3) + \frac{3(2x+3)^{-1}}{4}$	dM1
	Attempts limits = $\left[\frac{x(2x+3)^{-1}}{-2} + \frac{1}{4}\ln(2x+3)\right]_{0}^{12}$	$= \left[\frac{1}{4}\ln(2x+3) + \frac{3(2x+3)^{-1}}{4}\right]_{0}^{12}$	ddM1
	Correct un simplified answer $= -\frac{2}{9} + \frac{1}{4} \ln 27 - \frac{1}{4} \ln 3$	$=\frac{1}{4}\ln 27 + \frac{1}{36} - \frac{1}{4}\ln 3 - \frac{1}{4}$	A1
	Collects log terms $= -\frac{2}{9} + \frac{1}{4} \ln\left(\frac{27}{3}\right)$	$=-\frac{2}{9}+\frac{1}{4}\ln\left(\frac{27}{3}\right)$	M1
	$=\frac{1}{2}\ln 3 - \frac{2}{9}$		A0*
			(7)

Mathematics C34

WMA02

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Leave blank 10. A population of insects is being studied. The number of insects, N, in the population, is modelled by the equation $N = \frac{300}{3+17\mathrm{e}^{-0.2t}} \quad t \in \mathbb{R}, \ t \ge 0$ where *t* is the time, in weeks, from the start of the study. Using the model, (a) find the number of insects at the start of the study, (1) (b) find the number of insects when t = 10, (2) (c) find the time from the start of the study when there are 82 insects. (Solutions based entirely on graphical or numerical methods are not acceptable.) (4) (d) Find, by differentiating, the rate, measured in insects per week, at which the number of insects is increasing when t = 5. Give your answer to the nearest whole number. (3)



Qu	Scheme	Marks	
10. (a)	When $t = 0$ $N = 15$	B1 (1)	
(b)	Puts $t = 10$ so $N = 56.6$ (accept 56 or 57)	M1A1 (2)	
(c)	$82 = \frac{300}{3 + 17e^{-0.2t}} \implies e^{-0.2t} = \frac{54}{1394} = awrt \ 0.039$	M1 A1	
	$-0.2t = \ln\left(\frac{54}{1394}\right) \Longrightarrow t =$	dM1	
	t = awrt 16.3	A1 (4)	
(d)	$\frac{dN}{dt} = (-0.2) \times 300 \times (-1) \times 17e^{-0.2t} (3 + 17e^{-0.2t})^{-2}$	M1 A1	
	=4.38 so 4 insects per week	A1 cso (3)	
		(10 marks)	
(a) B1: 15 cao (b)			
M1: Substitutes $t = 10$ into the correct formula. Sight of $N = \frac{300}{3+17e^{-0.2\times 10}}$ is fine			
A1: Accept (c)	t 56 or 57 or awrt 56.6. These values would imply the M.		
M1: Subst	itutes 82 and proceeds to obtain $e^{\pm 0.2t} = C$ Condone slips on the power $27 \qquad 0.2t \qquad 697$ Accept decimals Eq. (9.2t) $= 0.020$ or $0.2t \qquad 0.020$		
$\mathbf{dM1} \cdot \mathbf{Depe}$	$=\frac{1}{697}$ $\stackrel{\text{oe}}{=}$ $=\frac{1}{27}$ $\stackrel{\text{oe}}{=}$ $\stackrel{\text{oe}}{=}$ $=\frac{1}{27}$ $\stackrel{\text{oe}}{=}$ $\stackrel{\text{oe}}{$		
A1: awrt	16.3 Accept 16 (weeks), 16.25 (weeks), 16 weeks 2 days or 17 weeks following correct log (1204)	g work and	
acceptable	accuracy. Accept $t = 5\ln\left(\frac{1394}{54}\right)oe$ for this mark		
It is possible to answer this by taking ln's at the point $1394e^{-0.2t} = 54$ M1A1 $\ln(1394) - 0.2t = \ln(54)$ dM1 A1 As scheme			
(d)			
M1: Differentiates to give a form equivalent to $\frac{dN}{dt} = ke^{-0.2t}(3+17e^{-0.2t})^{-2}$ (may use quotient rule)			
A1: Correct derivative which may be unsimplified $\frac{dN}{dt} = 1020e^{-0.2t}(3+17e^{-0.2t})^{-2}$			
A1: Obtain	s awrt 4 following a correct derivative . This is cso		

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This resource was created and owned by Pearson Edexcel WMA02 Past Paper Leave blank 11. (a) Express $35 \sin x - 12 \cos x$ in the form $R \sin(x - \alpha)$, where R > 0 and $0 < \alpha < \frac{\pi}{2}$ Give the exact value of R, and give the value of α , in radians, to 4 significant figures. (3) (b) Hence solve, for $0 \leq x < 2\pi$, $70 \sin x - 24 \cos x = 37$ (Solutions based entirely on graphical or numerical methods are not acceptable.) (4) $y = \frac{7000}{31 + (35\sin x - 12\cos x)^2},$ x > 0(c) Use your answer to part (a) to calculate (i) the minimum value of y, (ii) the smallest value of x, x > 0, at which this minimum value occurs. (4)



Ou	Scheme	Marks	
11. (a)	R = 37	B1	
	12		
	$\tan \alpha = \frac{1}{35} \implies \alpha = \text{awrt } 0.3303$	MI AI	
		(3)	
(b)	$\sin(x-\alpha) = \frac{37}{2R}$ (= 0.5)	M1	
	$x = \arcsin\left(\frac{37}{2 \times \text{their "}37"}\right) + \text{their "}0.3303"$	M1	
	x = awrt 0.854 or awrt 2.95	A1	
	x = awrt 0.854 and awrt 2.95	A1 (4)	
(c)(i)	Find $y = \frac{7000}{31 + (\pm "R")^2} = 5$	M1 A1	
(c)(ii)	$x - \alpha = \frac{\pi}{2} \implies x = 1.90$	M1 A1	
	-	(4) (11 marks)	
(a)		(11 marks)	
B1 : $R =$	37 no working needed. Condone $R = \pm 37$		
M1: tan	$\alpha = \pm \frac{12}{35}$ or $\tan \alpha = \pm \frac{35}{12}$ with an attempt to find alpha. Accept decimal attempts from		
tan a	$\alpha = \operatorname{awrt} \pm 0.343$ or $\tan \alpha = \operatorname{awrt} \pm 2.92$ If <i>R</i> is used allow $\sin \alpha = \pm \frac{12}{R}$ OR $\cos \alpha = \pm \frac{35}{R}$ with an	attempt to	
find	alpha R R		
A1 : α	= awrt 0.3303. Answers in degrees are A0		
(b)	37		
M1 : (Us	es part (a) to solve equation) $\sin(x \pm \alpha) = \frac{37}{2 \times theirR}$		
M1: oper	rations undone in the correct order to give $x = \dots$ Accept $sin(x \pm \alpha) = k \Rightarrow x = \arcsin k \pm \alpha$		
A1: one	correct answer to within required accuracy. Allow 0.272π or 0.938π .		
Cond	done for this mark only both $\frac{\pi}{6}$ + 0.3303 and $\frac{5\pi}{6}$ + 0.3303		
A1: both values (and no extra values in the range) correct to within required accuracy. Allow 0.272π , 0.938π			
(c) (i)			
M1: For	an attempt at $\frac{7000}{31 + (+"R")^2}$		
A1: 5	$JI + (\pm K)$		
(c)(ii)			
M1: Uses x – their $\alpha = (2n+1)\frac{\pi}{2}$ to find x This may be implied by 1.57 ± their 0.33 stated or calculated (2dp)			
A1: Awrt 1.90 but condone 1.9 for this answer			
Answers in degrees, withhold the first time seen, usually part (a). FYI (a) 18.92° (b) $48.9^{\circ}, 168.9^{\circ}$ (c)(ii) 108.9°			

WMA02 Leave

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12. In freezing temperatures, ice forms on the surface of the water in a barrel. At time t hours after the start of freezing, the thickness of the ice formed is x mm. You may assume that the thickness of the ice is uniform across the surface of the water.

At 4pm there is no ice on the surface, and freezing begins.

At 6pm, after two hours of freezing, the ice is 1.5 mm thick.

In a simple model, the rate of increase of x, in mm per hour, is assumed to be constant for a period of 20 hours.

Using this simple model,

- (a) express t in terms of x,
- (b) find the value of *t* when x = 3

(1)

(3)

(1)

(2)

(2)

In a second model, the rate of increase of x, in mm per hour, is given by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\lambda}{(2x+1)} \text{ where } \lambda \text{ is a constant and } 0 \leqslant t \leqslant 20$$

Using this second model,

(c) solve the differential equation and express t in terms of x and λ ,

(d) find the exact value for λ ,

(e) find at what time the ice is predicted to be 3 mm thick.

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Qu	Scheme		Ma	rks
	Way 1:	Way 2:		
12. (a)	Uses $x = kt$ or $t = cx$ and $x = 1.5$ when $t = 2$	Uses $x = kt + c$ with $x = 0, t = 0$ and with $x = 1.5$ when $t = 2$ as k	M1	
	so $k = $ or $c = $	with $x = 1.5$ when $t = 2$ so $k = 4$	A1	
	$t = \frac{1}{3}x$	$t = \frac{1}{3}x$		(2)
(b)	t = 4		B1	
	J., 1			(1)
(c)	$\frac{dx}{dt} = \frac{\lambda}{(2x+1)}$ so separate variables to give $\int dt$	$(2x+1)\mathrm{d}x = \int \lambda\mathrm{d}t$	M1	
	$x^{2} + x = \lambda t(+c')$ or $(2x+1)^{2} = \lambda t(+c)$ so $t =$		M1	ľ
	4		A 1	
	(When $t = 0, x = 0$ so $c = \frac{1}{4}$ or $c' = 0$) so $t = \frac{1}{4}$	$=\frac{x+x}{\lambda}$	AI	(3)
		λ	DI	
(d)	Uses $x = 1.5$ when $t = 2$ to give $\lambda = \frac{15}{2}$		BI	(1)
	8			(_)
(e)	$r^{2} + r = 12$			
	$t = \frac{x + x}{\lambda} = \frac{12}{\lambda} = 6.4$ hours later so		M1	
	<u>10.24pm or 22.24</u>		Δ 1	
			$\frac{\overline{A1}}{(2)}$	
			(9 ma	rks)
Mark (a) and (b) together (a)				
M1: Uses	correct $x = kt$ or $t = cx$ and $x = 1.5$ when $t = 2$	to find their constant (may not be $k \text{ or } c$)		
This may be the result of a differential equation $\frac{dx}{dt} = k$				
A1: $t = \frac{4}{3}x$ or such as $t = \frac{x}{0.75}$ or even $t = \frac{x}{\frac{3}{4}}$ Just this with no working is M1 A1				
(b) B1 : $t = 4$				
Mark (c),(d	l) and (e) together			
(c) M1: Correc	ct separation but condone missing integral sign	s		
M1: Correc	ct form for both integrals- may not find c or even	en include a c		
A1: Obtains a correct answer for t in terms of x and λ by using either $x = 0, t = 0 \implies t = \frac{x^2 + x}{\lambda}$ or				
$t = \frac{(2x+1)^2 - 1}{4x^2}$ oe. Alternatively uses $x = 1.5, t = 2 \implies t = \frac{4x^2 + 4x + 8\lambda - 15}{4x^2}$ oe				
Condone	correct responses where ' c ' seems to have been	$\frac{4\pi}{1}$ either cancelled out or ignored		
(d)				
B1 : $\lambda = \frac{15}{8}$ or decimal i.e. 1.875				
(e)				
M1: Substitutes $x = 3$ into their expression for <i>t</i> . Implied by $t = \frac{12}{2}$				
A1: 10.24pm or 22:24 only				

Past Paper

Mathematics C34

WMA02

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The curve C shown in Figure 4 has parametric equations

 $x = 1 + \sqrt{3} \tan \theta$, $y = 5 \sec \theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

The curve C crosses the y-axis at A and has a minimum turning point at B, as shown in Figure 4.

- (a) Find the exact coordinates of A.
- (b) Show that $\frac{dy}{dx} = \lambda \sin \theta$, giving the exact value of the constant λ . (4)
- (c) Find the coordinates of *B*.
- (d) Show that the cartesian equation for the curve *C* can be written in the form

$$y = k\sqrt{(x^2 - 2x + 4)}$$

where *k* is a simplified surd to be found.

(3)

(3)

(2)



Qu	Scheme	Marks	
13 (a)	Puts $x = 0$ and obtains $\theta = -\frac{\pi}{6}$	B1	
	Substitutes their θ to obtain $y = \frac{10\sqrt{3}}{3}$ or $\left(0, \frac{10\sqrt{3}}{3}\right)$	M1 A1	(3)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}} = \frac{5\sec\theta\tan\theta}{\sqrt{3}\sec^2\theta}$	M1 A1	
	$=\frac{5\times\sin\theta/\cos\theta}{\sqrt{3}\times\frac{1}{\cos\theta}}$	B1	
	$= \frac{5}{\sqrt{3}} \sin \theta \text{or} \lambda = \frac{5}{\sqrt{3}} \text{ oe}$	A1	(4)
(c)	Puts $\frac{dy}{dx} = 0$ and obtains θ and calculates x and y or deduces correct answer	M1	
	Obtains (1, 5)	A1	(2)
(d)	$\tan \theta = \frac{x-1}{\sqrt{3}}$ and $\sec \theta = \frac{y}{5}$	M1	
	Uses $1 + \tan^2 \theta = \sec^2 \theta$ to give $1 + "\left(\frac{x-1}{\sqrt{3}}\right)^2 " = "\left(\frac{y}{5}\right)^2 "$	M1	
	$\frac{3+x^2-2x+1}{3} = \left(\frac{y}{5}\right)^2 \text{so} y = \frac{5}{3}\sqrt{3}\sqrt{x^2-2x+4} \text{*}$	A1*	(3)
Alt 1(d)		(12 marks)	
()	$y = 5\sqrt{1 + \tan^2 \theta}, = 5\sqrt{1 + \left(\frac{x - 1}{\sqrt{3}}\right)}$	M1, M1	
	$y = \frac{5}{3}\sqrt{3}\sqrt{(x^2 - 2x + 4)}$ *	A1*	(3)
	Assume $y = k \sqrt{x^2 - 2x + 4}$ and sub both $x = 1 + \sqrt{3} \tan \theta$ and $y = 5 \sec \theta$		
	$5 \sec \theta = k \times \sqrt{3 + 3 \tan^2 \theta}$	M1	
Alt 2 (d)	$5 \sec \theta = k \times \sec \theta \sqrt{3}$	M1	
	$k = \frac{5}{3}\sqrt{3}$ AND conclusion "hence true"	A1*	
			(3)
		1	(-)

(a)

B1: For $\theta = -\frac{\pi}{6}$ or -30° or awrt -0.52 but may be awarded for $\cos \theta = \frac{\sqrt{3}}{2}$ or $\sec \theta = \frac{2}{\sqrt{3}}$ if θ is not explicitly found **M1:** Substitutes their θ (or their $\cos \theta$ or $\sec \theta$) found from an attempt at x = 0 to give y **A1:** cao. Accept $y = \frac{10}{\sqrt{3}}$ Correct answer with no incorrect working scores all 3 marks. Note that $\theta = \frac{\pi}{6}$ also gives $y = \frac{10\sqrt{3}}{3}$ but scores B0 M1 A0

QuSchemeMarks(b)M1: Attempts to differentiate both x and y wrt
$$\theta$$
 and establishes $\left(\frac{dy}{dx}\right) = \frac{dx}{dt}$ $\frac{dy}{dt} = \frac{dx}{dt}$ A1: Correct derivatives and correct fractionB1: For either $\lambda = \frac{5}{3}\sqrt{3}$ (seen explicitly stated or implied) or use of $\sec \theta = \frac{1}{\cos \theta}$ An alternative to seeing $\sec \theta = \frac{1}{\cos \theta}$ is $\frac{1}{\sec \theta} = \cos \theta$ A1: Fully correct solution showing all relevant steps with correct notation, no mixed variables and no errors. $\frac{\tan \theta}{\sec \theta}$ cannot just be written as $\sin \theta$ without an intermediate line of working $\frac{(\cos \theta)^{-2} \sin \theta}{\sec^{2} \theta}$ cannot just be written as $\sin \theta$ without an intermediate line of working $\frac{(\cos \theta)^{-2} \sin \theta}{\cos \theta} = \tan \theta$ (c)M1: Sets their $\frac{dy}{dx} = 0$ and proceeds to find (x, y) from their θ A1: for $(1, 5)$ or $x = 1, y = 5$ (d)M1: Attempt to obtain $\tan \theta$ and $\sec \theta$ in terms of x and y respectively. Allow $\tan \theta = \frac{x \pm 1}{\sqrt{3}} \sec \theta = \frac{y}{5}$ M1: Uses $1 + \tan^{2} \theta = \sec^{2} \theta$ with their expressions for $\tan \theta$ and $\sec \theta$ in terms of x and y respectivelyA1*: Obtains printed answer with no errors and with $k = \frac{5}{3}\sqrt{5}$ onlyYou do not need to see k explicitly stated as $\frac{5}{3}\sqrt{5}$, it is fine to be embedded within the formula

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14. <i>ABCD</i> is a parallelogram with <i>AB</i> parallel to <i>DC</i> and <i>AD</i> parallel to <i>BC</i> .	L b
The position vectors of A , B , C , and D relative to a fixed origin O are \mathbf{a} , \mathbf{b} , \mathbf{c} are respectively.	nd d
Given that	
a = i + j - 2k, $b = 3i - j + 6k$, $c = -i + 3j + 6k$	
(a) find the position vector d ,	(3)
(b) find the angle between the sides AB and BC of the parallelogram,	(4)
(c) find the area of the parallelogram <i>ABCD</i> .	(2)
The point E lies on the line through the points C and D , so that D is the midpoint of	CE.
(d) Use your answer to part (c) to find the area of the trapezium <i>ABCE</i> .	(2)

Qu	Scheme	Marks	
14 (a)	Attempts $\overrightarrow{BA} = \mathbf{a} - \mathbf{b} = -2\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}$ or $\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = -4\mathbf{i} + 4\mathbf{j}$ either way around	M1	
	Finds $\overrightarrow{OD} = \mathbf{a} - \mathbf{b} + \mathbf{c} = (-2\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}) + (-\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}) = -3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$	M1 A1	
			(3)
(b)	$\overrightarrow{BA} = \mathbf{a} - \mathbf{b} = -2\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}$ and $\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = -4\mathbf{i} + 4\mathbf{j}$	M1	
	$\cos \theta = \frac{\begin{pmatrix} -2\\2\\-8 \end{pmatrix} \begin{pmatrix} -4\\4\\0 \end{pmatrix}}{\sqrt{(-2)^2 + 2^2 + (-8)^2} \sqrt{(-4)^2 + 4^2}} = \frac{16}{\sqrt{72}\sqrt{32}} = \frac{1}{3}$ So angle is 1.23 radians or 70.5 degrees	dM1 A1 A1	(4)
(c)	Area = $\sqrt{72}\sqrt{32}\sin\theta = 45.3$ or $32\sqrt{2}$ oe	M1A1	(2)
(d)	Area = $\frac{3}{2} \times "45.3" = 67.9$ or $48\sqrt{2}$ oe	M1 A1	(2)
		(11 marks	s)

(a)

M1: For attempting one of $\mathbf{b} - \mathbf{a}$ or $\mathbf{a} - \mathbf{b}$ or $\mathbf{c} - \mathbf{b}$ or $\mathbf{b} - \mathbf{c}$. It must be correct for at least one of the components. Condone coordinate notation for the first two M marks

M1: For attempting $\mathbf{d} = \mathbf{a} - \mathbf{b} + \mathbf{c} = \mathbf{I}$ must be correct for at least one of the components.

A1: cao. Correct answer no working scores all 3 marks. It must be the vector (either form) and not a coordinate Note this can be attempted by finding the mid point *E* of *AC* and then using $\mathbf{d} = \mathbf{b} + 2 \overrightarrow{BE}$ but it must be a full method M1 Attempts $mp_{AC} = (0, 2, 2)$ and uses M1 Attempts $(3, -1, 6) + 2 \times (-3, 3, -4)$ A1

(b)

M1: Uses correct pair of vectors, so $\pm k \overrightarrow{BA}$ and $\pm k \overrightarrow{BC}$. Each must be correct for at least one of the components **dM1**: A clear attempt to use the dot product formula to find $\cos\theta = k, -1 < k < 1$. It is dependent upon having

chosen the correct pair of vectors. Allow for arithmetical slips in both their dot product calculation and the moduli, but the process must be correct.

It could also be found using the cosine rule. $\frac{72 + 32 - 72}{2\sqrt{72}\sqrt{32}} =$

(M1 is for attempt at all three lengths, so $\pm \overrightarrow{BA}$, $\pm \overrightarrow{BC}$, $\pm \overrightarrow{AC}$ and dM1 correct angle attempted using the correct formula)

A1: For 1/3 or -1/3 or equivalent - may be implied by 70.5 or 109.5 or 1.23 radians or 1.91 radians A1: cso for awrt 70.5 degrees or 1.23 radians. (Note that invcos(-1/3)=109.5 followed by 70.5 is A0 unless accompanied by a convincing argument that the angle 109.5 is the exterior angle, and therefore the interior angle is 70.5. It is not awarded for simply finding the acute angle. A diagram with correct angles would be ok) (c)

M1: Uses correct area formula for parallelogram.

You may see the area of the triangle ABC doubled which is fine.

A1: Obtains awrt 45.3. Allow this from an angle of 109.5

(d)

M1: Realises connection with part (c) and uses 1.5 times answer to the area of *ABCD* (It can be implied by 67.9) A1: awrt 67.9