Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C34

Advanced

Tuesday 17 January 2017 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. Find an equation of the tangent to the curve

$$x^3 + 3x^2y + y^3 = 37$$

at the point (1, 3). Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Qu	Scheme	Marks
1	Differentiate wrt x $\quad \underline{3x^2} + \underline{\underline{6xy + 3x^2\frac{dy}{dx}}} + \underline{3y^2\frac{dy}{dx}} = \underline{0}$	M1 A1 B1
	Substitutes (1, 3) **AND** rearranges to get $\frac{dy}{dx}\left(=-\frac{7}{10}\right)$	M1
	$(y-3) = -\frac{7}{10}(x-1)$ so $7x + 10y - 37 = 0$	M1A1 **(6)**
		(6 marks)

M1 : Differentiates implicitly to include either $3x^2\frac{dy}{dx}$ **or** $3y^2\frac{dy}{dx}$ **term**

Accept $3x^2\frac{dy}{dx}$ appearing as $3x^2y'$ or $3y^2\frac{dy}{dx}$ as $3y^2y'$

A1: Differentiates $y^3 \to 3y^2\frac{dy}{dx}$ **and** $x^3 \to 3x^2$ **and** $37 \to 0$

B1: Uses the product rule to differentiate $3x^2y$ giving $\underline{\underline{6xy + 3x^2\frac{dy}{dx}}}$

Do not penalise students who write $3x^2dx + 6xydx + 3x^2dy + 3y^2dy = 0$

M1: Substitutes $x = 1,\ y = 3$ into their expression (correctly each at least once) to find a 'numerical' value for $\frac{dy}{dx}$ (may be incorrect). Note that $\frac{dy}{dx} = \frac{-3x^2 - 6xy}{3x^2 + 3y^2}$

M1: Use of $(y-3) = m(x-1)$ where m is their numerical value of $\frac{dy}{dx}$

Alternatively uses $y = mx + c$ with $(1,3)$ and their m as far as c =..

A1: Accept integer multiples of the answer i.e. $7kx + 10ky - 37k = 0$ for example $21x + 30y - 111 = 0$

Note: If the gradient $-\frac{7}{10}$ just appears (from a graphical calculator) only M3 may be awarded

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2.

$$f(x) = x^3 - 5x + 16$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = (ax + b)^{\frac{1}{3}}$$

giving the values of the constants a and b.

(2)

The equation $f(x) = 0$ has exactly one real root α, where $\alpha = -3$ to one significant figure.

(b) Starting with $x_1 = -3$, use the iteration

$$x_{n+1} = (ax_n + b)^{\frac{1}{3}}$$

with the values of a and b found in part (a), to calculate the values of x_2, x_3 and x_4, giving all your answers to 3 decimal places.

(3)

(c) Using a suitable interval, show that $\alpha = -3.17$ correct to 2 decimal places.

(2)

Qu	Scheme	Marks
2(a)	$f(x)=x^3-5x+16=0$ so $x^3=5x-16$	M1
	$\Rightarrow x=\sqrt[3]{5x-16}$	A1
		(2)
(b)	$x_2=\sqrt[3]{5\times-3-16}$	M1
	$x_2=-3.141$ awrt	A1
	$x_3=-3.165$ awrt **and** $x_4=-3.169$ awrt	A1
		(3)
(c)	$f(-3.175)=-0.130984375..,\ \ f(-3.165)=0.120482875$ Sign change (and as $f(x)$ is continuous) therefore a root α lies in the interval $[-3.175,\ -3.165]\Rightarrow\alpha=-3.17$ (2 dp)	M1A1
		(2) **(7 marks)**

(a) Way 1:
M1: Must state f(x) = 0 (or imply by writing $x^3-5x+16=0$) and reach $x^3=\pm5x\pm16$
A1: completely correct with all lines including f(x) = 0 stated or implied (see above), $x^3=5x-16$ and $x=\sqrt[3]{5x-16}$ oe with or without a = 5, b = −16. Isw after a correct answer

If a candidate writes $x^3=5x-16\Rightarrow x=(5x-16)^{1/3}$ then they can score 1 0 for a correct but incomplete solution.

Similarly if a candidate writes $x^3-5x+16=0\Rightarrow x=(5x-16)^{1/3}$

Way 2:
M1: starts with answer, cubes and reaches a =.., b = .
A**1**: Completely correct reaching equation and stating hence f(x) = 0

(b)
Ignore subscripts in this part, just mark as the first, second and third values given.
M1: An attempt to substitute $x_1=-3$ into **their** iterative formula. E.g. Sight of $\sqrt[3]{-31}$, or can be implied by $x_2=awrt-3.14$
A1: $x_2=\text{awrt}-3.141$
A1: $x_3=\text{awrt}-3.165$ **and** $x_4=\text{awrt}-3.169$

(c)
M1: Choose suitable interval for x, e.g. $[-3.175,\ -3.165]$ and at least one attempt to evaluate f(x). Evidence would be the values embedded within an expression or one value correct. A minority of candidates may choose a tighter range which should include -3.1698 (alpha to 4dp). This would be acceptable for both marks, provided the conditions for the A mark are met. Some candidates may use an adapted $f(x)=0$, for example $g(x)=x-\sqrt[3]{(5x-16)}$ This is also acceptable even if it is called f, but you must see it defined. For your information $g(-3.175)=-0.004, g(-3.165)=(+)0.004$ If the candidate states an f (without defining it) it must be assumed to be $f(x)=x^3-5x+16$
A1: needs (i) both evaluations correct to 1 sf, (either rounded or truncated)
(ii) sign change stated (>0, <0 acceptable as would a negative product) and
(iii) some form of conclusion which may be $\Rightarrow\alpha=-3.17$ or "so result shown" or qed or tick or equivalent

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3. (a) Express $\dfrac{9 + 11x}{(1 - x)(3 + 2x)}$ in partial fractions.

(3)

(b) Hence, or otherwise, find the series expansion of

$$\frac{9 + 11x}{(1 - x)(3 + 2x)}, \quad |x| < 1$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a simplified fraction.

(6)

Qu	Scheme	Marks
3(a)	$\dfrac{9+11x}{(1-x)(3+2x)} = \dfrac{A}{1-x} + \dfrac{B}{3+2x}$ and attempt to find A or B	M1
	$A = 4$, $B = -3$	A1, A1
		(3)
(b)	$(1-x)^{-1} = 1 + x + x^2 + x^3 + ...$	B1
	$(3+2x)^{-1} = \dfrac{1}{3}\times\left(1+(-1)\left(\dfrac{2}{3}x\right)+\dfrac{(-1)(-2)}{2}\left(\dfrac{2}{3}x\right)^2+\dfrac{(-1)(-2)(-3)}{6}\left(\dfrac{2}{3}x\right)^3...\right)$	B1 M1
	Attempts $'4'\times(.....)+'-3'\times(.....)$	M1
	$= 3 + \dfrac{14}{3}x + \dfrac{32}{9}x^2 + \dfrac{116}{27}x^3...$	A1, A1
		(6)
		(9 marks)

(a)
M1: For expression in markscheme or $9 + 11x = A(3+2x) + B(1-x)$ and use of substitution or comparison of coefficients in an attempt to find A or B (Condone slips on the terms)
A1: One correct value (this implies the M1)
A1: Both correct values (attached to the correct fraction).
You do not explicitly need to see the expression rewritten in PF form.
(b)
B1: Correct expansion $(1-x)^{-1} = 1 + x + x^2 + x^3 + ...$ with or without working. Must be simplified
B1: For taking out a factor of 3^{-1} Evidence would be seeing either 3^{-1} or $\dfrac{1}{3}$ before the bracket or could be implied by the candidate multiplying their B by $\dfrac{1}{3}$.
M1: For the form of the binomial expansion with $n=-1$ and a term of $\left(\pm\dfrac{2}{3}x\right)$.
To score M1 it is sufficient to see just any two terms of the expansion. eg. $1+....+\dfrac{(-1)(-2)}{2}\left(\pm\dfrac{2}{3}x\right)^2$
M1: Attempts to combine the two series expansions. Condone slips on signs but there must have been some attempt to combine terms (at least once) and to use both their coefficients
A1: Two terms correct which may be unsimplified.
A1: All four terms correct. (cao) Could be mixed number fraction form. ISW after a correct answer

Alternative use of binomial in line 2 of scheme: ie. $3^{-1} + (-1)(3)^{-2}(2x)$
B1: For seeing either 3^{-1} or $\dfrac{1}{3}$ as the first term
M1: It is sufficient to see just the first two terms (unsimplified) then marks as before

Way 2: Otherwise method: Use of $(9+11x)(1-x)^{-1}(3+2x)^{-1}$: B1 B1 M1 : as before
Then M1: Attempt to multiply three brackets and obtain 3 + .. A1: two terms correct A1: All four correct
Way 3: Use of $(9+11x)\left(3-\left(x+2x^2\right)\right)^{-1}$ or alternatives is less likely – send to review

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4. Given that

$$f(x) = \frac{4}{3x+5}, \quad x > 0$$

$$g(x) = \frac{1}{x}, \qquad x > 0$$

(a) state the range of f, **(2)**

(b) find $f^{-1}(x)$, **(3)**

(c) find $fg(x)$. **(1)**

(d) Show that the equation $fg(x) = gf(x)$ has no real solutions. **(4)**

Qu	Scheme	Marks
4.(a)	$0 < \mathrm{f}(x) < \frac{4}{5}$	M1A1 **(2)**
(b)	$y = \dfrac{4}{3x+5} \qquad \Rightarrow (3x+5)y = 4$	M1
	$\Rightarrow x = \dfrac{4-5y}{3y}$	dM1
	$\mathrm{f}^{-1}(x) = \dfrac{4-5x}{3x} \qquad \left(0 < x < \dfrac{4}{5}\right)$	A1o.e. **(3)**
(c)	$\mathrm{fg}(x) = \dfrac{4}{\frac{3}{x}+5}$	B1 (1)
(d)	$\dfrac{3x+5}{4} = \dfrac{4}{\frac{3}{x}+5}$	M1
	$15x^2 + 18x + 15 = 0$	A1
	Uses $18^2 < 4\times15\times15$ and so deduce no real roots	M1 A1 **(4)** **(10 marks)**

(a)
M1: One limit such as $y > 0$ or $y < 0.8$. Condone for this mark both limits but with x (not y) or with the boundary included. For example $[0, 0.8], 0 < x < 0.8, 0 \leqslant y \leqslant 0.8$

A1: Fully correct so accept $0 < \mathrm{f}(x) < \frac{4}{5}$ and exact equivalents $0 < y < \frac{4}{5}$ $(0, 0.8)$

(b)
M1: Set $y = \mathrm{f}(x)$ or $x = \mathrm{f}(y)$ and multiply both sides by denominator.
dM1:Make x (or a swapped y) the subject of the formula. Condone arithmetic slips

A1: o.e for example $y/\mathrm{f}^{-1}(x) = \dfrac{1}{3}\left(\dfrac{4}{x} - 5\right)$ or $y = \left(\frac{4}{x} - 5\right)\big/3$ - do not need domain for this mark. ISW after a correct answer.

(c) Mark parts c and d together

B1: $\mathrm{fg}(x) = \dfrac{4}{\frac{3}{x}+5}$ - allow any correct form then isw

(d)
M1: Sets $\mathrm{fg}(x) = \mathrm{gf}(x)$ with **both sides correct** (but may be unsimplified) and forms a quadratic in x. Do not withhold this mark if fg or gf was originally correct but was "simplified" incorrectly and set equal to a correct gf
A1: Correct 3TQ. It need not be all on one side of the equation. The =0 can be implied by later work
M1: Attempts the discriminant or attempts the formula or attempts to complete the square.
A1: Completely correct work (cso) and conclusion. If $b^2 - 4ac$ has been found it must be correct (-576)

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5.

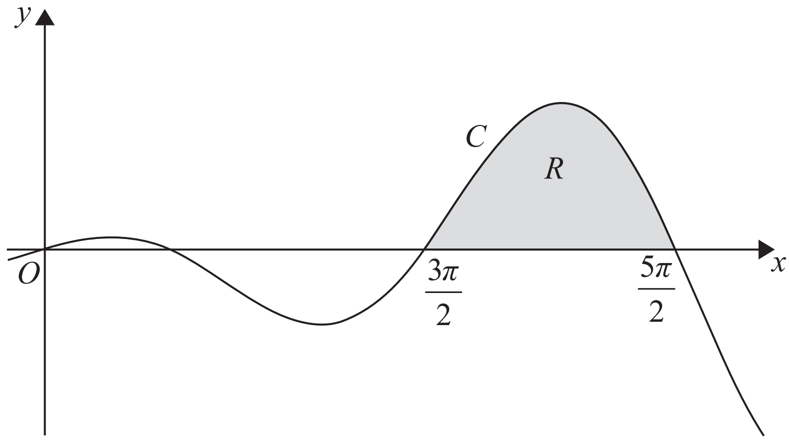

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = x\cos x, \quad x \in \mathbb{R}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve C and the x-axis for $\frac{3\pi}{2} \leqslant x \leqslant \frac{5\pi}{2}$

(a) Complete the table below with the exact value of y corresponding to $x = \frac{7\pi}{4}$ and with the exact value of y corresponding to $x = \frac{9\pi}{4}$

x	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$
y	0		2π		0

(1)

(b) Use the trapezium rule, with all five y values in the completed table, to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

(c) Find

$$\int x\cos x \, dx$$

(3)

(d) Using your answer from part (c), find the exact area of the region R.

(2)

Qu	Scheme	Marks
5.(a)	$\frac{7\pi}{4\sqrt{2}}$ or equivalent e.g. $\frac{7\pi\sqrt{2}}{8}$ AND $\frac{9\pi}{4\sqrt{2}}$ or equivalent e.g. $\frac{9\pi\sqrt{2}}{8}$	B1 **(1)**
(b)	$\frac{1}{2}\times\frac{\pi}{4}\times\{.........................\}$	B1 oe
	$\frac{1}{2}\times h\times\left\{0+0+2\left(\text{"}\frac{7\pi}{4\sqrt{2}}\text{"}+2\pi+\text{"}\frac{9\pi}{4\sqrt{2}}\text{"}\right)\right\}$	M1
	$= 11.91$ (only)	A1 **(3)**
(c)	$\int x\cos x\,dx \quad = [x\sin x]-\int \sin x\,dx$	M1
	$= x\sin x + \cos x\ (+c)$	dM1 A1 **(3)**
(d)	$[x\sin x+\cos x]_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} \quad = \frac{5\pi}{2}+\frac{3\pi}{2}=4\pi$	M1 A1 **(2)**
		(9 marks)

(a)
B1: Both correct (as above) Must be exact and not decimal
(b)
B1: See $\frac{1}{2}\times\frac{\pi}{4}\times$ as part of trapezium rule or $h=\frac{\pi}{4}$ stated or used. This can be scored if 'h' is in an unsimplified form.
M1: Correct structure of the bracket in the trapezium rule.

You may not see the zero's Eg $2\left(\text{"}\frac{7\pi}{4\sqrt{2}}\text{"}+2\pi+\text{"}\frac{9\pi}{4\sqrt{2}}\text{"}\right)$

A1: 11.91 only. This may be a result of using the decimal equivalents. Sight of 11.91 will score all 3 marks
(c)
M1: For a correct attempt at integration by parts to give an expression of the form $[\pm x\sin x]-\int\pm\sin x\,dx$
If you see such an expression you would only withhold the mark if there is evidence of an incorrect formula (seen or implied) Eg $\int u\,dv = uv+\int v\,du$
dM1: For $\pm\, x\sin x \pm \cos x$ following line one
A1: cso
Allow all three marks for candidate who just writes down the correct answer with no working

Watch for candidates who write down methods like this.
$$\begin{array}{cc} D & I \\ x & \cos x \\ 1 & \sin x \\ 0 & -\cos x \end{array}$$
and then write $x\sin x-(-\cos x)$

This is a commonly taught algorithm (They differentiate down the lh column and integrate on the rh column. The answer is found by $D1\times I2-D2\times I3$ where $D2$ is the second entry in the D column. This can score full marks for the answer $x\sin x+\cos x$ but also pick up methods for slips.
If they attempt $D1\times I2+D2\times I3$ it is M0 as they are implying an incorrect formula. Ask your TL if unclear.
(d)
M1: Using limits $\frac{5\pi}{2}$ and $\frac{3\pi}{2}$ correctly in their answer to part (c) - substituting (seen correctly in all terms or implied in all terms) and subtracting either way around
A1: 4π or equivalent single term. CSO. It must have been derived from $x\sin x+\cos x$

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6. (i) Differentiate $y = 5x^2 \ln 3x, \quad x > 0$

(2)

(ii) Given that

$$y = \frac{x}{\sin x + \cos x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(1+x)\sin x + (1-x)\cos x}{1 + \sin 2x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

(4)

Qu	Scheme	Marks
6.(i)	$\frac{dy}{dx} = 5x^2 \times \frac{3}{3x} + \ln(3x) \times 10x$	M1 A1
		(2)
(ii)	$\frac{dy}{dx} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{(\sin x + \cos x)^2}$	M1
	$\frac{dy}{dx} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{(\sin^2 x + \cos^2 x) + (2\sin x \cos x)} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{1 + \sin 2x}$	B1, B1
	$\frac{dy}{dx} = \frac{(1+x)\sin x + (1-x)\cos x}{1 + \sin 2x}$ *	A1 *
		(4)
		(6 marks)

(i)

M1: Applies the Product rule to $y = 5x^2 \ln 3x$

Expect $\frac{dy}{dx} = Ax + Bx\ln(3x)$ for this mark (A, B positive constant)

A1: cao- need not be simplified

(ii)

M1: Applies the Quotient rule, a form of which appears in the formula book, to $y = \frac{x}{\sin x + \cos x}$

Expect $\frac{dy}{dx} = \frac{(\sin x + \cos x)1 - x(\pm\cos x \pm \sin x)}{(\sin x + \cos x)^2}$ for M1

Condone invisible brackets for the M and an attempted incorrect 'squared' term on the denominator Eg $\sin^2 x + \cos^2 x$

B1: Denominator should be expanded to $\sin^2 x + \cos^2 x + ...$ and $(\sin^2 x + \cos^2 x) \to 1$

B1: Denominator should be expanded to $... + k\sin x \cos x$ and $(k\sin x \cos x) \to \frac{k}{2}\sin 2x$.

For example sight of $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x = 1 + \sin 2x$ without the intermediate line on the Denominator is B0 B1

A1: cso – answer is given. This mark is withheld if there is poor notation $\cos x \leftrightarrow \cos$ $\sin^2 x \leftrightarrow \sin x^2$

If the only error is the omission of $(\sin^2 x + \cos^2 x) \to 1$ then this final A1* can be awarded.

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Use of product rule or implicit differentiation needs to be applied correctly with possible sign errors differentiating functions for M1, then other marks as before. If quoted the product rule must be correct

Product rule $\frac{dy}{dx} = (\sin x + \cos x)^{-1} \times 1 \pm x \times (\sin x + \cos x)^{-2}(\pm\cos x \pm \sin x)$

Implicit differentiation $(\sin x + \cos x)y = x \Rightarrow (\sin x + \cos x)\frac{dy}{dx} + y(\pm\cos x \pm \sin x) = 1$

To score the B's under this method there must have been an attempt to write $\frac{dy}{dx}$ as a single fraction

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7.

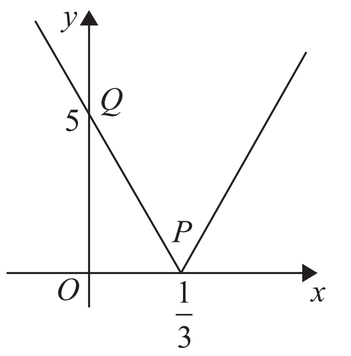

Figure 2

Figure 2 shows a sketch of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The point $P\left(\frac{1}{3}, 0\right)$ is the vertex of the graph.

The point $Q\,(0, 5)$ is the intercept with the y-axis.

Given that $\mathrm{f}(x) = |ax + b|$, where a and b are constants,

(a) (i) find all possible values for a and b,

(ii) hence find an equation for the graph.

(4)

(b) Sketch the graph with equation

$$y = \mathrm{f}\left(\frac{1}{2}x\right) + 3$$

showing the coordinates of its vertex and its intercept with the y-axis.

(3)

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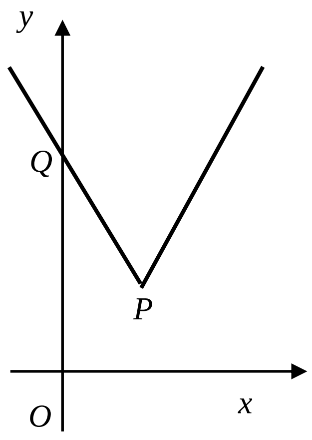
Qu	Scheme	Marks
7 (a)(i)	Substitute (0, 5) to give $\lvert b \rvert = 5$ so $b = \pm 5$	B1
	Substitute $\left(\frac{1}{3}, 0\right)$ to give $\left\lvert \frac{1}{3}a + b \right\rvert = 0$ so $a = \mp 15$	M1 A1
(ii)	Gives equation as $y = \lvert -15x+5 \rvert$ or $y = \lvert 15x-5 \rvert$	B1
		(4)
(b)	 V shape correct way up P at $\left(\frac{2}{3}, 3\right)$ Q at (0, 8)	B1 B1 B1
		(3)
		(7 marks)

(a)
(i)
B1: For both $b = \pm 5$ not just $\lvert b \rvert = 5$
M1: Substitute $\left(\frac{1}{3}, 0\right)$ to give $\left\lvert \frac{1}{3}a + b \right\rvert = 0$. This mark is implied by $a = \pm 3 \times$ value of b
A1: $a = 15$ corresponding to $b = -5$ **and** $a = -15$ corresponding to $b = 5$

If they write down an equation rather than giving values of a and b then
Just $y = \lvert -15x+5 \rvert$ **or** $y = \lvert 15x-5 \rvert$ scores B0, M1, A0
Both $y = \lvert -15x+5 \rvert$ **and** $y = \lvert 15x-5 \rvert$ scores B1, M1, A1
Linear equations $y = -15x+5$ and/or $y = 15x-5$ (without the modulus) only score B0 M1 A0

(ii) Note that this is an A1 mark on e-pen

B1: $y = \lvert -15x + 5 \rvert$ or $y = \lvert 15x - 5 \rvert$ or allow equations such as for this mark only $\mathrm{f}(x) = \begin{cases} 15x-5 & x \geqslant \frac{1}{3} \\ -15x+5 & x < \frac{1}{3} \end{cases}$

If candidates don't state (i), (ii) and write down just $y = \lvert -15x+5 \rvert$ they would score (i) B0 M1 A0 (ii) B1
(b) There must be a sketch to score any of these marks.
B1: V shape the correct way up any position but not on the x- axis. Accept V's that don't have symmetry
B1: P at $\left(\frac{2}{3}, 3\right)$ Score if the coordinates are stated within the text OR marked on the axes. If they appear in both then the graph takes precedence.
B1: For **crossing** the y- axis at (0, 8). Accept 8 marked on the correct axis. Condone (8,0) marked on the correct axis

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8. (a) Using the trigonometric identity for $\tan(A + B)$, prove that

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}, \qquad x \neq (2n+1)30°, \qquad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $-30° < x < 30°$,

$$\tan 3x = 11 \tan x$$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

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Qu	Scheme	Marks
8 (a)	$\tan(2x+x)=\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$	M1
	$=\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x}$	dM1
	$=\dfrac{2\tan x+\tan x(1-\tan^2 x)}{1-\tan^2 x-2\tan^2 x}$ OR $=\dfrac{\dfrac{2\tan x+\tan x-\tan^3 x}{1-\tan^2 x}}{\dfrac{1-\tan^2 x-2\tan^2 x}{1-\tan^2 x}}$ oe	A1
	So $\tan 3x=\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}$ *	A1*cso
		(4)
(b)	Put $\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}=11\tan x$ so $3\tan x-\tan^3 x=11\tan x(1-3\tan^2 x)$	M1
	$32\tan^3 x=8\tan x$	A1
	So $\tan x=\pm\dfrac{1}{2}$ or $0\Rightarrow x=..$	dM1
	$\Rightarrow x=$ awrt $26.6°$, $-26.6°$, 0	A1 A1
		(5) **(9 marks)**

(a)

M1: Expands $\tan(2x+x)=\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$ condoning sign errors

dM1: Uses the correct double angle formula $\dfrac{2\tan x}{1-\tan^2 x}$ both times within their expression for $\tan(2x+x)$

A1: Multiplies both numerator and denominator by $1-\tan^2 x$ to obtain a correct intermediate line

Eg $=\dfrac{2\tan x+\tan x(1-\tan^2 x)}{1-\tan^2 x-2\tan^2 x}$ or similar.

Alternatively they write both numerator and denominator as single correct fractions.
They cannot just write down the final given answer for this mark

A1*: Correct printed answer achieved with no errors and all of the lines in the markscheme (c.s.o.)
Withhold the final A1 for candidates who use poor notation or mixed variables.

Examples of poor notation would include $\tan \leftrightarrow \tan x$ $\tan^2 x \leftrightarrow \tan x^2$ $\tan 2x=\dfrac{2\tan\theta}{1-\tan^2\theta}$

(b)

M1: Attempts to use the given identity and multiplies by $1-3\tan^2 x$. Condone slips

A1: Obtain $32\tan^3 x=8\tan x$ or equivalent. Accept $32\tan^2 x=8$ for this mark

dM1: Obtains one value of x from $\tan x=...$ using a correct method for their equation. The order of operations to find x must be correct but can be scored from $\tan x=0\Rightarrow x=0$

A1: Either one of $x=26.6°$ or $-26.6°$ or in radians ± 0.46

A1: CAO $x=$ awrt $26.6°$, awrt $-26.6°$, 0 (do not need degrees symbol) with no extras within the range

Note: Answers only scores 0 marks. Answers from a correct cubic/quadratic scores M1 A1 dM1 (implied) then as scheme

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9.

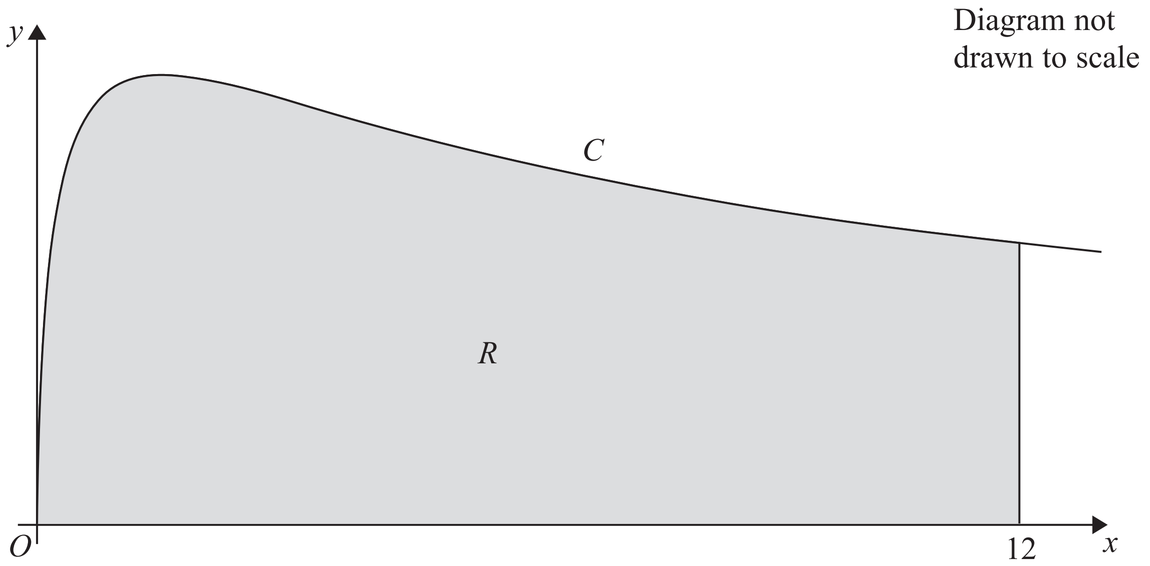

Figure 3

(a) By using the substitution $u = 2x + 3$, show that

$$\int_0^{12} \frac{x}{(2x+3)^2}\, dx = \frac{1}{2}\ln 3 - \frac{2}{9}$$

(7)

The curve C has equation

$$y = \frac{9\sqrt{x}}{(2x+3)}, \quad x > 0$$

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis and the line with equation $x = 12$. The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Use the result of part (a) to find the exact value of the volume of the solid generated.

(2)

Qu	Scheme	Marks
9(a)	$\frac{du}{dx}=2$	B1
	$\left(\int \frac{x}{(2x+3)^2}\,dx\right)=\int \frac{u-3}{4u^2}du$	M1
	$=\int \frac{1}{4}u^{-1}-\frac{3}{4}u^{-2}\,du$	dM1
	$=\frac{1}{4}\ln u+\frac{3}{4}u^{-1}$	ddM1 A1
	$\left[\frac{1}{4}\ln u+\frac{3}{4}u^{-1}\right]_3^{27}=\frac{1}{4}\ln 27+\frac{3}{4}\times\frac{1}{27}-\left(\frac{1}{4}\ln 3+\frac{3}{4}\times\frac{1}{3}\right)=\frac{1}{4}\ln 9-\frac{8}{36}$	M1
	$=\frac{1}{2}\ln 3-\frac{2}{9}$ *	A1*
		(7)
(b)	$V=\pi\times\int_0^{12}\left(\frac{9\sqrt{x}}{2x+3}\right)^2 dx$	M1
	$=81\pi\left(\frac{1}{2}\ln 3-\frac{2}{9}\right)$	A1
		(2)
		(9 marks)

(a)

B1: States or uses $\frac{du}{dx}=2$ or equivalent such as $\frac{dx}{du}=0.5$ or $dx=\frac{1}{2}du$

M1: Expression of the form $k\int \frac{u-3}{u^2}du$ and allow missing du and/or missing integral sign

d M1: Splits into the form $...u^{-1}\pm...u^{-2}$ and again allow missing du and/or missing integral sign.

Alternatively they could use integration by parts at this stage $\int \frac{u-3}{4u^2}du=\pm a\frac{(u-3)}{u}\pm b\int\frac{1}{u}du$ A small number of candidates will also use partial fractions that gives the same answer as the main scheme.

ddM1: For $...\ln u\pm...u^{-1}$ or 'obviously $...\ln 4u\pm...u^{-1}$ or by parts $\pm a\frac{(u-3)}{u}\pm b\ln u$

A1: $\frac{1}{4}\ln u+\frac{3}{4}u^{-1}$ This answer or equivalent such as $\frac{1}{4}\ln 4u+\frac{3}{4}u^{-1}$ or by parts $-\frac{(u-3)}{4u}+\frac{1}{4}\ln u$

M1: Applies limits of 27 and 3 to the result of integrating their function in u, subtracts the correct way around and combines the ln terms correctly. Alternatively using $u=2x+3$ applies the limits x = 12 and 0 to the result of their adapted function subtracts the correct way round and combines the ln terms correctly.

A1*: given answer achieved correctly without errors. The only omission that would be allowed could be the dM1 line which could be implied. You need to see an intermediate step with correct ln work before the final answer is reached.

(b)

M1: Attempts to use part (a) to find the exact volume. Accept $\pi\times\int_0^{12}\left(\frac{9\sqrt{x}}{2x+3}\right)^2 dx$

Condone only the omission of π or 81 or a bracket for this M mark so accept $81\left(\frac{1}{2}\ln 3-\frac{2}{9}\right)$ or $\pi\left(\frac{1}{2}\ln 3-\frac{2}{9}\right)$ or $\frac{1}{2}\ln 3-\frac{2}{9}\times 81\pi$ as evidence

A1: Any correct exact equivalent in terms of ln3 and π Accept for example $81\pi\left(\ln\sqrt{3}\right)-18\pi$

A correct answer implies both marks. Remember to isw after a correct answer.

It is possible to do 9(a) by parts or via partial fractions **without** using the given substitution. This does not satisfy the demands of the question but should score some marks. A fully correct solution via either method scores 5 out of 7

Qu	Scheme		Marks
9(a)	By Parts	By Partial Fractions	B0
		$\int x(2x+3)^{-2}\,dx = \int \frac{\frac{1}{2}}{(2x+3)} + \frac{-\frac{3}{2}}{(2x+3)^2}\,dx$	
	$\int x(2x+3)^{-2}\,dx = \frac{x(2x+3)^{-1}}{-2} + \int \frac{(2x+3)^{-1}}{2}\,dx$	One term of $= \frac{1}{4}\ln(2x+3) + \frac{3(2x+3)^{-1}}{4}$	M1
	$= \frac{x(2x+3)^{-1}}{-2} + \frac{1}{4}\ln(2x+3)$	Both terms of $= \frac{1}{4}\ln(2x+3) + \frac{3(2x+3)^{-1}}{4}$	dM1
	Attempts limits $= \left[\frac{x(2x+3)^{-1}}{-2} + \frac{1}{4}\ln(2x+3)\right]_0^{12}$	$= \left[\frac{1}{4}\ln(2x+3) + \frac{3(2x+3)^{-1}}{4}\right]_0^{12}$	ddM1
	Correct un simplified answer $= -\frac{2}{9} + \frac{1}{4}\ln 27 - \frac{1}{4}\ln 3$	$= \frac{1}{4}\ln 27 + \frac{1}{36} - \frac{1}{4}\ln 3 - \frac{1}{4}$	A1
	Collects log terms $= -\frac{2}{9} + \frac{1}{4}\ln\left(\frac{27}{3}\right)$	$= -\frac{2}{9} + \frac{1}{4}\ln\left(\frac{27}{3}\right)$	M1
	$= \frac{1}{2}\ln 3 - \frac{2}{9}$		A0*
			(7)

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10. A population of insects is being studied. The number of insects, N, in the population, is modelled by the equation

$$N = \frac{300}{3 + 17e^{-0.2t}} \qquad t \in \mathbb{R},\ t \geqslant 0$$

where t is the time, in weeks, from the start of the study.

Using the model,

(a) find the number of insects at the start of the study,
(1)

(b) find the number of insects when $t = 10$,
(2)

(c) find the time from the start of the study when there are 82 insects.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)
(4)

(d) Find, by differentiating, the rate, measured in insects per week, at which the number of insects is increasing when $t = 5$. Give your answer to the nearest whole number.
(3)

Qu	Scheme	Marks
10.(a)	When $t = 0$ $N = 15$	B1 **(1)**
(b)	Puts $t = 10$ so $N =$ 56.6 (accept 56 or 57)	M1A1 **(2)**
(c)	$82 = \dfrac{300}{3+17e^{-0.2t}} \Rightarrow e^{-0.2t} = \dfrac{54}{1394} = \text{awrt } 0.039$	M1 A1
	$-0.2t = \ln\left(\dfrac{54}{1394}\right) \Rightarrow t =$	dM1
	t = awrt 16.3	A1 **(4)**
(d)	$\dfrac{dN}{dt} = (-0.2)\times 300\times(-1)\times 17e^{-0.2t}(3+17e^{-0.2t})^{-2}$	M1 A1
	=4.38 so 4 insects per week	A1 cso **(3)**
		(10 marks)

(a)
B1: 15 cao
(b)
M1: Substitutes $t = 10$ into the correct formula. Sight of $N = \dfrac{300}{3+17e^{-0.2\times 10}}$ is fine
A1: Accept 56 or 57 or awrt 56.6. These values would imply the M.
(c)
M1: Substitutes 82 and proceeds to obtain $e^{\pm 0.2t} = C$ Condone slips on the power
A1: For $e^{-0.2t} = \dfrac{27}{697}$ oe $e^{0.2t} = \dfrac{697}{27}$ oe Accept decimals Eg $e^{-0.2t} = \text{awrt } 0.039$ or $e^{0.2t} = \text{awrt } 25.8$
dM1: Dependent upon previous M, scored for taking ln's (of a positive value) and proceeding to $t =$
A1: awrt 16.3 Accept 16 (weeks), 16.25 (weeks), 16 weeks 2 days or 17 weeks following correct log work and acceptable accuracy. Accept $t = 5\ln\left(\dfrac{1394}{54}\right)$ *oe* for this mark

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It is possible to answer this by taking ln's at the point $1394e^{-0.2t} = 54$
M1A1 $\ln(1394) - 0.2t = \ln(54)$ dM1 A1 As scheme

(d)
M1: Differentiates to give a form equivalent to $\dfrac{dN}{dt} = ke^{-0.2t}(3+17e^{-0.2t})^{-2}$ (may use quotient rule)
A1: Correct derivative which may be unsimplified $\dfrac{dN}{dt} = 1020e^{-0.2t}(3+17e^{-0.2t})^{-2}$
A1: Obtains awrt 4 **following a correct derivative**. This is cso

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11. (a) Express $35 \sin x - 12 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R, and give the value of α, in radians, to 4 significant figures.
(3)

(b) Hence solve, for $0 \leqslant x < 2\pi$,

$$70 \sin x - 24 \cos x = 37$$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
(4)

$$y = \frac{7000}{31 + (35\sin x - 12\cos x)^2}, \quad x > 0$$

(c) Use your answer to part (a) to calculate

(i) the minimum value of y,

(ii) the smallest value of x, $x > 0$, at which this minimum value occurs.
(4)

Qu	Scheme	Marks
11. (a)	$R = 37$	B1
	$\tan\alpha = \frac{12}{35} \Rightarrow \alpha = \text{awrt } 0.3303$	M1 A1
		(3)
(b)	$\sin(x-\alpha) = \frac{37}{2R} \quad (= 0.5...)$	M1
	$x = \arcsin\left(\frac{37}{2\times\text{their "37"}}\right) + \text{their"0.3303"}$	M1
	$x =$ awrt 0.854 or awrt 2.95	A1
	$x =$ awrt 0.854 and awrt 2.95	A1
		(4)
(c)(i)	Find $y = \frac{7000}{31+(\pm\text{"}R\text{"})^2} = 5$	M1 A1
(c)(ii)	$x - \alpha = \frac{\pi}{2} \Rightarrow x = 1.90$	M1 A1
		(4)
		(11 marks)

(a)

B1: $R = 37$ no working needed. Condone $R = \pm 37$

M1: $\tan\alpha = \pm\frac{12}{35}$ or $\tan\alpha = \pm\frac{35}{12}$ with an attempt to find alpha. Accept decimal attempts from $\tan\alpha = \text{awrt } \pm 0.343$ or $\tan\alpha = \text{awrt } \pm 2.92$ If R is used allow $\sin\alpha = \pm\frac{12}{R}$ OR $\cos\alpha = \pm\frac{35}{R}$ with an attempt to find alpha

A1: $\alpha = \text{awrt } 0.3303$. Answers in degrees are A0

(b)

M1: (Uses part (a) to solve equation) $\sin(x \pm \alpha) = \frac{37}{2\times theirR}$

M1: operations undone in the correct order to give $x = ...$ Accept $\sin(x\pm\alpha) = k \Rightarrow x = \arcsin k \pm \alpha$

A1: one correct answer to within required accuracy. Allow 0.272π or 0.938π.

Condone for this mark only **both** $\frac{\pi}{6} + 0.3303$ **and** $\frac{5\pi}{6} + 0.3303$

A1: both values (and no extra values in the range) correct to within required accuracy. Allow $0.272\pi, 0.938\pi$

(c) (i)

M1: For an attempt at $\frac{7000}{31+(\pm\text{"}R\text{"})^2}$

A1: 5

(c)(ii)

M1: Uses $x - \text{their } \alpha = (2n+1)\frac{\pi}{2}$ to find x This may be implied by $1.57 \pm \text{their } 0.33$ stated or calculated (2dp)

A1: Awrt 1.90 but condone 1.9 for this answer

Answers in degrees, withhold the first time seen, usually part (a). FYI (a) $18.92°$ (b) $48.9°, 168.9°$ (c)(ii) $108.9°$

Leave blank

12. In freezing temperatures, ice forms on the surface of the water in a barrel. At time t hours after the start of freezing, the thickness of the ice formed is x mm. You may assume that the thickness of the ice is uniform across the surface of the water.

At 4pm there is no ice on the surface, and freezing begins.

At 6pm, after two hours of freezing, the ice is 1.5 mm thick.

In a simple model, the rate of increase of x, in mm per hour, is assumed to be constant for a period of 20 hours.

Using this simple model,

(a) express t in terms of x,

(2)

(b) find the value of t when $x = 3$

(1)

In a second model, the rate of increase of x, in mm per hour, is given by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\lambda}{(2x+1)} \quad \text{where } \lambda \text{ is a constant and } 0 \leqslant t \leqslant 20$$

Using this second model,

(c) solve the differential equation and express t in terms of x and λ,

(3)

(d) find the exact value for λ,

(1)

(e) find at what time the ice is predicted to be 3 mm thick.

(2)

Qu	Scheme		Marks
12. (a)	Way 1: Uses $x = kt$ or $t = cx$ and $x = 1.5$ when $t = 2$ so $k =$ or $c =$	Way 2: Uses $x = kt + c$ with $x = 0$, $t = 0$ and with $x = 1.5$ when $t = 2$ so $k =$	M1
	$t = \frac{4}{3}x$	$t = \frac{4}{3}x$	A1 **(2)**
(b)	$t = 4$		B1 **(1)**
(c)	$\frac{dx}{dt} = \frac{\lambda}{(2x+1)}$ so separate variables to give $\int (2x+1)\,dx = \int \lambda\,dt$		M1
	$x^2 + x = \lambda t (+c')$ or $\frac{(2x+1)^2}{4} = \lambda t (+c)$ so $t =$		M1
	(When $t = 0$, $x = 0$ so $c = ¼$ or $c' = 0$) so $t = \frac{x^2 + x}{\lambda}$		A1 **(3)**
(d)	Uses $x = 1.5$ when $t = 2$ to give $\lambda = \frac{15}{8}$		B1 **(1)**
(e)	$t = \frac{x^2 + x}{\lambda} = \frac{12}{\lambda}$ = 6.4 hours later <u>so</u> <u>10.24pm or 22.24</u>		M1 <u>A1</u> **(2)**
			(9 marks)

Mark (a) and (b) together

(a)

M1: Uses correct $x = kt$ or $t = cx$ *and* $x = 1.5$ when $t = 2$ to find their constant (may not be *k or c*)

This may be the result of a differential equation $\frac{dx}{dt} = k$

A1: $t = \frac{4}{3}x$ oe such as $t = \frac{x}{0.75}$ or even $t = \frac{x}{3/4}$ Just this with no working is M1 A1

(b)

B1: $t = 4$

Mark (c),(d) and (e) together

(c)

M1: Correct separation but condone missing integral signs

M1: Correct form for both integrals- may not find *c* or even include a *c*

A1: Obtains a correct answer for *t* in terms of *x* and λ by using either $x = 0, t = 0 \Rightarrow t = \frac{x^2 + x}{\lambda}$ or

$t = \frac{(2x+1)^2 - 1}{4\lambda}$ oe. Alternatively uses $x = 1.5, t = 2 \Rightarrow t = \frac{4x^2 + 4x + 8\lambda - 15}{4\lambda}$ oe

Condone correct responses where '*c*' seems to have been either cancelled out or ignored

(d)

B1: $\lambda = \frac{15}{8}$ or decimal i.e. 1.875

(e)

M1: Substitutes $x = 3$ into their expression for t. Implied by $t = \frac{12}{"\lambda"}$

A1: 10.24pm or 22:24 only

Leave blank

13.

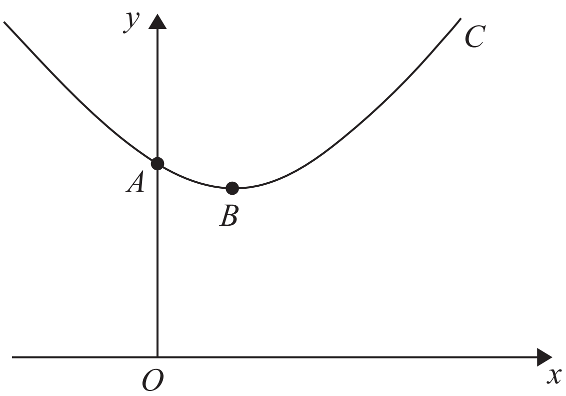

Figure 4

The curve C shown in Figure 4 has parametric equations

$$x = 1 + \sqrt{3}\tan\theta, \qquad y = 5\sec\theta, \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

The curve C crosses the y-axis at A and has a minimum turning point at B, as shown in Figure 4.

(a) Find the exact coordinates of A.

(3)

(b) Show that $\frac{dy}{dx} = \lambda \sin\theta$, giving the exact value of the constant λ.

(4)

(c) Find the coordinates of B.

(2)

(d) Show that the cartesian equation for the curve C can be written in the form

$$y = k\sqrt{(x^2 - 2x + 4)}$$

where k is a simplified surd to be found.

(3)

Qu	Scheme	Marks
13 (a)	Puts $x = 0$ and obtains $\theta = -\frac{\pi}{6}$	B1
	Substitutes their θ to obtain $y = \frac{10\sqrt{3}}{3}$ or $\left(0, \frac{10\sqrt{3}}{3}\right)$	M1 A1 **(3)**
(b)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{5\sec\theta\tan\theta}{\sqrt{3}\sec^2\theta}$	M1 A1
	$= \frac{5 \times \sin\theta/\cos\theta}{\sqrt{3} \times 1/\cos\theta}$	B1
	$= \frac{5}{\sqrt{3}}\sin\theta$ or $\lambda = \frac{5}{\sqrt{3}}$ oe	A1 **(4)**
(c)	Puts $\frac{dy}{dx} = 0$ and obtains θ and calculates x and y **or** deduces correct answer	M1
	Obtains (1, 5)	A1 **(2)**
(d)	$\tan\theta = \frac{x-1}{\sqrt{3}}$ and $\sec\theta = \frac{y}{5}$	M1
	Uses $1+\tan^2\theta = \sec^2\theta$ to give $1+"\left(\frac{x-1}{\sqrt{3}}\right)^2" = "\left(\frac{y}{5}\right)^2"$	M1
	$\frac{3+x^2-2x+1}{3} = \left(\frac{y}{5}\right)^2$ **so** $y = \frac{5}{3}\sqrt{3}\sqrt{(x^2-2x+4)}$ *	A1* **(3)**
		(12 marks)
Alt 1(d)	$y = 5\sqrt{1+\tan^2\theta}, = 5\sqrt{1+\left(\frac{x-1}{\sqrt{3}}\right)^2}$	M1, M1
	$y = \frac{5}{3}\sqrt{3}\sqrt{(x^2-2x+4)}$ *	A1* (3)
Alt 2 (d)	Assume $y = k\sqrt{(x^2-2x+4)}$ and sub both $x = 1+\sqrt{3}\tan\theta$ and $y = 5\sec\theta$	
	$5\sec\theta = k \times \sqrt{3+3\tan^2\theta}$	M1
	$5\sec\theta = k \times \sec\theta\sqrt{3}$	M1
	$k = \frac{5}{3}\sqrt{3}$ AND conclusion "hence true"	A1* (3)

(a)

B1: For $\theta = -\frac{\pi}{6}$ or $-30°$ or awrt -0.52 but may be awarded for $\cos\theta = \frac{\sqrt{3}}{2}$ or $\sec\theta = \frac{2}{\sqrt{3}}$ if θ is not explicitly found

M1: Substitutes their θ (or their $\cos\theta$ or $\sec\theta$) found from an attempt at $x = 0$ to give y

A1: cao. Accept $y = \frac{10}{\sqrt{3}}$ Correct answer with no incorrect working scores all 3 marks.

Note that $\theta = \frac{\pi}{6}$ also gives $y = \frac{10\sqrt{3}}{3}$ but scores B0 M1 A0

Qu	Scheme	Marks
	(b) **M1:** Attempts to differentiate both x and y wrt θ and establishes $\left(\frac{dy}{dx}\right) = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ **A1:** Correct derivatives and correct fraction **B1:** For either $\lambda = \frac{5}{3}\sqrt{3}$ (seen explicitly stated or implied) or use of $\sec\theta = \frac{1}{\cos\theta}$ An alternative to seeing $\sec\theta = \frac{1}{\cos\theta}$ is $\frac{1}{\sec\theta} = \cos\theta$ **A1:** Fully correct solution showing all relevant steps with correct notation, no mixed variables and no errors. $\frac{\tan\theta}{\sec\theta}$ cannot just be written as $\sin\theta$ without an intermediate line of working $\frac{(\cos\theta)^{-2}\sin\theta}{\sec^2\theta}$ cannot just be written as $\sin\theta$ without an intermediate line of working However it is acceptable to write down $\tan\theta\cos\theta$ as $\sin\theta$ due to this being a version of $\frac{\sin\theta}{\cos\theta} = \tan\theta$ (c) **M1:** Sets their $\frac{dy}{dx} = 0$ and proceeds to find (x, y) from their θ **A1:** for (1, 5) or $x = 1$, $y = 5$ (d) **M1:** Attempt to obtain $\tan\theta$ and $\sec\theta$ in terms of x and y respectively. Allow $\tan\theta = \frac{x \pm 1}{\sqrt{3}}$ $\sec\theta = \frac{y}{5}$ **M1:** Uses $1 + \tan^2\theta = \sec^2\theta$ with their expressions for $\tan\theta$ and $\sec\theta$ in terms of x and y respectively **A1*:** Obtains printed answer with no errors and with $k = \frac{5}{3}\sqrt{3}$ only You do not need to see k explicitly stated as $\frac{5}{3}\sqrt{3}$, it is fine to be embedded within the formula	

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14. *ABCD* is a parallelogram with *AB* parallel to *DC* and *AD* parallel to *BC*.
The position vectors of *A*, *B*, *C*, and *D* relative to a fixed origin *O* are **a**, **b**, **c** and **d** respectively.

Given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + 6\mathbf{k}, \quad \mathbf{c} = -\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$$

(a) find the position vector **d**,
(3)

(b) find the angle between the sides *AB* and *BC* of the parallelogram,
(4)

(c) find the area of the parallelogram *ABCD*.
(2)

The point *E* lies on the line through the points *C* and *D*, so that *D* is the midpoint of *CE*.

(d) Use your answer to part (c) to find the area of the trapezium *ABCE*.
(2)

Qu	Scheme	Marks
14 (a)	Attempts $\overrightarrow{BA}=\mathbf{a}-\mathbf{b}=-2\mathbf{i}+2\mathbf{j}-8\mathbf{k}$ or $\overrightarrow{BC}=\mathbf{c}-\mathbf{b}=-4\mathbf{i}+4\mathbf{j}$ either way around	M1
	Finds $\overrightarrow{OD}=\mathbf{a}-\mathbf{b}+\mathbf{c}=(-2\mathbf{i}+2\mathbf{j}-8\mathbf{k})+(-\mathbf{i}+3\mathbf{j}+6\mathbf{k})=-3\mathbf{i}+5\mathbf{j}-2\mathbf{k}$	M1 A1
		(3)
(b)	$\overrightarrow{BA}=\mathbf{a}-\mathbf{b}=-2\mathbf{i}+2\mathbf{j}-8\mathbf{k}$ and $\overrightarrow{BC}=\mathbf{c}-\mathbf{b}=-4\mathbf{i}+4\mathbf{j}$	M1
	$\cos\theta=\dfrac{\begin{pmatrix}-2\\2\\-8\end{pmatrix}.\begin{pmatrix}-4\\4\\0\end{pmatrix}}{\sqrt{(-2)^2+2^2+(-8)^2}\sqrt{(-4)^2+4^2}}=\dfrac{16}{\sqrt{72}\sqrt{32}}=\dfrac{1}{3}$	dM1 A1
	So angle is 1.23 radians or 70.5 degrees	A1
		(4)
(c)	Area = $\sqrt{72}\sqrt{32}\sin\theta=45.3$ **or** $32\sqrt{2}$ **oe**	M1A1
		(2)
(d)	Area = $\dfrac{3}{2}\times"45.3"=67.9$ or $48\sqrt{2}$ **oe**	M1 A1
		(2)
		(11 marks)

(a)
M1: For attempting one of **b – a** or **a – b or c – b** or **b – c.** It must be correct for at least one of the components.
Condone coordinate notation for the first two M marks
M1: For attempting $\mathbf{d}=\mathbf{a}-\mathbf{b}+\mathbf{c}$ = It must be correct for at least one of the components.
A1: cao. Correct answer no working scores all 3 marks. It must be the vector (either form) and not a coordinate
Note this can be attempted by finding the mid point E of AC and then using **d** =**b**+2$\overrightarrow{BE}$ but it must be a full method M1 Attempts $mp_{AC}=(0,2,2)$ **and uses** .... M1 Attempts $(3,-1,6)+2\times(-3,3,-4)$ A1
(b)
M1: Uses correct pair of vectors , so $\pm k\overrightarrow{BA}$ and $\pm k\overrightarrow{BC}$. Each must be correct for at least one of the components
dM1: A clear attempt to use the dot product formula to find $\cos\theta=k, -1<k<1$. It is dependent upon having chosen the correct pair of vectors. Allow for arithmetical slips in both their dot product calculation and the moduli, but the process must be correct.

It could also be found using the cosine rule. $\dfrac{72+32-72}{2\sqrt{72}\sqrt{32}}=$

(M1 is for attempt at all three lengths, so $\pm\overrightarrow{BA}, \pm\overrightarrow{BC}, \pm\overrightarrow{AC}$ and dM1 correct angle attempted using the correct formula)
A1: For 1/3 or – 1/3 or equivalent - may be implied by 70.5 or 109.5 or 1.23 radians or 1.91 radians
A1: cso for awrt 70.5 degrees or 1.23 radians. (Note that invcos(– 1/3)=109.5 followed by 70.5 is A0 .... unless accompanied by a convincing argument that the angle 109.5 is the exterior angle, and therefore the interior angle is 70.5. It is not awarded for simply finding the acute angle. A diagram with correct angles would be ok)
(c)
M1: Uses correct area formula for parallelogram.
You may see the area of the triangle ABC doubled which is fine.
A1: Obtains awrt 45.3. Allow this from an angle of 109.5
(d)
M1: Realises connection with part (c) and uses 1.5 times answer to the area of $ABCD$ (It can be implied by 67.9)
A1: awrt 67.9