Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C34
Advanced

Tuesday 21 June 2016 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. (a) Express $3\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

(b) Hence solve, for $0 \leqslant \theta < 360°$, the equation

$$3\cos\theta + 5\sin\theta = 2$$

Give your answers to one decimal place.

(4)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$3\cos\theta - 5\sin\theta = 2$$

(2)

Question Number	Scheme	Notes	Marks
1.(a)	$R = \sqrt{34}$	Cao (Must be exact but score when first seen and ignore decimal value (5.83…))	B1
	$\tan\alpha = \pm\frac{5}{3}, \tan\alpha = \pm\frac{3}{5} \Rightarrow \alpha = ...$ (Allow $\cos\alpha = \pm\frac{5}{\sqrt{34}}$ or $\pm\frac{3}{\sqrt{34}}$, $\sin\alpha = \pm\frac{5}{\sqrt{34}}$ or $\pm\frac{3}{\sqrt{34}} \Rightarrow \alpha = ...$) Where $\sqrt{34}$ is their R		M1
	$\alpha = 59.04°$	awrt 59.04°	A1
			(3)
(b)	$\sqrt{34}\cos(\theta - 59.04) = 2 \Rightarrow \cos(\theta - 59.04) = \frac{2}{\sqrt{34}}(0.343)$ Attempts to use part (a) $"\sqrt{34}"\cos(\theta - "59.04") = 2$ and proceeds to $\cos(\theta \pm "59.04") = K, \quad \lvert K\rvert \leq 1$ May be implied by $\theta - "59.04" = 69.94...°$ or $\theta - "59.04"\cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right)$ The $\theta - "59.04"$ must be seen here or implied later		M1
	$\theta_1 - 59.04 = 69.94 \Rightarrow \theta_1 = \text{awrt } 129.0°$		A1
	$\theta_2 \pm 59.04 = 360 - '69.94' \Rightarrow \theta_2 = ...$ Correct attempt at a second solution in the range. It is **dependent** upon having scored the previous M. Usually for $\theta -$ their $59.04 = 360 -$ their $'69.94' \Rightarrow \theta = ...$		**d**M1
	$\theta_2 = 349.1°$	awrt 349.1°	A1
	For solutions in (b) that are otherwise fully correct, if there are extra answers in range, deduct the final A mark.		
			(4)
(c)	$\theta + \text{their } 59.04 = \cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right) \Rightarrow \theta = ...$ Allow $\theta - \text{their } 59.04 = \cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right) \Rightarrow \theta = ...$ if they have $\theta +$ .. in (b) Evidence that use is being made of parts (a) and (b) to obtain a value for θ. This can be implied by the use of their answers to part (b).		M1
	$\theta = 10.9°$	awrt 10.9	A1
			(2)
			(9 marks)

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2. The point P with coordinates $\left(\frac{\pi}{2}, 1\right)$ lies on the curve with equation

$$4x\sin x = \pi y^2 + 2x, \qquad \frac{\pi}{6} \leqslant x \leqslant \frac{5\pi}{6}$$

Find an equation of the normal to the curve at P.

(6)

Question Number	Scheme	Notes	Marks
2	$\frac{d(4x\sin x)}{dx} = 4x\cos x + 4\sin x$	Applies product rule to $4x\sin x$ to give $\frac{d(4x\sin x)}{dx} = \pm 4x\cos x + 4\sin x$	M1
	$\frac{d(\pi y^2)}{dy} = 2\pi y\frac{dy}{dx}$	Applies chain rule to πy^2 to give $\frac{d(\pi y^2)}{dy} = Ay\frac{dy}{dx}$	M1
	$4x\sin x = \pi y^2 + 2x \Rightarrow 4x\cos x + 4\sin x = 2\pi y\frac{dy}{dx} + 2$ Fully correct differentiation. oe Accept $4x\cos x dx + 4\sin x dx = 2\pi y dy + 2dx$		A1
	For the differentiation ignore any spurious $"\frac{dy}{dx} ="$		
	Alternative for first 3 marks using explicit differentiation: $y = \left(\frac{1}{\sqrt{\pi}}\right)(4x\sin x - 2x)^{\frac{1}{2}}$		
	$\frac{dy}{dx} = \left(\frac{1}{2\sqrt{\pi}}\right)(4x\sin x - 2x)^{-\frac{1}{2}}(4x\cos x + 4\sin x - 2)$ M1: $\frac{d(4x\sin x)}{dx} = \pm 4x\cos x + 4\sin x$ (as before) M1: $(4x\sin x - 2x)^{\frac{1}{2}} \to k(4x\sin x - 2x)^{-\frac{1}{2}}$		M1 M1
	Allow omission of π and sign errors when rearranging for the M marks		
	$\frac{dy}{dx} = \frac{1}{2\sqrt{\pi}}(4x\sin x - 2x)^{-\frac{1}{2}}(4x\cos x + 4\sin x - 2)$ oe		A1
	$x = \frac{\pi}{2}, y = 1$ $\Rightarrow 4 = 2\pi\frac{dy}{dx} + 2 \Rightarrow \frac{dy}{dx} = \ldots\left(\frac{1}{\pi}\right)$	Uses $x = \frac{\pi}{2}$ and $y = 1$ to obtain a value for $\frac{dy}{dx}$ (may be implied). For implicit differentiation, there must be a dy/dx and there must be x's and y's. Explicit differentiation just requires use of $x = \frac{\pi}{2}$.	M1
	$y - 1 = "-\pi"\left(x - \frac{\pi}{2}\right)$ or $y = "-\pi"x + c \Rightarrow c = 1 + \frac{\pi^2}{2}$ Uses normal gradient $-1\big/\frac{dy}{dx}$ and $x = \frac{\pi}{2}, y = 1$ to find equation of normal. Must use $-1\big/\left(\text{their } \frac{dy}{dx}\right)$ and $x = \frac{\pi}{2}$ and $y = 1$ must be correctly placed. If using $y = mx + c$ must reach as far as $c = \ldots$		M1
	$y - 1 = -\pi\left(x - \frac{\pi}{2}\right)$ oe	Allow 3sf or more decimal equivalent answers e.g. $y = -3.14x + 5.93$, $y - 1 = -3.14(x - 1.57)$ etc.	A1cso
			(6 marks)

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3. (a) Find the binomial expansion of

$$(1 + ax)^{-3}, \quad |ax| < 1$$

in ascending powers of x, up to and including the term in x^3, giving each coefficient as simply as possible in terms of the constant a.

(3)

$$f(x) = \frac{2 + 3x}{(1 + ax)^3}, \quad |ax| < 1$$

In the series expansion of $f(x)$, the coefficient of x^2 is 3

Given that $a < 0$

(b) find the value of the constant a,

(4)

(c) find the coefficient of x^3 in the series expansion of $f(x)$, giving your answer as a simplified fraction.

(2)

Question Number	Scheme	Notes	Marks
3(a)	$(1+ax)^{-3}=1+(-3)(ax)+\frac{(-3)(-4)}{2!}(ax)^2+\frac{(-3)(-4)(-5)}{3!}(ax)^3+...$ Uses the binomial expansion with $n=-3$ and $'x'=ax$. Minimum for M1 is $1+(-3)(ax)$ but can be scored for a correct 3rd or 4th term e.g. $\frac{(-3)(-4)}{2!}(ax)^2$ or $\frac{(-3)(-4)(-5)}{3!}(ax)^3$		M1
	$=1-3ax+6a^2x^2-10a^3x^3+...$ or $=1-3ax+6(ax)^2-10(ax)^3+...$	A1: Three of the four terms correct and simplified A1: All four terms correct and simplified and seen in part (a).	A1A1
			(3)
(b)	$f(x)=\frac{2+3x}{(1+ax)^3}=(2+3x)(1-3ax+6a^2x^2-10a^3x^3)$ Writes $f(x)$ as $(2+3x)(1-3ax+6a^2x^2-10a^3x^3)$ using their expansion from part (a). This may be implied by their expansion. Do not condone 'invisible' brackets around $2+3x$ or part(a) unless their presence is implied by later work and allow to recover in (b) from missing brackets in (a) e.g. ax^2 now becoming a^2x^2		M1
	NB $f(x)=2+(3-6a)x+(12a^2-9a)x^2+(18a^2-20a^3)x^3$		
	$12a^2-9a=3$	Multiplies out and sets their coefficient of x^2 (which comes from exactly 2 terms from their expansion – the two terms may have been combined earlier) = 3.	dM1
	$4a^2-3a-1=(4a+1)(a-1)\Rightarrow a=...$ Correct method of solving a 3TQ. If working is shown see general guidance for correct methods. If no working is shown then you may need to check their values if their quadratic is incorrect.		ddM1
	$a=-\frac{1}{4}$	Cao. Accept equivalent answers but must come from the **correct quadratic** and must be clearly identified.	A1
			(4)
(c)	$18\left(-\frac{1}{4}\right)^2-20\left(-\frac{1}{4}\right)^3$	Subs their $a=-\frac{1}{4}$ (positive or negative) into their coefficient of x^3 (which comes from exactly 2 terms from their expansion)	M1
	Coefficient of x^3 is $\frac{23}{16}$	Cao. Allow $\frac{23}{16}x^3$	A1
			(2)
			9 marks

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4.

$$g(x) = \frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12}, \qquad x > 3, \quad x \in \mathbb{R}$$

(a) Given that

$$\frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} \equiv x^2 + A + \frac{B}{x - 3}$$

find the values of the constants A and B.

(4)

(b) Hence, or otherwise, find the equation of the tangent to the curve with equation $y = g(x)$ at the point where $x = 4$. Give your answer in the form $y = mx + c$, where m and c are constants to be determined.

(5)

Question Number	Scheme	Notes	Marks
4 (a)	$$x^2+x-12\overline{\big)x^4+x^3-7x^2+8x-48}$$ quotient x^2+5; $\underline{x^4+x^3-12x^2}$; $5x^2+8x-48$; $\underline{5x^2+5x-60}$; $3x+12$ M1: Divides $x^4+x^3-7x^2+8x-48$ by x^2+x-12 to get a quadratic quotient and a remainder of the form $\alpha x+\beta$ where α and β are not both zero A1: Correct quotient and remainder		M1A1
	$$\frac{x^4+x^3-7x^2+8x-48}{x^2+x-12}\equiv x^2+5+\frac{3(x+4)\text{ or }3x+12}{(x+4)(x-3)}$$ Writes their answer as $$\frac{x^4+x^3-7x^2+8x-48}{x^2+x-12}\equiv\text{Their Quotient}+\frac{\text{Their Remainder}}{(x+4)(x-3)}$$		M1
	$\equiv x^2+5+\dfrac{3}{(x-3)}$ or states $A=5$, $B=3$		A1
			(4)

	Alternatives to part (a) by dividing by linear factors	
	M1: Divides by $(x-3)$ first then divides by $(x+4)$: $\left(x^4+x^3-7x^2+8x-48\right)\div(x-3): Q_1=x^3+4x^2+5x+23, R_1=21$ $\left(x^3+4x^2+5x+23\right)\div(x+4): Q_2=x^2+5, R_2=3$ For the M1, first division requires Q_1 to be a cubic and R_1 a constant and the second division to give a quadratic Q_2 and constant R_2 A1: Correct quotients and remainders	M1A1
	$\frac{x^4+x^3-7x^2+8x-48}{(x+4)(x-3)} \equiv x^2+5+\frac{3}{x+4}+\frac{21}{(x-3)(x+4)}$ Writes their answer as $Q_2+\frac{R_2}{x+4}+\frac{R_1}{(x-3)(x+4)}$	M1
	$\equiv x^2+5+\frac{3}{(x-3)}$ or states A = 5, B = 3	A1
	M1: Divides by $(x+4)$ first then divides by $(x-3)$: $\left(x^4+x^3-7x^2+8x-48\right)\div(x+4): Q_1=x^3-3x^2+5x-12, R_1=0$ $\left(x^3-3x^2+5x-12\right)\div(x-3): Q_2=x^2+5, R_2=3$ For the M1, first division requires Q_1 to be a cubic and R_1 a constant and the second division to give a quadratic Q_2 and constant R_2 A1: Correct quotients and remainders	M1A1
	$\frac{x^4+x^3-7x^2+8x-48}{(x+4)(x-3)} \equiv x^2+5+\frac{3}{x-3}(+0)$ Writes their answer as $Q_2+\frac{R_2}{x-3}+\frac{R_1}{(x-3)(x+4)}$	M1
	$\equiv x^2+5+\frac{3}{(x-3)}$ or states A = 5, B = 3	A1

	Alternative by comparing coefficients		
	$x^4+x^3-7x^2+8x-48 \equiv (x^2+A)(x^2+x-12)+B(x+4)$ Multiplies through by (x^2+x-12) to obtain correct lhs and one of $(x^2+A)(x^2+x-12)$ or $B(x+4)$ on the rhs If $(x^2+A)(x^2+x-12)$ is expanded, must see both $x^2(x^2+x-12)+A(x^2+x-12)$		M1
	2 correct equations e.g. $x^2 \Rightarrow A-12=-7,\quad x \Rightarrow A+B=8,\quad \text{const} \Rightarrow -12A+4B=-48$		A1
	$A=5,\ B=3$	M1: Solves to obtain one of A or B A1: Both values correct	M1A1
	Alternative by substitution		
	$\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12} \equiv x^2+A+\dfrac{B}{x-3}$ $x=0 \Rightarrow 4=A-\dfrac{B}{3},\ x=1 \Rightarrow \dfrac{45}{10}=1+A-\dfrac{B}{2}$ M1: Substitutes 2 values for x A1: 2 correct equations **Multiplying through before substitution must satisfy the condition for multiplying through in the previous alternative.**		M1A1
	$A=5,\ B=3$	M1: Solves to obtain one of A or B A1: Both values correct	M1A1

(b)	$g'(x)=2x-\frac{3}{(x-3)^2}$	M1: $x^2+A+\frac{B}{x-3}\rightarrow 2x\pm\frac{B}{(x-3)^2}$ A1: $x^2+A+\frac{B}{x-3}\rightarrow 2x-\frac{B}{(x-3)^2}$ Follow through their B or the letter B or a made up B.	M1A1ft
	Special Case: If they write g(x) as $x^2+5+\frac{3x+12}{(x-3)}$ and correctly attempt to differentiate as $2x$ + the quotient rule on $\frac{3x+12}{(x-3)}$ then the M mark is available but **not** the A1ft. It must be the correct quotient rule and the numerator must be a linear expression.		
	$g'(4)=2\times4-\frac{3}{(4-3)^2}(=5)$	Substitutes $x=4$ into their derivative	M1
	Uses $m=g'(4)=(5)$ with $(4,g(4))=(4,24)$ to form eqn of tangent		
	$y-24=5(x-4)$	Correct method of finding an equation of the tangent. The gradient must be $g'(4)$ and the point must be an attempt on $(4,g(4))$	M1
	$y=5x+4$	Cso. This mark may be withheld for an incorrect "A" earlier or any incorrect work leading to a correct gradient.	A1
			(5)
			(9 marks)
	Alternative to part (b) for first 3 marks		
	$g'(x)=\frac{(x^2+x-12)(4x^3+3x^2-14x+8)-(x^4+x^3-7x^2+8x-48)(2x+1)}{(x^2+x-12)^2}$ M1: Correct use of the quotient rule – there must be evidence of the application of $\frac{vu'-uv'}{v^2}$ or this formula quoted and attempted. A1: Correct derivative		M1A1
	$g'(4)=\frac{8\times256-192\times9}{8^2}(=5)$	Substitutes $x=4$ into their derivative	M1

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5. Use integration by parts to find the exact value of

$$\int_0^2 x2^x \, dx$$

Write your answer as a single simplified fraction.

(6)

Question Number	Scheme	Notes	Marks
	Note that 2^x can be replaced by $e^{x\ln 2}$ throughout and allow omission of "dx" throughout		
5	$\int x2^x \mathrm{d}x = x\frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2}\mathrm{d}x$	M1: Integrates by parts the right way around to obtain an expression of the form $ax2^x - \int b2^x \mathrm{d}x$. Allow a = 1 and/or b = 1. A1: $x\frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2}\mathrm{d}x$ (Does not need to be seen all on one line)	M1A1
	$\int x2^x \mathrm{d}x = x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}$	dM1: Completes to obtain an expression of the form $...-k2^x$ A1: $x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}$	dM1A1
	$\left[x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}\right]_0^2 = \left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right) - \left(\frac{0\times 2^0}{\ln 2} - \frac{2^0}{(\ln 2)^2}\right)$ Uses the limits 0 and 2 and subtracts the right way round. F(0) may be implied by e.g. $\frac{1}{(\ln 2)^2}$ But $\left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right) - (0)$ or just $\left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right)$ is ddM0		ddM1
	$\left(= \frac{8}{\ln 2} - \frac{4}{(\ln 2)^2} + \frac{1}{(\ln 2)^2}\right)$		
	$= \frac{8\ln 2 - 3}{(\ln 2)^2}$	Correct simplified fraction. Allow equivalent simplified forms e.g. $\frac{\ln 256 - 3}{(\ln 2)^2}, \frac{\ln 2^8 - 3}{(\ln 2)^2}$ Allow denominator as (ln2)(ln2) and $\ln^2 2$ but not as $\ln 2^2$	A1
			(6 marks)

	Alternative by substitution:		
	$u = 2^x \Rightarrow \int x2^x \mathrm{d}x = \int \frac{\ln u}{\ln 2}.u.\frac{1}{u\ln 2}\mathrm{d}u = \int \frac{\ln u}{(\ln 2)^2}\,\mathrm{d}u$		
	$\int \frac{\ln u}{(\ln 2)^2}\,\mathrm{d}u = \frac{1}{(\ln 2)^2}\left(u\ln u - \int \mathrm{d}u\right)$	M1: Integrates by parts the right way around to obtain an expression of the form $au\ln u - \int b\mathrm{d}u$. Allow a = 1 and/or b = 1. A1: $\frac{1}{(\ln 2)^2}\left(u\ln u - \int \mathrm{d}u\right)$	M1A1
	$\int \frac{\ln u}{(\ln 2)^2}\,\mathrm{d}u = \frac{1}{(\ln 2)^2}(u\ln u - u)$	dM1: Completes to obtain an expression of the form $... - ku$ A1: $\frac{1}{(\ln 2)^2}(u\ln u - u)$	dM1A1
	$\left[\frac{1}{(\ln 2)^2}(u\ln u - u)\right]_1^4 = \frac{1}{(\ln 2)^2}(4\ln 4 - 4) - (\ln 1 - 1)$ Uses the limits 1 and 4 and subtracts the right way round.		M1
	$= \frac{4\ln 4 - 3}{(\ln 2)^2}$	Correct simplified fraction. Allow equivalent simplified forms e.g. $\frac{\ln 256 - 3}{(\ln 2)^2}, \frac{\ln 2^8 - 3}{(\ln 2)^2},$ Allow denominator as (ln2)(ln2) and $\ln^2 2$ but not as $\ln 2^2$	A1

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6. Given that a and b are constants and that $a > b > 0$

(a) on separate diagrams, sketch the graph with equation

(i) $y = |x - a|$

(ii) $y = |x - a| - b$

Show on each sketch the coordinates of each point at which the graph crosses or meets the x-axis and the y-axis.

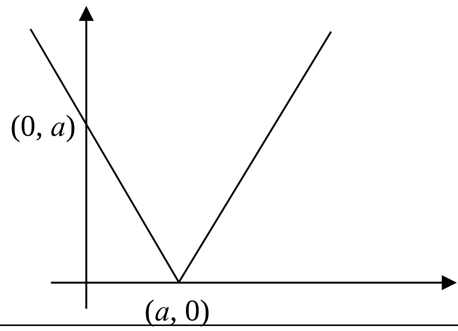
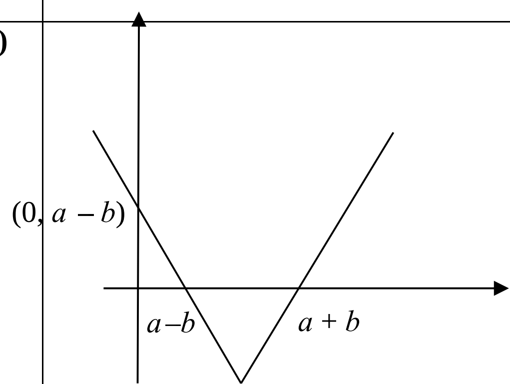
(5)

(b) Hence or otherwise find the complete set of values of x for which

$$|x - a| - b < \frac{1}{2}x$$

giving your answer in terms of a and b.

(4)

Question Number	Scheme	Notes	Marks
6(a)(i)		V shape with vertex on *x*-axis but **not** at the origin.	B1
		Correct V shape with (0, *a*) or just *a* and (*a*, 0) or just *a* marked in the correct places. Left branch must cross or touch the *y*-axis. Allow coordinates the wrong way round if marked in the correct place.	B1
			(2)
(a)(ii)		Their part (i) translated down (by any amount) but clearly not left or right, or the correct shape i.e. a V with the vertex in 4th quadrant.	B1ft
		A *y*-intercept of *a* – *b* on the positive *y*-axis or intercepts of *a* – *b* and *a* + *b* on the positive *x*-axis with *a* + *b* to the right of *a* – *b*	B1
		A fully correct diagram.	B1
			(3)
(b)	$x-a-b=\frac{1}{2}x \Rightarrow x=\ldots$ **or** $-x+a-b=\frac{1}{2}x \Rightarrow x=\ldots$	Solves $x-a-b=\frac{1}{2}x$ **or** solves $-x+a-b=\frac{1}{2}x$ as far as $x=\ldots$ Allow $<$ or $>$ for $=$.	M1
	$x-a-b=\frac{1}{2}x \Rightarrow x=\ldots$ **and** $-x+a-b=\frac{1}{2}x \Rightarrow x=\ldots$	Solves $x-a-b=\frac{1}{2}x$ **and** solves $-x+a-b=\frac{1}{2}x$ as far as $x=\ldots$ Allow $<$ or $>$ for $=$.	M1
	$\frac{2}{3}(a-b)<x<2(a+b)$	ddM1: Chooses inside region. A1: Allow alternatives e.g. $x<2(a+b)$ **and** $x>\frac{2}{3}(a-b)$, $x<2(a+b) \cap x>\frac{2}{3}(a-b)$, $\left(\frac{2}{3}(a-b),\ 2(a+b)\right)$ but not $x<2(a+b)$, $x>\frac{2}{3}(a-b)$	ddM1A1
			(4)
			(9 marks)

	Attempts at squaring in (b)		
	$(x-a)^2 = \left(\frac{1}{2}x+b\right)^2$		
	$(x-a)^2 = \left(\frac{1}{2}x+b\right)^2 \Rightarrow 3x^2 - 4x(2a+b) + 4\left(a^2-b^2\right) = 0$ Squares both sides and obtains 3TQ = 0		M1
	$x = \frac{4(2a+b) \pm 4(a+2b)}{6}$ $\left(= 2(a+b),\ \frac{2}{3}(a-b)\right)$	Attempt to solve 3TQ applying usual rules	M1
	$\frac{2}{3}(a-b) < x < 2(a+b)$	ddM1: Chooses inside region. **Dependent on both previous M marks.** A1: Allow alternatives e.g. $x < 2(a+b)$ **and** $x > \frac{2}{3}(a-b)$, $\left(\frac{2}{3}(a-b),\ 2(a+b)\right)$ but not $x < 2(a+b)$**,** $x > \frac{2}{3}(a-b)$ Expressions must have just one term in *a* and one term in *b*.	ddM1A1

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7.

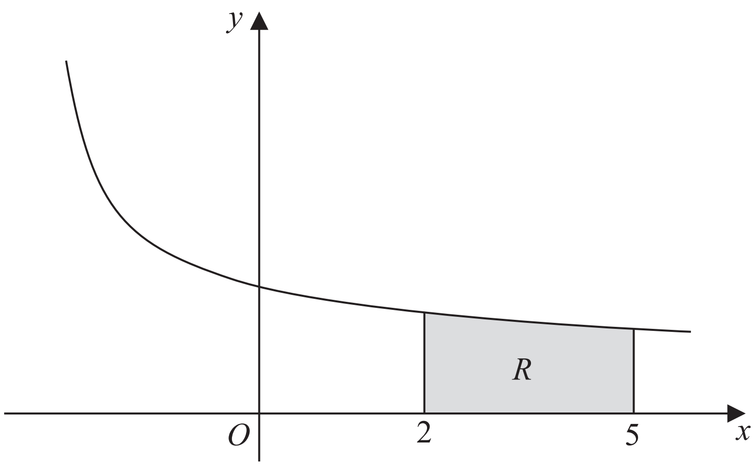

Diagram not drawn to scale

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{1}{\sqrt{2x+5}}$, $\quad x > -2.5$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = 2$ and $x = 5$

(a) Use the trapezium rule with three strips of equal width to find an estimate for the area of R, giving your answer to 3 decimal places.

(4)

(b) Use calculus to find the exact area of R.

(4)

(c) Hence calculate the magnitude of the error of the estimate found in part (a), giving your answer to one significant figure.

(1)

Question Number	Scheme	Notes	Marks
7 (a)	Strip width = 1	May be implied by their trapezium rule.	B1
	Area $\approx \frac{1}{2}\left(\frac{1}{\sqrt{9}}+\frac{1}{\sqrt{15}}+2\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}\right)\right)$ $\approx \frac{1}{2}(0.33...+0.25...+2(0.30...+0.27...))$	M1: Correct structure for the y values. Look for (y at x = 2) + (y at x = 5) + 2(sum of other y values). A1: Correct numerical expression. If decimals are used, look for awrt 1dp initially, however a correct final answer would imply this mark.	M1 A1
	Awrt 0.875		A1
			(4)
	May use separate trapezia:		
	Area $\approx \frac{1}{2}\left(\frac{1}{\sqrt{9}}+\frac{1}{\sqrt{11}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{15}}\right)$		
	B1: Strip width = 1 M1: Correct structure for the y values as above A1: Correct expression as described above A1: Awrt 0.875		
(b)	$\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x=(2x+5)^{\frac{1}{2}}$	M1: $\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x=k(2x+5)^{\frac{1}{2}}$ A1: $\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x=(2x+5)^{\frac{1}{2}}$	M1A1
	$\int_2^5 \frac{1}{\sqrt{2x+5}}\mathrm{d}x=(2(5)+5)^{\frac{1}{2}}-(2(2)+5)^{\frac{1}{2}}$	Substitutes 5 and 2 and subtracts the right way round. May be implied by the correct exact answer but not by a decimal answer **only** e.g. 0.8729… and not by work in decimals e.g. 3.872… -3 unless the substitution of 5 and 2 is explicitly seen.	dM1
	$=\sqrt{15}-\sqrt{9}\ (=\sqrt{15}-3)$	$\sqrt{15}-\sqrt{9}$ or $\sqrt{15}-3$	A1
			(4)

	Alternative to (b) by substitution $u=2x+5$		
	$u=2x+5\Rightarrow\int\frac{1}{\sqrt{2x+5}}\mathrm{d}x=\int\frac{1}{\sqrt{u}}\frac{1}{2}\mathrm{d}u$	M1: $\int\frac{1}{\sqrt{2x+5}}\mathrm{d}x=ku^{\frac{1}{2}}$ A1: $\int\frac{1}{\sqrt{2x+5}}\mathrm{d}x=u^{\frac{1}{2}}$	M1A1
	$\int_2^5\frac{1}{\sqrt{2x+5}}\mathrm{d}x=(15)^{\frac{1}{2}}-(9)^{\frac{1}{2}}$	Substitutes 15 and 9 and subtracts the right way round. May be implied by the correct exact answer but not by a decimal answer **only** e.g. 0.8729… and not by work in decimals e.g. 3.872… -3 unless the substitution of 15 and 9 is explicitly seen.	dM1
	$=\sqrt{15}-\sqrt{9}\left(=\sqrt{15}-3\right)$	$\sqrt{15}-\sqrt{9}$ or $\sqrt{15}-3$	A1
	Alternative to (b) by substitution $u=(2x+5)^{\frac{1}{2}}$		
	$u=(2x+5)^{\frac{1}{2}}\Rightarrow\int\frac{1}{u}.u\,\mathrm{d}u=\int u\,\mathrm{d}u$	M1: $\int\frac{1}{\sqrt{2x+5}}\mathrm{d}x=ku$ A1: $\int\frac{1}{\sqrt{2x+5}}\mathrm{d}x=u$	M1A1
	$\int_2^5\frac{1}{\sqrt{2x+5}}\mathrm{d}x=(15)^{\frac{1}{2}}-(9)^{\frac{1}{2}}$	Substitutes √15 and 3 and subtracts the right way round. May be implied by the correct exact answer but not by a decimal answer **only** e.g. 0.8729… and not by work in decimals e.g. 3.872… -3 unless the substitution of √15 and 3 is explicitly seen.	dM1
	$=\sqrt{15}-\sqrt{9}\left(=\sqrt{15}-3\right)$	$\sqrt{15}-\sqrt{9}$ or $\sqrt{15}-3$	A1
(c)	$\pm(\text{correct}(a)-\text{correct}(b))=\pm0.002$ or $\pm\frac{\text{correct}(a)-\text{correct}(b)}{\text{correct}(b)}\times100=\pm0.2\%$	Finds the magnitude of the error and writes as ±0.002 or $\pm2\times10^{-3}$ or ±0.2% Or finds the percentage error and writes as ±0.2%	B1
			(1)
			(9 marks)

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8. (a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $0 \leqslant \theta < \frac{\pi}{2}$

(i) $\sin 2\theta - \tan\theta = \sqrt{3}\cos 2\theta$

(ii) $\tan(\theta + 1)\cos(2\theta + 2) - \sin(2\theta + 2) = 2$

Give your answers in radians to 3 significant figures, as appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(7)

Question Number	Scheme		Marks
8 (a)	$\sin 2x - \tan x \equiv 2\sin x\cos x - \dfrac{\sin x}{\cos x}$	Uses a **correct** identity for sin2x	M1
	$\equiv \dfrac{2\sin x\cos x\cos x}{\cos x} - \dfrac{\sin x}{\cos x}$	Obtains common denominator. This is **NOT** dependent upon the previous M so accept expressions like, $\sin 2x - \tan x \equiv \sin 2x - \dfrac{\sin x}{\cos x}$ $\equiv \dfrac{\sin 2x\cos x - \sin x}{\cos x}$	M1
	$\equiv \dfrac{2\cos^2 x\sin x - \sin x}{\cos x}$	Correct fraction with just sin x and cos x	A1
	$\equiv \dfrac{(2\cos^2 x - 1)\sin x}{\cos x} \equiv \cos 2x\tan x$*	Uses a correct identity for cos2x and completes correctly with no errors. An error could be for example, mixed variables used or loss of an x along the way.	A1*
			(4)
	Alternative 1 for (a)		
	$\sin 2x - \tan x \equiv 2\sin x\cos x - \dfrac{\sin x}{\cos x}$	Uses a **correct** identity for sin2x	M1
	$\dfrac{\sin x}{\cos x}\left(2\cos^2 x - 1\right)$	M1: Takes out a factor of $\dfrac{\sin x}{\cos x}$ A1: Correct expression	M1A1
	$\equiv \tan x\cos 2x$*	Completes correctly with no errors.	A1*
	Alternative 2 for (a)		
	$2\sin x\cos x - \dfrac{\sin x}{\cos x} \equiv \dfrac{\sin x}{\cos x}\left(\cos^2 x - \sin^2 x\right)$	Uses a **correct** identity for sin2x	M1
	$2\sin x\cos^2 x - \sin x \equiv \sin x\left(\cos^2 x - \sin^2 x\right)$	Multiplies **both sides** by cos x	M1
	$2\cos^2 x - 1 \equiv \left(\cos^2 x - \sin^2 x\right)$	Correct identity	A1
	This is true*	Conclusion provided	A1*
	Alternative 3 for (a)		
	$\tan x\cos 2x \equiv \dfrac{\sin x}{\cos x}\left(2\cos^2 x - 1\right)$	Uses a **correct** identity for cos2x	M1
	$\equiv 2\sin x\cos x - \dfrac{\sin x}{\cos x}$	M1: Multiplies out A1: Correct expression	M1A1
	$\equiv \sin 2x - \tan x$*	A1: Obtains lhs with no errors	A1*

8(b)(i)	$\sin 2\theta - \tan\theta = \sqrt{3}\cos 2\theta \Rightarrow \tan\theta\cos 2\theta = \sqrt{3}\cos 2\theta$		
	$\tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} = (\text{awrt } 1.05)$	M1: $\tan\theta = \pm\sqrt{3} \Rightarrow \theta = ...$ A1: $\theta = \frac{\pi}{3}$ Accept awrt 1.05. Ignore solutions outside the range but withhold the A mark for extra solutions in range.	M1A1
	$\cos 2\theta = 0 \Rightarrow \theta = \frac{\pi}{4}\ (\text{awrt } 0.785)$	M1: $\cos 2\theta = 0 \Rightarrow \theta = ...$ A1: $\theta = \frac{\pi}{4}$ Accept awrt 0.785. Ignore solutions outside the range but withhold the A mark for extra solutions in range.	M1A1
(b)(ii)	$\tan(\theta+1)\cos(2\theta+2) - \sin(2\theta+2) = 2 \Rightarrow \tan(\theta+1) = -2$ M1: $\tan(\theta+1) = \pm 2$		M1
	$\Rightarrow \theta = \arctan(-2) - 1$	Correct order of operations i.e. $\theta = \arctan(\pm 2) - 1$. This may be implied by $\theta = -2.1...$	dM1
	$\Rightarrow \theta = 1.03$	awrt $\theta = 1.03$. Ignore solutions outside the range but withhold the A mark for extra solutions in range.	A1
			(7)
			(11 marks)

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9.

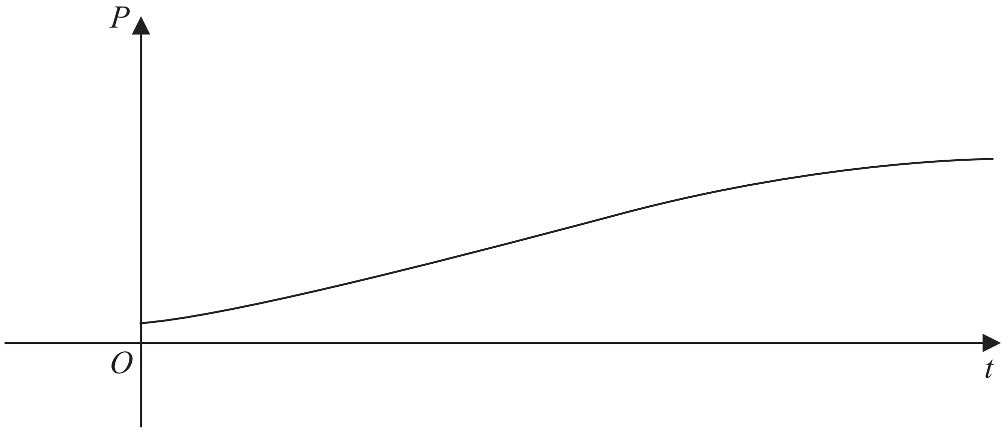

Figure 2

The population of a species of animal is being studied. The population P, at time t years from the start of the study, is assumed to be

$$P = \frac{9000e^{kt}}{3e^{kt} + 7}, \qquad t \geqslant 0$$

where k is a positive constant.

A sketch of the graph of P against t is shown in Figure 2.

Use the given equation to

(a) find the population at the start of the study,
(2)

(b) find the value for the upper limit of the population.
(1)

Given that $P = 2500$ when $t = 4$

(c) calculate the value of k, giving your answer to 3 decimal places.
(5)

Using this value for k,

(d) find, using $\frac{dP}{dt}$, the rate at which the population is increasing when $t = 10$

Give your answer to the nearest integer.
(3)

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Question Number	Scheme		Marks
9.(a)	$t=0 \Rightarrow P=\frac{9000}{3+7}=900$	M1: Sets t = 0, may be implied by $e^0=1$ or may be implied by $\frac{9000}{3+7}$ or by a correct answer of 900.	M1A1
		A1: 900	
			(2)
(b)	$t \to \infty \quad P \to \frac{9000}{3}=3000$	Sight of 3000	B1
			(1)
(c)	$t=4,\ P=2500 \Rightarrow 2500=\frac{9000e^{4k}}{3e^{4k}+7}$	Correct equation with $t=4$ and $P=2500$	B1
	$e^{4k}=\frac{17500}{1500}=(\text{awrt } 11.7 \text{ or } 11.6)$ or $e^{-4k}=\frac{1500}{17500}=(\text{awrt } 0.857)$	M1: Rearranges the equation to make $e^{\pm 4k}$ the subject. They need to multiply by the $3e^{4k}+7$ term, and collect terms in e^{4k} or e^{-4k} reaching $e^{\pm 4k}=C$ where C is a constant.	M1A1
		A1: Achieves intermediate answer of $e^{4k}=\frac{17500}{1500}=(\text{awrt } 11.7 \text{ or } 11.6)$ or $e^{-4k}=\frac{1500}{17500}=(\text{awrt } 0.857)$	
	$k=\frac{1}{4}\ln\left(\frac{35}{3}\right)$ **or** awrt 0.614	**d**M1: Proceeds from $e^{\pm 4k}=C,\ \ C>0$ by correctly taking ln's and then making k the subject of the formula. Award for e.g. $e^{4k}=C \Rightarrow 4k=\ln(C) \Rightarrow k=\frac{\ln(C)}{4}$	**d**M1A1
		A1: cao: Awrt 0.614 or the correct exact answer (or equivalent)	
			(5)

	Alternative correct work in (c):		
	$t=4,\ P=2500 \Rightarrow 2500=\frac{9000e^{4k}}{3e^{4k}+7}$	Correct equation with $t=4$ and $P=2500$	B1
	$7500e^{4k}+17500=9000e^{4k}$		
	$1500e^{4k}=17500$		
	$\ln 1500+\ln e^{4k}=\ln 17500$	M1: Takes ln's correctly	M1A1
		A1: Correct equation	
	$\ln e^{4k}=\ln 17500-\ln 1500$		
	$4k=\ln 17500-\ln 1500$		
	$k=\frac{\ln 17500-\ln 1500}{4}$	Makes k the subject	M1A1
	$k=\frac{1}{4}\ln\left(\frac{35}{3}\right)$ **or** awrt 0.614	cao: Awrt 0.614 or the correct exact answer (or equivalent)	

<table>
<tr><td rowspan="4">(d)</td><td colspan="2">$\frac{dP}{dt}=\frac{(3e^{kt}+7)\times 9000ke^{kt}-9000e^{kt}\times 3ke^{kt}}{(3e^{kt}+7)^2}\left(=\frac{63000ke^{kt}}{(3e^{kt}+7)^2}\right)$
Differentiates using the quotient rule to achieve
$\frac{dP}{dt}=\frac{(3e^{kt}+7)\times Pe^{kt}-9000e^{kt}\times Qe^{kt}}{(3e^{kt}+7)^2}$
or
$\frac{dP}{dt}=9000ke^{kt}\left(3e^{kt}+7\right)^{-1}-9000e^{kt}\left(3e^{kt}+7\right)^{-2}\times 3ke^{kt}$
Differentiates using the product rule to achieve
$\frac{dP}{dt}=Pe^{kt}\left(3e^{kt}+7\right)^{-1}-9000e^{kt}\left(3e^{kt}+7\right)^{-2}\times Qe^{kt}$
or
$\frac{dP}{dt}=63000ke^{-kt}\left(3+7e^{-kt}\right)^{-2}$
Differentiates using the chain rule on $P=9000\left(3+7e^{-kt}\right)^{-1}$ to achieve
$\frac{dP}{dt}=\pm De^{-kt}\left(3+7e^{-kt}\right)^{-2}$
Watch for $e^{kt}\to kte^{kt}$ which is M0</td><td>M1</td></tr>
<tr><td>Sub t =10 and $k=0.614\Rightarrow\frac{dP}{dt}=...$</td><td>Substitutes t = 10 and their k to obtain a value for $\frac{dP}{dt}$. If the value for $\frac{dP}{dt}$ is incorrect then the substitution of t = 10 must be seen explicitly.</td><td>dM1
(A1 on Epen)</td></tr>
<tr><td>$\frac{dP}{dt}=9$</td><td>Awrt 9 (NB $\frac{dP}{dt}=9.1694...$)</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(3)</td></tr>
<tr><td></td><td colspan="2"></td><td>(11 marks)</td></tr>
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10. (a) Given that $-\frac{\pi}{2} < g(x) < \frac{\pi}{2}$, sketch the graph of $y = g(x)$ where

$$g(x) = \arctan x, \qquad x \in \mathbb{R}$$

(2)

(b) Find the exact value of x for which

$$3g(x+1) - \pi = 0$$

(3)

The equation $\arctan x - 4 + \frac{1}{2}x = 0$ has a positive root at $x = \alpha$ radians.

(c) Show that $5 < \alpha < 6$

(2)

The iteration formula

$$x_{n+1} = 8 - 2\arctan x_n$$

can be used to find an approximation for α

(d) Taking $x_0 = 5$, use this formula to find x_1 and x_2, giving each answer to 3 decimal places.

(2)

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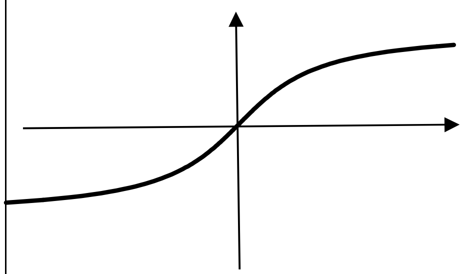
Question Number	Scheme		Marks
10(a)		M1: Curve not a straight line through (0, 0) in quadrants 1 and 3 only. A1: Grad $\to 0$ as $x \to \pm\infty$	M1A1
			(2)
(b)	$3\arctan(x+1)-\pi=0$ $\Rightarrow \arctan(x+1)=\frac{\pi}{3}$	Substitutes $g(x+1)=\arctan(x+1)$ in $3g(x+1)-\pi=0$ and makes $\arctan(x+1)$ the subject. Do not condone missing brackets unless later work implies their presence.	M1
	$\Rightarrow x=\tan\left(\frac{\pi}{3}\right)-1=\sqrt{3}-1$	dM1: Takes tan and makes x the subject e.g. allow $x=\sqrt{3}\pm1$. Note that $\tan\left(\frac{\pi}{3}\right)$ does not need to be evaluated for this mark. May be implied by e.g. $x = 0.732\ldots$ A1: $\sqrt{3}-1$	dM1A1
			(3)
(c)	Sub $x=5$ **and** $x=6$ into $\pm\left(\arctan x-4+\frac{1}{2}x\right) \Rightarrow -0.126..,+0.405...$ **and obtains at least one answer correct to 1sf**		M1
	Both values correct (to one sig fig), change of sign + conclusion Allow equivalent statements e.g. positive, negative therefore root etc. but this mark may be withheld if there are any contradictory statements e.g. therefore root lies between g(5) and g(6)		A1
	If $-\left(\arctan x-4+\frac{1}{2}x\right)$ is used to give $0.126..,-0.405...$, allow both marks if a conclusion is given.		
			(2)
(d)	$x_1=8-2\arctan 5$	Score for $x_1=8-2\arctan 5=\ldots$ This may be implied by awrt 5.3 (radians) or awrt -149 (degrees) for x_1	M1
	$x_1=5.253,\quad x_2=5.235$	$x_1=$ awrt 5.253, $x_2=$ awrt 5.235 Ignore any subsequent iterations and ignore labelling if answers are clearly the second and third terms.	A1
			(2)
			(9 marks)

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11. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 7 \\ 4 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -6 \\ -7 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 4 \\ b \end{pmatrix}$$

where λ and μ are scalar parameters and b is a constant.

Given that l_1 and l_2 meet at the point X,

(a) show that $b = -3$ and find the coordinates of X.

(5)

The point A lies on l_1 and has coordinates (6, 3, 5)

The point B lies on l_2 and has coordinates (14, 9, – 9)

(b) Show that angle $AXB = \arccos\left(-\frac{1}{10}\right)$

(4)

(c) Using the result obtained in part (b), find the exact area of triangle AXB.

Write your answer in the form $p\sqrt{q}$ where p and q are integers to be determined.

(3)

Question Number	Scheme		Marks
11 (a)	$\begin{pmatrix}7\\4\\9\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\4\end{pmatrix}=\begin{pmatrix}-6\\-7\\3\end{pmatrix}+\mu\begin{pmatrix}5\\4\\b\end{pmatrix}\Rightarrow$ $7+1\lambda=-6+5\mu$, $4+1\lambda=-7+4\mu$, $9+4\lambda=3+b\mu$ any two of Writes down any two equations for the coordinates of the point of intersection. There must be an attempt to set the coordinates equal but condone slips.		M1
	Full method to find both λ and μ from equations 1 and 2 and uses these values and equation 3 to find a value for b		**d**M1
	$(1)-(2)\Rightarrow 3=1+\mu\Rightarrow\mu=2$		
	Sub $\mu=2$ into (1) $\Rightarrow 7+1\lambda=-6+10\Rightarrow\lambda=-3$		
	Put values in 3rd equation $9-12=3+2b\Rightarrow b=-3$* Completely correct work including $\lambda=-3,\ \mu=2$ and substitution into **both** sides of the third equation to give $b=-3$		A1
	Position vector of intersection is $\begin{pmatrix}7\\4\\9\end{pmatrix}+-3\begin{pmatrix}1\\1\\4\end{pmatrix}$ or $\begin{pmatrix}-6\\-7\\3\end{pmatrix}+2\begin{pmatrix}5\\4\\-3\end{pmatrix}$ Substitutes their value of λ into l_1 to find the coordinates or position vector of the point of intersection. Alternatively substitutes their value of μ into l_2 to find the coordinates or position vector of the point of intersection. **May be implied by at least 2 correct coordinates for X**		**d**M1
	$X=(4,\ 1,\ -3)$	Correct coordinates or vector. Correct coordinates implies M1A1 Marks for finding the coordinates of X can score anywhere in the question.	A1
			(5)
	(b) Way 1		
(b)	$\pm\overrightarrow{XA}=\pm\begin{pmatrix}2\\2\\8\end{pmatrix},\quad \pm\overrightarrow{XB}=\pm\begin{pmatrix}10\\8\\-6\end{pmatrix}$	Attempts the difference between the coordinates X and A, X and B. This could be implied by the calculation of the lengths AX and BX. Allow slips but must be subtracting.	M1
	$\pm\overrightarrow{XA}.\pm\overrightarrow{XB}=\lvert XA\rvert\lvert XB\rvert\cos\theta\Rightarrow 20+16-48=\sqrt{72}\sqrt{200}\cos\theta$ M1: Attempt the scalar product of $\overrightarrow{XA}$ and $\overrightarrow{XB}$ or $\overrightarrow{AX}$ and $\overrightarrow{BX}$ or $\overrightarrow{XA}$ and $\overrightarrow{BX}$ or $\overrightarrow{AX}$ and $\overrightarrow{XB}$ Allow $\cos\theta=\dfrac{\begin{pmatrix}2\\2\\8\end{pmatrix}\bullet\begin{pmatrix}10\\8\\-6\end{pmatrix}}{\sqrt{72}\sqrt{200}}$ for M1 but not A1 unless the numerator is evaluated A1: A correct un-simplified expression $20+16-48=\sqrt{72}\sqrt{200}\cos\theta$ oe		dM1A1
	$\cos\theta=\dfrac{-12}{\sqrt{72}\times\sqrt{200}}\Rightarrow\theta=\arccos\left(-\dfrac{1}{10}\right)$*	This is a given answer. There must be an intermediate line with $\cos\theta=$.. or $\theta=\ldots$	A1*
			(4)

	(b) Way 2		
(b)	$\mathbf{d}_1 = \begin{pmatrix}1\\1\\4\end{pmatrix}, \quad \mathbf{d}_2 = \begin{pmatrix}5\\4\\-3\end{pmatrix}$	Uses b = -3 and the direction vectors **or multiples of the direction vectors**	M1
	$\mathbf{d}_1.\mathbf{d}_2 = \lvert\mathbf{d}_1\rvert\lvert\mathbf{d}_2\rvert\cos\theta \Rightarrow 5+4-12=\sqrt{18}\sqrt{50}\cos\theta$ M1: Attempt the scalar product of the direction vectors Allow $\cos\theta = \frac{\begin{pmatrix}1\\1\\4\end{pmatrix}\bullet\begin{pmatrix}5\\4\\-3\end{pmatrix}}{\sqrt{18}\sqrt{50}}$ for M1 but not A1 unless the numerator is evaluated A1: A correct un-simplified expression $5+4-12=\sqrt{18}\sqrt{50}\cos\theta$ oe		dM1A1
	$\cos\theta = \frac{-3}{\sqrt{18}\times\sqrt{50}} \Rightarrow \theta = \arccos\left(-\frac{1}{10}\right)$*	This is a given answer. There must be an intermediate line with $\cos\theta$=.. or θ= …	A1*

	(b) Way 3		
(b)	$\pm\overrightarrow{XA} = \pm\begin{pmatrix}2\\2\\8\end{pmatrix}, \quad \pm\overrightarrow{XB} = \pm\begin{pmatrix}10\\8\\-6\end{pmatrix}$	Attempts the difference between the coordinates X and A, X and B. This could be implied by the calculation of the lengths AX and BX. Allow slips but must be subtracting.	M1
	$\lvert AB\rvert^2 = \lvert XA\rvert^2 + \lvert XB\rvert^2 - 2\lvert XA\rvert\lvert XB\rvert\cos\theta \Rightarrow 8^2+6^2+14^2 = 72+200-2\sqrt{72}\sqrt{200}\cos\theta$ M1: Uses $\overrightarrow{AB}$ with a correct attempt at the cosine rule A1: A correct un-simplified expression $8^2+6^2+14^2 = 72+200-2\sqrt{72}\sqrt{200}\cos\theta$ oe		dM1A1
	$\cos\theta = \frac{-24}{2\sqrt{72}\times\sqrt{200}} \Rightarrow \theta = \arccos\left(-\frac{1}{10}\right)$*	This is a given answer. There must be an intermediate line with $\cos\theta$=.. or θ= …	A1*
(c)	$\cos\theta = -\frac{1}{10} \Rightarrow \sin\theta = \frac{\sqrt{99}}{10}$	oe e.g. $\sqrt{\frac{99}{100}}, \frac{3\sqrt{11}}{10}$. May be implied by a correct exact area.	B1
	Area of triangle = $\frac{1}{2}XA\times XB\times\sin\theta \quad A=\frac{1}{2}\times6\sqrt{2}\times10\sqrt{2}\times\frac{3\sqrt{11}}{10}$ Uses Area of triangle = $\frac{1}{2}XA\times XB\times\sin\theta$ This mark can be scored for e.g. $\frac{1}{2}(\text{their } XA)\times(\text{their } XB)\times\sin\left(\cos^{-1}\left(-\frac{1}{10}\right)\right)$ or $\frac{1}{2}(\text{their } XA)\times(\text{their } XB)\times\sin(95.7391...)$ Must be using the angle given by $\cos^{-1}\left(-\frac{1}{10}\right)$		M1
	$A=18\sqrt{11}$ oe	Accept for example $A=9\sqrt{44}, \sqrt{3564}$	A1
	Note that $A=\frac{1}{2}\times6\sqrt{2}\times10\sqrt{2}\times\sin(95.7391...)=18\sqrt{11}$ **scores all 3 marks**		
			(3)
			(12 marks)

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12.

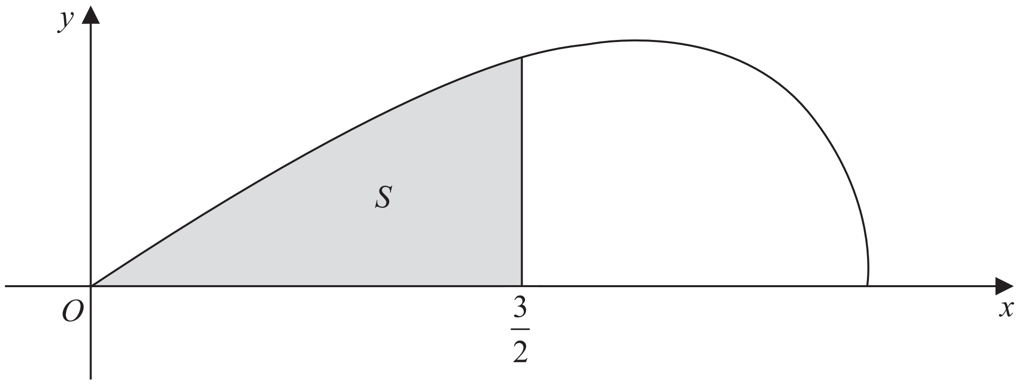

Figure 3

Figure 3 shows a sketch of the curve with parametric equations

$$x = 3\sin t, \quad y = 2\sin 2t, \quad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The finite region S, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = \frac{3}{2}$

The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume of the solid of revolution is given by

$$k\int_0^a \sin^2 t \cos^3 t \, dt$$

where k and a are constants to be given in terms of π.

(5)

(b) Use the substitution $u = \sin t$, or otherwise, to find the exact value of this volume, giving your answer in the form $\frac{p\pi}{q}$ where p and q are integers.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="5">12.(a)</td><td colspan="2">$V = \int y^2 \mathrm{d}x = \int y^2 \frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t = \int (2\sin 2t)^2 3\cos t \,\mathrm{d}t$
M1: Attempts $\int y^2 \mathrm{d}x = \int y^2 \frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t$ where $\frac{\mathrm{d}x}{\mathrm{d}t} = \pm k\cos t$
May be implied by e.g. $\int (2\sin 2t)^2 3\cos t$
A1: $= \int (2\sin 2t)^2 3\cos t\,(\mathrm{d}t)$ (dt can be missing as long as the M is scored)</td><td>M1A1</td></tr>
<tr><td>$= \int (4\sin t\cos t)^2 3\cos t \,\mathrm{d}t$</td><td>Uses sin2$t$ = 2sin$t$cos$t$</td><td>M1</td></tr>
<tr><td>$x = \frac{3}{2} \Rightarrow t = \frac{\pi}{6}$ or $k = 48$</td><td>Correct value for a (must be exact) or a correct value for k</td><td>B1</td></tr>
<tr><td>$V = \int \pi y^2 \mathrm{d}x = 48\pi \int_0^{\frac{\pi}{6}} \sin^2 t \cos^3 t \,\mathrm{d}t$ *</td><td>Achieves printed answer including "dt" (even if lost earlier) with correct limits and 48π in place with no errors. Or achieves the printed answer with the letters a and k and states the correct values of a and k.</td><td>A1*</td></tr>
<tr><td colspan="2"></td><td>(5)</td></tr>
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(b)	$u = \sin t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \cos t$	States $\frac{\mathrm{d}u}{\mathrm{d}t} = \cos t$ or equivalent. May be implied.	B1
	$V = k\int \sin^2 t \cos^3 t\,\mathrm{d}t = k\int u^2 \cos^2 t\,\mathrm{d}u = k\int u^2 (1-\sin^2 t)\,\mathrm{d}u = k\int u^2(1-u^2)\,\mathrm{d}u$ M1: Substitutes **fully** including for dt using $u = \sin t$ and $\cos^2 t = \pm 1 \pm \sin^2 t$ to produce an integral just in terms of u. A1ft: Fully correct integral in terms of u - follow through on incorrect k's and ignore inclusion or omission of π so look for e.g. $k\int u^2(1-u^2)\,\mathrm{d}u$ or equivalent and allow the letter k.		M1A1ft
	$= k\left[\frac{u^3}{3} - \frac{u^5}{5}\right]$	Multiplies out to form a polynomial in u and integrates with $u^n \to u^{n+1}$ for at least one of their powers of u.	M1
	Volume = $48\pi\left[\frac{u^3}{3} - \frac{u^5}{5}\right]_0^{\frac{1}{2}} = \frac{17\pi}{10}$	**d**M1: All methods must have been scored. It is for using the limits 0 and $\frac{1}{2}$ and subtracting or for using the limits 0 and $\frac{\pi}{6}$ if they return to sin t. However, in both cases the substitution of 0 does not need not be seen. A1: $V = \frac{17\pi}{10}$ oe such as $V = \frac{51\pi}{30}$	**d**M1A1
			(6)
	If $\frac{\mathrm{d}u}{\mathrm{d}t} = -\cos t$ is used, maximum B0M1A0M1M1A0 is possible		
			(11 marks)

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13.

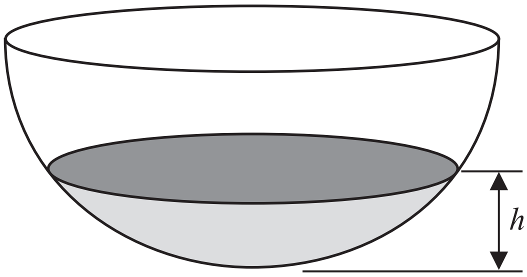

Figure 4

Figure 4 shows a hemispherical bowl containing some water.

At t seconds, the height of the water is h cm and the volume of the water is V cm^3, where

$$V = \frac{1}{3}\pi h^2(30 - h), \qquad 0 < h \leqslant 10$$

The water is leaking from a hole in the bottom of the bowl.

Given that $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -\dfrac{1}{10}V$

(a) show that $\dfrac{\mathrm{d}h}{\mathrm{d}t} = -\dfrac{h(30-h)}{30(20-h)}$

(5)

(b) Write $\dfrac{30(20-h)}{h(30-h)}$ in partial fraction form.

(3)

Given that $h = 10$ when $t = 0$,

(c) use your answers to parts (a) and (b) to find the time taken for the height of the water to fall to 5 cm. Give your answer in seconds to 2 decimal places.

(6)

Question Number	Scheme		Marks
13(a)	$V=\frac{1}{3}\pi h^2(30-h)=10\pi h^2-\frac{1}{3}\pi h^3 \Rightarrow \frac{dV}{dh}=20\pi h-\pi h^2$ or $V=\frac{1}{3}\pi h^2(30-h) \Rightarrow \frac{dV}{dh}=\frac{2}{3}\pi h(30-h)-\frac{1}{3}\pi h^2$		M1A1
	M1: Attempts $\frac{dV}{dh}$ either by multiplying out and differentiating each term to give a derivative of the form $\alpha h-\beta h^2$ or by the product rule to give a derivative of the form $\alpha h(30-h)\pm\beta h^2$. **A1:** Any correct (possibly un-simplified) form for $\frac{dV}{dh}$		
	Uses $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt} \Rightarrow -\frac{1}{10}V=\left(20\pi h-\pi h^2\right)\times\frac{dh}{dt}$		M1
	Uses a **correct** form of the chain rule, e.g. $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ or uses $\frac{dh}{dV}\times\frac{dV}{dt}$ with their $\frac{dV}{dh}$ and $\frac{dV}{dt}=-\frac{1}{10}V$.		
	$\Rightarrow -\frac{1}{10}\times\frac{1}{3}\pi h^2(30-h)=\pi h(20-h)\times\frac{dh}{dt}\left(\Rightarrow\frac{dh}{dt}=...\right)$		M1
	Substitutes $V=\frac{1}{3}\pi h^2(30-h)$ and rearranges to obtain $\frac{dh}{dt}$ in terms of h		
	$\frac{dh}{dt}=-\frac{h(30-h)}{30(20-h)}$ *	This is a given answer. There must have been intermediate lines and correct factorisation and no errors and "$\frac{dh}{dt}=$" must be seen at some point.	A1*
			(5)
(b)	$\frac{30(20-h)}{h(30-h)}\equiv\frac{A}{h}+\frac{B}{30-h}$	Correct form for the partial fractions	B1
	$30(20-h)\equiv A(30-h)+Bh$ $h=30\Rightarrow 30B=-300\Rightarrow B=-10$ and $h=0\Rightarrow 30A=600\Rightarrow A=20$ Attempts to get both constants by a correct method e.g. substituting, comparing coefficients, cover up rule		M1
	$\frac{30(20-h)}{h(30-h)}\equiv\frac{20}{h}-\frac{10}{30-h}$	Correct partial fractions (or states "A" = 20, "B" = -10)	A1
			(3)

(c)	**Way 1**		
	$\frac{dh}{dt} = -\frac{h(30-h)}{30(20-h)} \Rightarrow \int \frac{30(20-h)}{h(30-h)} dh = -1\int dt$ A correct statement which may be implied by subsequent work. Condone the omission of "d*h*" and "d*t*" provided the intention is clear but the minus sign must be present on one side or the other.		B1
	$20\ln h + 10\ln(30-h)$	M1: Integrates their partial fractions to obtain $\pm P\ln h \pm Q\ln(30-h)$ A1: Correct integration for their partial fractions of the form $\frac{A}{h} + \frac{B}{30-h}$ following through their "*A*" and "*B*".	M1A1ft
	$t = 0, h = 10 \Rightarrow c = 20\ln 10 + 10\ln 20$	Substitutes h = 10 and t = 0 to find a value for c. NB c = 76.0…	M1
	$h = 5 \Rightarrow t = 20\ln 10 + 10\ln 20 - 10\ln 25 - 20\ln 5$ Substitutes h = 5 and uses their value of c to find a value for t.		ddM1
	$t = 11.63$ (secs)	Awrt 11.63 only	A1cso
			(6)
			(14 marks)
	(c) Way 2		
	$\frac{dh}{dt} = -\frac{h(30-h)}{30(20-h)} \Rightarrow \int \frac{30(20-h)}{h(30-h)} dh = -1\int dt$ A correct statement which may be implied by subsequent work. Condone the omission of "d*h*" and "d*t*" provided the intention is clear but the minus sign must be present on one side or the other.		B1
	$20\ln h + 10\ln(30-h)$	M1: Integrates their partial fractions to obtain $\pm P\ln h \pm Q\ln(30-h)$ A1: Correct integration for their partial fractions of the form $\frac{A}{h} + \frac{B}{30-h}$ following through their "*A*" and "*B*".	M1A1ft
	$(t =)[20\ln h + 10\ln(30-h)]_5^{10}$ or $(t =)[20\ln h + 10\ln(30-h)]_{10}^{5}$	Attempts the limits 5 and 10 for h. Either statement as shown is sufficient.	M1
	$(t =)[20\ln 10 + 10\ln 20] - [20\ln 5 + 10\ln 25]$	Substitutes h = 5 and h = 10 to find a value for t.	ddM1
	t = 11.63	Awrt 11.63 only	A1cso
			(6)